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Strategic Export Subsidies under a Budget Constraint: Ad Valorem versus Specific

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Abstract

This note shows that in the Brander-Spencer model of export subsidy, if there is no cost of financing subsidies, either a specific export subsidy or an ad valorem export subsidy can be used to achieve the same maximum welfare level. If, however, there is a binding budget constraint, a specific subsidy dominates an ad valorem subsidy. (JEL Classification: F13)

I. Introduction

Strategic export subsidies has received considerable attention in the trade literature. Such attention comes out of the observed use of export subsidies by many governments, and out of the difficulties of explaining these

policies in the neoclassical framework. An interesting model is provided by Brander and Spencer [1985] which explains how in the presence of international oligopolistic competition an export subsidy can raise domestic welfare. Their work is followed by other's work which investigates further the effects of export subsidies.[1]

Nearly all the work in the literature that examines strategic export subsidies is restricted to using subsidies of specific rates. In the real world, however, export subsidies are usually specified in ad valorem terms. This thus raises the practical question of whether specific and ad valorem export subsidies are equivalent.

It is well known from the public finance literature that with competitive markets specific taxes/subsidies and ad valorem taxes/subsidies are equivalent as long as they drive the same wedge between consumer prices and producer prices; see, for example, Stiglitz [1988: 425-426]. However, the equivalence between specific and ad valorem taxes breaks down if the markets are characterized by monopolists Stiglitz [1988: 434-436] . The absence of such equivalence has also been shown for some open economies; for example, tariffs on a foreign monopolist Helpman and Krugman [1988]: 65-66; and Wong [1995]: 527-528 , or tariffs in the presence of a trade war Lockwood and Wong [1996] .

This paper examines the equivalence (or non-equivalence), from a domestic country's point of view, between specific and ad valorem export subsidies in the Brander-Spencer framework. This paper is different from those mentioned earlier in that export subsidies are considered and that there are two oligopolistic firms.

In the present analysis, two cases are distinguished and compared. In this first one, financing an export subsidy is costless in the sense that the government can easily raise enough of revenue to finance any export subsidy it

beyond a revenue level that is smaller than what the Brander-Spencer model suggests. We show that in the former case, specific and ad valorem export subsidies are equivalent strategically in the sense that either of them can be used to achieve the same maximum welfare level. In the latter case, in which government budget constraint is binding because of the cost of raising enough of revenue, we show that a specific export subsidy dominates an ad valorem export subsidy.

The rest of this note is organized as follows. Section II sets up the model and provides comparative static analysis. Section III compares the welfare effects of ad valorem and specific subsidies when the domestic government may or may not be subject to a budget constraint. Concluding remarks are furnished in section IV.

II. The Model

Following a practice in the literature, we assume that there are two countries which are labeled 'domestic' and 'foreign,' each of which has one firm producing an identical commodity and exporting it to a third market. The government in the domestic country, which has perfect information, sets a credible specific or ad valorem subsidy on export prior to the quantity decisions made by the domestic and the foreign firms.

The inverse demand for the commodity in the third country is specified as

$$p = p(q + q^*), \tag{1}$$

where p is the price, q is the output (export) of the domestic firm and q^* is the output (export) of the foreign firm. The domestic government has the options of imposing a subsidy of specific rate of s or a subsidy of ad valorem rate of r on the export of the domestic firm. The foreign government, how-

where $R(\cdot)$ and $R^*(\cdot)$ are the revenue functions and $C(q)$ and $C^*(q^*)$ are the cost functions of the domestic and the foreign firms, respectively. Note that for convenience we include both types of export subsidies in condition (2a), but it should be understood that in the present analysis, we consider cases in which at most one of these two types of subsidies is imposed, meaning that in (2a) at least r or s is zero.

The two firms are assumed to play Cournot in an output game. The first-order conditions for profit maximization (2) are given as follows:

$$\pi_q = (1 + r)R_q - C_q + s = 0 \tag{3a}$$

$$\pi^*_{q^*} = R^*_{q^*} - C_{q^*} = 0. \tag{3b}$$

Equation (3) can be used to solve for $q = q(r, s)$ and $q^* = q^*(r, s)$. Substituting these equations into the revenue functions yields $R = R(q(r, s), q^*(r, s))$ and $R^* = R^*(q(r, s), q^*(r, s))$.

Before computing the welfare effects of r and s, we need to derive the comparative static effects of r and s on q and q^*. These effects can be derived by totally differentiating (3):

$$q_r = \frac{dq}{dr} = -\frac{R_q \overset{*}{q^* q^*}}{D} > 0 \tag{4a}$$

$$q^*_r = \frac{dq^*}{dr} = -\frac{R_q \overset{*}{q^* q}}{D} < 0 \tag{4b}$$

$$q_s = \frac{dq}{ds} = -\frac{\overset{*}{q^* q^*}}{D} > 0 \tag{4c}$$

$$q^*_s = \frac{dq^*}{dr} = -\frac{\overset{*}{q^* q}}{D} < 0, \tag{4d}$$

III. Welfare Comparison

As pointed out by Brander and Spencer, the domestic government has a unilateral incentive to offer an export subsidy to the domestic firm: The optimal subsidy moves the industry equilibrium to what would, in the absence of a subsidy, be the Stackelberg leader-follower position in output space with the domestic firm as a leader. In their model, Brander and Spencer assume the use of a specific export subsidy by the domestic government. We now want to show that the domestic government can impose instead an ad valorem export subsidy to reach the same maximum welfare level:[2]

Proposition 1: If in the Brander-Spencer model unlimited revenues can be raised costlessly, the domestic government can use either a specific or an ad valorem export subsidy to improve the national welfare to the same maximum level. In other words, in the absence of any government budget constraints specific and ad valorem export subsidies are equivalent.

Proof: Let us define the welfare function under an ad valorem subsidy program as

$$W^a = \pi - rR.$$

The first-order condition for welfare maximization is:

$$\frac{dW^a}{dr} = (1+r)R_{q^*}q_r^* - r(R_q q_r + R_{q^*}q_r^*) = 0.$$

The above equation after arrangement can be written as

$$r = \frac{R_{q^*}}{R_q}\frac{q_r^*}{q_r} = \frac{R_{q^*}}{R_q}\frac{dq^*}{dq},$$

where dq^*/dq is the slope of the reaction function of the foreign firm. Substi-

$$_q = R_q + R_{q^*}\frac{dq^*}{dq} - C_q = 0,$$

which is exactly the Stackelberg leader-follower equilibrium with the domestic firm as a leader. From the work of Brander and Spencer, we know that an optimal specific export subsidy also shifts the industry to the same equilibrium. Because under either export subsidy program, national welfare is equal to the profit