Trade Policies and Outsourcing for Market Dominance

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Abstract

This paper attempts to analyze the effect of trade policies on the prices of both final and intermediate goods, when outsourcing part of production is a means used to obtain the market dominance in the host country. The market dominance could be granted by the host country to a foreign firm as a reward for its transferring of technology through outsourcing. It is found that when the subsidy by the host country on the production of intermediate inputs is substantial and if the demand curve is concave, a reduction in tariff on the final goods would lead to a rise in the price of both the final and intermediate goods. Consequently, this produces an anti-competitive result.

- **JEL classification**: F12, F13
- **Keywords**: Outsourcing, Cournot duopoly, Market dominance

I. Introduction

As is commonly known, Boeing and Airbus are the only two competing producers of gigantic passenger airplanes in the world. According to Business Week (2004) and The Economist (2004), Boeing decided to let Japanese manufacturing companies such as Mitsubishi, Kawasaki, and Fuji to cooperatively produce a third of the brand-new airplane model 787. The mutual benefits for the two sides in the deal is evident: Boeing wants Japanese airlines to buy the new airplanes and thus increases its market share, while the Japanese want the transferring of
technology to enhance their competence to produce airplanes by themselves. It is no wonder that the project to produce some components for Boeing is also subsidized by the Japanese government. To counteract, Airbus has pinned its hope on China by offering to leave some components of its airplanes produced by the state-owned companies in China.

Taking into account the motive to increase market share in a country as one of reasons for outsourcing and at the same time considering the great subsidy placed on the production of intermediate inputs by the host country, it is important in this paper to deal with two issues below:

1. Under what conditions will there exist an equilibrium of outsourcing for market dominance?

2. With substantial subsidy in place, whether reducing tariffs as a WTO member will also produce the same anti-competitive result as discussed in Chen, Ishikawa, and Yu (2004)?

The issues related to outsourcing and trade have attracted the attention of a number of authors (e.g., Arndt and Kierzkowski (2001), Kleinert (2003), Kohler (2004), Grossman and Helpman (2005), Lin and Chang (2005), Spencer (2005), and Bitzer and Geishecker (2006)). Furthermore, recently developed theoretical models explaining the causes of outsourcing mainly offer two kinds of answers: one is outsourcing for cost reduction (see Hanson (1996) and Zhao(2001)); the other is for strategic consideration (see Chen, Ishikawa, and Yu (2004)). This paper considers another reason for outsourcing. That is outsourcing to obtain market dominance in a certain industry of a country where the government can influence directly or indirectly the market demand in that industry. Market dominance could be achieved in two ways: one is through the subsidy on the production of intermediate goods that is offered by the government in the host country; the other is by granting the outsourcing firm the status of leadership in the final goods market. In this paper, both will be considered. Since Boeing was earlier than Airbus to leave some jet components produced in China, the perspective of outsourcing for market dominance could explain why Boeing’s current market share in China is 60% but only 30% for Airbus.

With the new cause behind outsourcing in mind, it is also interesting to explore

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1The recent trend in 2006 indicates that China has agreed to buy at least 150 A320 airplanes from Airbus. As a further move to increase its market share in China from 30% to 50%, Airbus has decided to have an assembly plant for A320 installed in Tianjin.

2The value is estimated to be 730 million US dollars for the period from 1981 to 2005.
further the possible side effect of subsidy that is used by the host country to nurture its own newly emerging industries.

This paper is organized as follows: An analytical model based on Cournot competition for duopoly is introduced in Section 2. The equilibrium with non-outsourcing is analyzed in section 3. In section 4, the existence of equilibrium with outsourcing is analyzed and the effects of trade policies on the prices of intermediate and final goods are discussed. The conclusions are in section 5.

II. The Model

Assume that firm A and B, which are situated in two different countries, initially engage in Cournot competition in the final goods market of a country C. The market demand for country C is described below:

\[ P(Q) = Q_A + Q_B \]  

where \( P' < 0, P'' \leq 0 \), and \( P''' = 0 \). \( Q \) is the total quantity of final goods supplied by firm A and B, denoted \( Q_A \) and \( Q_B \), respectively. Country C will impose the same tariff \( t_c \) on the final goods imported from firm A and B. Another assumption is that firm A and B produce intermediate inputs at the same marginal cost \( m \).

To simplify analysis, the competition between the two firms for the granting of market dominance in country C will be excluded. Firm B is assumed to have had a similar arrangement regarding outsourcing with another country. It is possible for firm A and country C to come to terms with each other about outsourcing, only if the profit for firm A with the granted market leadership is to be larger than that obtained from Cournot competition with firm B, and if the profit for the local firm in country C with the transferred technology to produce the intermediate inputs is at least nonnegative.

If firm A decides to outsource, the government in country C will give a subsidy \( s \) on the per unit production of intermediate inputs, and country A will impose tariff \( t_A \) on the imports of intermediate inputs from country C. It is also assumed that if the local firm in country C acquires the technology to produce intermediate inputs, it also can produce the intermediate inputs at constant marginal cost \( m \).

Besides the subsidy placed on the production of intermediate inputs that can increase the market share of firm A, the government in country C may also induce domestic customers that are state-owned or state-funded companies to favor firm A
by letting firm A satisfy their demand first. Firm B thus can only meet residual demand. As a result, firm A becomes the market leader while firm B as a follower, and their interaction with each other belongs to Stackelberg competition. The government in country C has the incentive to support firm A, since that indirectly helps domestic intermediate producers to grow.

The structure of this paper’s analysis can be shown in a three-stage game: Given trade policies \( t_c, t_A, \) and \( s \) in the first stage, firm A decides whether or not to let country C own the technology to produce intermediate inputs and buy intermediate inputs from firm C. If firm A agrees to, then in the second stage, country C will grant firm A the status of market leader, while firm C sets the price of intermediate inputs \( w \). In the third stage, given firm A’s decision to outsource, firm A and B engage in Stackelberg competition; otherwise, they engage in Cournot competition.

In the following analysis, the non-outsourcing equilibrium is described first, then the outsourcing equilibrium and finally the effects of changes in trade policies on the prices of both intermediate and final goods are discussed.

### III. Analysis of Non-outsourcing Equilibrium

The equilibrium of Cournot competition arising from the decision by firm A not to outsource is described below:

\[
\max_{Q_A} \pi_A = P(Q) \cdot Q_A - (m + t_C) \cdot Q_A
\]

\[
\max_{Q_B} \pi_B = P(Q) \cdot Q_B - (m + t_C) \cdot Q_B
\]

F.O.C. \( \frac{\partial \pi_A}{\partial Q_A} = 0 \Rightarrow P'(Q) \cdot Q_A + P(Q) - (m + t_C) = 0 \)

\[
\frac{\partial \pi_A}{\partial Q_A} = 0 \Rightarrow P'(Q) \cdot Q_A + P(Q) - (m + t_C) = 0
\]

The optimal profit for each firm in the equilibrium of Cournot competition is denoted \( \pi^*_A \) and \( \pi^*_B \), respectively. The effect of \( t_c \) on the optimal quantity of each firm denoted by \( Q^*_A \) and \( Q^*_B \) is shown below:

\[
\frac{\partial Q^*_A}{\partial t_c} = \frac{P'}{\Delta} < 0
\]
where the condition $Q_d^* = Q_b^*$ is applied and $\Delta = 3P'[P'+P''Q'] > 0$. Based on the result above, it is evident that when two firms engage in Cournot competition in the third country, a reduction in the tariff will lead to a decline in the price of the final goods.

IV. Analysis of Outsourcing Equilibrium

If firm A decides to outsource the production of intermediate goods to country C, it will be granted market dominance in country C, meaning that it is up to firm A to decide first the quantity of final goods it is willing to supply to satisfy the demand in country C. If firm A is granted market dominance, the competition between firm A and firm B is analogous to Stackelberg competition.

Through backward induction, given $Q_d$, from the optimal problem as stated in equation (3) that firm B faces, the supply function of firm B is derived as $Q_b^* = Q_b(Q_d,t_C)$. The influence of a change in $Q_d$ and $t_C$ on $Q_B$ respectively is shown below:

$$\frac{\partial Q_b}{\partial Q_d} = \frac{-(P'+P''Q_b)}{2P'+P''Q_b} < 0 \tag{6}$$

$$\frac{\partial Q_b}{\partial t_C} = \frac{1}{2P'+P''Q_b} < 0 \tag{7}$$

Equation (6) and (7) show that an increase in $Q_d$ will result in a decrease in $Q_b$ and a decrease in $t_C$ will lead to an increase in $Q_B$.

Given the reaction function of $Q_B$ derived above in response to $Q_d$, the optimal problem for firm A is shown below:

$$\max_{Q_d} \pi_A = P(Q_d + Q_b(Q_d,t_C)) \cdot Q_d - (w + t_d + t_C) \cdot Q_d \tag{8}$$

F.O.C. $\frac{\partial \pi_A}{\partial Q_d} = 0 \Rightarrow P' \left[1 + \frac{\partial Q_b}{\partial Q_d}\right] Q_d + P(Q) - (w + t_d + t_C) = 0$

$$\Rightarrow Q_d^* = Q_d(w,t_d,t_C)$$

The effect of a change in $Q_d$ on is shown below:
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where \( D = P^n \cdot \left(1 + \frac{\partial Q_A}{\partial Q'_A}\right)^2 Q_d + 2 P' \cdot \left(1 + \frac{\partial Q_B}{\partial Q'_A}\right) + P' Q_A \frac{\partial^2 Q_B}{\partial Q'_A} \leq 0 \) is required to satisfy the second order condition, and

\[
F = \frac{P'P^n Q_A(P'^2 + P^n Q_B) - (P'^2 + P^n Q_B)(2P' + P^n Q_B)}{(2P' + P^n Q_B)^3}
\]

whose sign is uncertain.

Based on the above result, a decline in both the tariff on intermediate goods and in its price will result in a rise in \( Q_A \). The effect of a change in \( t_C \) on \( Q_A \) is uncertain, since there are two forces offsetting each other: one is an increase in \( Q_A \) directly resulting from a reduction in \( t_C \); the other is a decrease in \( Q_A \) indirectly caused by a rise in \( Q_B \) that is also a result of a reduction in \( t_C \).

If firm A agrees to transfer the technology of producing intermediate goods to country C and buy intermediate goods from firm C, the optimal problem of firm C is described below:

\[
\max_w \pi^*_C = w \cdot Q_A - (m - s) \cdot Q_A \tag{11}
\]

\[
F. O. C. \frac{\partial \pi^*_C}{\partial w} = 0 \Rightarrow Q_A + (w - m + s) \cdot \frac{\partial Q_A}{\partial w} = 0 \\
\Rightarrow Q_A(w, t_A, t_C) + \frac{(w - m + s)}{D} = 0 \\
\Rightarrow w^* = w(w_A, t_C, s)
\]

The derived optimal pricing of firm C is a function of the tariff on both intermediate and final goods and the subsidy offered by the government in country C on the production of intermediate goods. If \( s = 0 \), it is clear that \( w > m \) must hold, since the marginal benefit will be larger than the marginal cost when \( w \leq m \).

According to the pricing function of firm C, the respective effect of a change in \( t_A, t_C \) and \( s \) on the price of intermediate goods is shown below:

\[
\frac{\partial w}{\partial t_A} = -\frac{\left[1 - \frac{(w - m + s)E}{D^2}\right]}{2 - \frac{(w - m + s)E}{D^2}} < 0 \tag{12}
\]
where $E \leq 0^1$ can make equation (12) hold. If $P'' = 0$, then $E=0$. If $P'' < 0$, $P' - P''B_B > 0$ is a sufficient but not necessary condition to ensure $E < 0$. The result of equation (12) shows that if the demand curve is linear or concave enough to make $P' - P''B_B > 0$, a reduction in tariff on the intermediate goods will cause a rise in the price of intermediate goods.

$$\frac{\partial w}{\partial s} = \frac{-1}{2 - \frac{(w - m + s)}{D^2} E} < 0$$

(13)

The result of equation (13) shows that if the demand curve is linear or concave enough, a rise in the subsidy on the production of intermediate goods will result in a decline in the price of intermediate goods.

$$\frac{\partial w}{\partial t_C} = \left[1 - \frac{(w - m + s)}{D} \right] \frac{G}{2 - \frac{(w - m + s)}{D^2} E} < 0$$

(14)

where $G \geq 0$ and $E \leq 0$ can make equation (14) hold. If $P'' = 0$, then $G=E=0$ and it is for sure that a reduction in the tariff on the final goods will cause a rise in the price of intermediate goods. If $P'' < 0$, then $P' - P''Q_B > 0$ ensures $E < 0$, while $F < 0$ and $\frac{\partial D}{\partial Q_B} < 0$ make $G > 0^4$.

According to the optimal problems in three stages, where the optimal profit for three individual firms is denoted $\pi_A^s, \pi_B^s$, and $\pi_C$ respectively, and based on the characteristics of results from comparative statics, the existence of outsourcing equilibrium is shown below:

**Proposition 1.** Regardless of the level of $t_C$, as $s$ is getting larger and $t_A$ is getting smaller, the more likely the existence of the outsourcing equilibrium, which means $\pi_A^s \geq \pi_A$ and $\pi_C \geq 0$.

The proof is in Appendix B.

The result of proposition 1 is within the expectation of intuition from our model. No matter how large the tariff on the final goods is, firm A and B compete equally

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1 Please see Appendix A.

4 Please see Appendix A.
if there is no outsourcing. To gain the upper hand over firm B, there exist incentives for firm A to let the intermediate inputs be produced in country C, if the subsidy offered by country C is substantial and the trade in intermediate goods is more liberalized.

In addition to the advantage of low labor cost in the labor-abundant country, proposition 1 also points to another advantage of outsourcing the production of intermediate inputs abroad. That is to secure a great amount of subsidy and a large order for final goods from the host country eager to develop its own high tech industry. According to The Economist (2006), Boeing has overtaken Airbus by receiving record orders for its 787 Dreamliner and 777 wide-bodied jets. One reason for Boeing’s better performance could be attributed to its open-mindedness toward outsourcing abroad while due to job consideration in France and Germany, that is still a no-go area for Airbus.

With the existence of outsourcing equilibrium from proposition 1, the effect of a change in tariff on intermediate inputs on its price is as stated in equation (12), and its effect on the price of final goods is shown below:

\[
\frac{\partial Q^*}{\partial t_A} = \left(1 + \frac{\partial Q^*}{\partial Q_A} \right) \cdot \left( \frac{\partial Q^*}{\partial w} \frac{\partial w}{\partial t_A} + \frac{\partial Q^*}{\partial t_A} \right) < 0
\]

\[
\Rightarrow \frac{\partial P}{\partial t_A} > 0
\]

Based on equation (12) and (16), we have the following proposition:

**Proposition 2.** If demand curve is linear or concave enough to make \(P' - P''Q_B > 0\), a reduction in the tariff on intermediate inputs will increase the price of intermediate inputs but decrease the price of final goods.

The intuition for the result of proposition 2 is that as the price of intermediate goods remains the same, a decline in tariff results in a lower unit cost that will increase the purchase of intermediate goods and the quantity of final goods supplied by firm A. A rise in the demand of intermediate goods leads to the increase in its price, while the increased supply of final goods by firm A is only partially offset by the reduced output of firm B, resulting in a rise in market supply of final goods and thus decreasing the price of final goods.
The effect of a change in subsidy on the production of intermediate inputs on the price of final goods is shown below:

\[
\frac{\partial Q^*}{\partial s} = \left(1 + \frac{\partial Q_B}{\partial P^*} \cdot \frac{\partial Q_A^*}{\partial w} \cdot \frac{\partial w}{\partial s}\right) > 0
\]

\[\Rightarrow \frac{\partial P^*}{\partial s} < 0\]  \hspace{1cm} (17)

From equation (13) and (17), we have the following proposition:

**Proposition 3.** If demand curve is linear or concave enough to make \(P' - P''Q_B > 0\), a rise in subsidy on the production of intermediate inputs will decrease the prices of both intermediate and final goods.

The intuition for the result of proposition 3 can also be easily understood. A rise in subsidy on the production of intermediate goods will reduce the cost to produce them. The reduction in the producing cost is passed on to firm A in the form of lower price charged for intermediate inputs, inducing firm A to increase its demand for intermediate inputs and its supply for final goods. The increase in supply is also at the expense of firm B but leads to increase in market supply anyway, that decreases the price of final goods.

The effect of a change in tariff on final goods on its price is shown below:

\[
\frac{\partial Q^*}{\partial t_C} = \left(1 + \frac{\partial Q_B}{\partial P^*} \cdot \frac{\partial Q_A^*}{\partial w} \cdot \frac{\partial w}{\partial t_C}\right) + \frac{\partial Q_B}{\partial t_C} > 0
\]

According to equation (18), except for \(\frac{\partial Q_B^*}{\partial w} < 0\), it is uncertain whether the term

\[
\left(\frac{\partial Q_A^*}{\partial w} \cdot \frac{\partial Q_A^*}{\partial t_C}\right)
\]

is larger than, equal to, or less than zero:

\[
\frac{\partial Q_A^*}{\partial w} \cdot \frac{\partial Q_A^*}{\partial t_C} = \frac{1}{D} \left[1 - \frac{(w-m+s)G}{2 - (w-m+s)E}\right] \frac{F}{D} < 0
\]

\[\Rightarrow 1 - \frac{(w-m+s)}{D} \frac{\partial D}{\partial Q_B} \frac{\partial Q_B^*}{\partial t_C} >= -2F\]  \hspace{1cm} (19)

Based on equation (19), there are following two situations to be discussed: the
first situation is \( 1 - \frac{(w-m+s)}{D} \frac{\partial D}{\partial Q^*_B} \frac{\partial Q^*_B}{\partial A_c} \leq -2F \). As a result, \( \frac{\partial Q^*_B}{\partial A_c} < 0 \) and \( \frac{\partial P^*}{\partial A_c} < 0 \).

One example of this situation is \( P'' = 0 \), that means the demand curve is linear.

**Proposition 4.** If the demand curve is linear, a reduction in the tariff on final goods will increase the price of intermediate inputs but decrease the price of final goods.

The second situation is \( 1 - \frac{(w-m+s)}{D} \frac{\partial D}{\partial Q^*_B} > -2F \) and \( \frac{\partial Q^*_B}{\partial A_c} \rightarrow 0 \). As a result, \( \frac{\partial Q^*_B}{\partial A_c} > 0 \) and \( \frac{\partial P^*}{\partial A_c} < 0 \). One example of this situation is an increasing \( s \) that leads to \( Q^*_B \) approaching zero, while \( P' \rightarrow -\infty \), that also results in \( \frac{\partial Q^*_B}{\partial A_c} \rightarrow 0 \). Based on that, we have the following lemma and proposition to be proved:

**Lemma 1.** As \( P' \rightarrow -\infty, F < 0 \) and \( E < 0 \) also hold.

The proof is in Appendix B.

**Proposition 5.** If the demand curve is concave and \( s \) is large enough to make \( Q^*_B \) approach zero, while \( P' \rightarrow -\infty \), then a reduction in the tariff on final goods will increase the prices of both intermediate and final goods.

The proof is in Appendix B.

As far as the effect of a change in tariff on the final goods is concerned, according to proposition 4, the intuition for the result in case of a linear demand curve is that if the tariff on final goods decreases, its effect on the supply of final goods by firm A is zero, since a direct rise in firm A’s supply of final goods is completely offset by the indirect decline due to an increase in the price of intermediate goods caused by a reduced tariff on final goods. Therefore, the increase in the supply of final goods is only from firm B.

According to proposition 5, the intuition for the result in case of a concave demand curve is as follows: an increasing subsidy on the production of intermediate goods will not only decrease the quantity sold by the non-outsourcing firm but also increase the total quantity sold in the host country. Since the demand curve is concave, an increase in the total quantity sold will also lead to a decline in the elasticity of demand for the final goods. When the demand for final goods is
very inelastic, if there is a reduction in tariff on the final goods, the extent of an increase in the quantity directly affected as a result will be smaller than that of a decline in the quantity caused by a rise in the price of intermediated goods that is also a result of a decline in the tariff on the final goods.

Besides the distortion stated in proposition 5, the subsidy offered by the host country could be a controversial issue under WTO framework, since it is indirectly beneficial for the outsourcing firm to have a lower price for intermediate inputs and thus have an upper hand over its rivals. That could be a reason to explain why in 2005, Airbus insisted on the inclusion of Japanese government’s subsidy on the production of wings for the 787 into negotiation between Airbus and Boeing to avert taking the subsidy-related complaint to WTO.

V. Conclusions

Besides using political and diplomatic interventions at their disposal, Airbus and Boeing have decided to outsource the production of some intermediate goods to a few countries. The purpose is for establishing a symbiotic relationship and thus obtaining market dominance in those countries. Growing at a steep rate, China is a country with the potential to become the largest airline market only second to America. Therefore, it is predictable that besides cost consideration, for the sake of dominating the market, the two giant commercial jet makers will let China be the main supplier of components for making airplanes.

Based on this real-life example, this paper offers another reason for outsourcing abroad. The reason is for market dominance in the host country. Through liberalization of trade in intermediate goods and substantial subsidy by the host country on the production of intermediate goods, there would be equilibrium of outsourcing for market dominance. According to proposition 1 and 5, the amount of subsidy on the intermediate goods is not only crucial in proving the existence of outsourcing equilibrium, but is also one of the conditions for lowering tariff on final goods to produce an anti-competitive result, that means a rise in the price of the final goods. Thus, the policy implication of the results derived in this paper is that cost reduction aside, a firm trying to outsource abroad might be expecting to gain an advantage over its rivals through the subsidy on the intermediate goods offered by the government in host country. The subsidy is offered in return possibly for improving the employment rate or for obtaining the vital technology. But with a substantial subsidy in place, there exists a possibility that a lower tariff on the final
goods might lead to a rise in its price.

As in CIY (2004), though the analysis of the optimal trade policies is not the focus of the study, what the optimal trade policies will be with the existence of outsourcing for market dominance is an issue worth pursuing. Further study on the besides competition between firms for market dominance is also needed, beyond this one on competition in the final goods market.

Appendix A

Footnote 3:

\[ E = 3P'' \left[ 1 + \frac{\partial Q_B}{\partial Q_A} \right] + 3 \left( P'' \left[ 1 + \frac{\partial Q_B}{\partial Q_A} \right] Q_A + P' \right) \frac{\partial Q_B}{\partial Q_A} + P'Q_A \frac{\partial^2 Q_B}{\partial Q_A^2} \]

where

\[ \frac{\partial Q_B}{\partial Q_A} = \frac{(P'')^2 \left( 1 + \frac{\partial Q_B}{\partial Q_A} \right) Q_B - P'P'' \frac{\partial Q_b}{\partial Q_A}}{[2P'' + P'' Q_B]^2} > 0 \]

and

\[ \frac{\partial^2 Q_B}{\partial Q_A^2} = \frac{\frac{(P'')^2 Q_B - P'P'' \frac{\partial Q_B}{\partial Q_A}}{(2P'' + P'' Q_B)^2}}{\frac{2 \left[ (P'')^2 \left( 1 + \frac{\partial Q_B}{\partial Q_A} \right) Q_B - P'P'' \frac{\partial Q_B}{\partial Q_A} \right] (P'P'' - (P'')^2 Q_B)}{(2P'' + P'' Q_B)^4} } > 0 \]

The condition \( P' - P'' Q_B > 0 \) makes \( \frac{\partial Q_B}{\partial Q_A} > 0 \), that sufficiently ensures \( E < 0 \).

Footnote 4:

\[ G = \frac{\partial D}{\partial c} = \frac{\partial D}{\partial Q_A} \cdot \frac{\partial Q_A}{\partial c} + \frac{\partial D}{\partial Q_B} \cdot \frac{\partial Q_B}{\partial c} = -\frac{EF}{D} + \frac{\partial D}{\partial Q_B} \cdot \frac{\partial Q_B}{\partial c} \]

\( G > 0 \), if \( E \leq 0, F < 0 \) and \( \frac{\partial D}{\partial Q_B} < 0 \), where:

\[ \frac{\partial D}{\partial Q_B} = 2P'' \left[ 1 + \frac{\partial Q_B}{\partial Q_A} \right] Q_A \frac{\partial Q_B}{\partial Q_A} + 2P'' \left[ 1 + \frac{\partial Q_B}{\partial Q_A} \right] + 2P' \frac{\partial^2 Q_B}{\partial Q_A^2} \frac{\partial Q_B}{\partial Q_B} \]

\[ + P'' Q_A \frac{\partial^2 Q_B}{\partial Q_A^2} + P' Q_A \frac{\partial^2 Q_B}{\partial Q_A^2} \]

If \( P' - P'' Q_B > 0 \), then:
\[ \frac{\partial^2 Q_B}{\partial Q_A \partial Q_B} = \frac{P^\prime(P^\prime Q_B - P^\prime)}{(2P^\prime + P^\prime Q_B)^2} > 0 \]

But the sign of \( \frac{\partial^2 Q_B}{\partial Q_A \partial Q_B} \) is uncertain, where:

\[
\frac{\partial^2 Q_B}{\partial Q_A \partial Q_B} = \frac{[P^\prime (P^\prime Q_B - P^\prime) + (P^\prime)^2 - P^\prime P^\prime (\partial Q_B^* / \partial Q_A)]}{(2P^\prime + P^\prime Q_B)^3} - 2 \left( \frac{1}{P^\prime} \frac{\partial Q_B}{\partial Q_A} \right) \left( P^\prime - P^\prime Q_B - P^\prime P^\prime \frac{\partial Q_B}{\partial Q_A} \right) \left( \frac{3P^\prime}{3} \right)
\]

**Appendix B**

**Proof of Proposition 1:**

According to equation (12) and (13), let \( t_A \to 0 \) and \( s \) be large enough to make \( w + t_A \) approach \( m \). By traditional duopoly theory, the profit for a market leader in Stackelberg competition must be larger than that from Cournot competition, when facing nearly the same cost as \( m + t_C \), that is to say:

\[ \pi_A(m + t_A + t_C) \to \pi_A(m + t_C) > \pi_A(m + t_C) \]

Since \( t_A \to 0 \) and \( w + t_A \to m, w \to m \). As a result, the profit for firm C, \( \pi_C = (w + s - m) \cdot Q_A^* \), becomes \( \pi_C = s \cdot Q_A^* \) that must be larger than or equal to zero.

**Proof of Lemma 1:**

As \( P' \to -\infty, F \to -\frac{1}{2} \), Based on equation (6) and footnote 3 in Appendix A, as \( P' \to -\infty, \frac{\partial Q_B}{\partial Q_A} \to -\frac{1}{2}, \frac{\partial^2 Q_B}{\partial Q_A^2} \to 0 \), and \( \frac{\partial^2 Q_B}{\partial Q_A^3} \to 0 \). Consequently, \( E \to \frac{3}{4} P'' < 0 \).

**Proof of Proposition 5:**

According to equation (13), (9), and (6), a rising \( s \) can make \( Q_B^* \) approach zero,
such that: 
\[
\frac{\partial^2 Q_B}{\partial Q_A^2} \rightarrow -\frac{1}{2}, \quad \frac{\partial^2 Q_B}{\partial Q_A^2} \rightarrow -\frac{P'P''}{4(P')^2} > 0, \quad \frac{\partial^2 Q_B}{\partial Q_A \partial Q_B} \rightarrow -\frac{P'P''}{4(P')^2} < 0,
\]
\[
\frac{\partial^2 Q_B}{\partial Q_B^2} \rightarrow \frac{5}{32} \left(\frac{P''}{(P')^2}\right)^2 < 0, \quad \text{and} \quad \frac{\partial^2 D}{\partial Q_B^2} \rightarrow \frac{P''(4P' - P''Q_A)}{8P'} - \frac{5}{32} Q_A \left(\frac{P''}{P'}\right)^2 > 0.
\]
As a consequence, 
\[
D - (w - m + s) \frac{\partial D}{\partial Q_B} \frac{\partial Q_B}{\partial \alpha_c} \rightarrow \frac{3P''Q_A + P'}{8} - (w - m + s) \left[ \frac{P''(4P' - P''Q_A)}{16(P')^2} - \frac{5}{64} \left(\frac{P''}{P'}\right)^2 Q_A \right] < -2F \cdot D \quad \text{(B1)}
\]
\[
-2F \cdot D \rightarrow \left(\frac{P''}{4P'} + 1\right) \left(\frac{3}{8} P''Q_A + P'\right) \quad \text{B(2)}
\]

Based on equation (B1) and (B2),
\[
D - (w - m + s) \frac{\partial D}{\partial Q_B} \frac{\partial Q_B}{\partial \alpha_c} \rightarrow -2F \cdot D
\]
\[
\Rightarrow - (w - m + s) \left[ \frac{P''(4P' - P''Q_A)}{16(P')^2} - \frac{5}{64} \left(\frac{P''}{P'}\right)^2 Q_A \right] > \frac{P''}{4P'} \left(\frac{3}{8} P''Q_A + P'\right)
\]

As \(P' \to \infty\), the left-hand side of the above inequality approaches zero, while the right-hand side becomes 
\[
- \frac{P''}{4P'} > 0,
\]
that means:
\[
D - (w - m + s) \frac{\partial D}{\partial Q_B} \frac{\partial Q_B}{\partial \alpha_c} \rightarrow -2F \cdot D
\]
\[
\Rightarrow 1 - \frac{(w - m + s)}{D} \frac{\partial D}{\partial Q_B} \frac{\partial Q_B}{\partial \alpha_c} \rightarrow -2F
\]
\[
\Rightarrow \frac{\partial Q_B^*}{\partial w} \frac{\partial \alpha_c}{\partial \alpha_c} + \frac{\partial Q_B^*}{\partial \alpha_c} > 0
\]

Equation (7) shows that the condition \(P' \to \infty\) also makes \(\frac{\partial Q_B^*}{\partial \alpha_c} \to 0\), with which
\[
\frac{\partial Q_B^*}{\partial w} \frac{\partial \alpha_c}{\partial \alpha_c} + \frac{\partial Q_B^*}{\partial \alpha_c} > 0 \quad \text{ensures} \quad \frac{\partial P}{\partial \alpha_c} < 0.
\]
According to lemma 1, \(F < 0\) and \(E < 0\) when
$P' \rightarrow -\infty$. From $\frac{\partial D}{\partial Q_b} \rightarrow P'' < 0$, $\frac{\partial w}{\partial c} < 0$ is inferred.

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References


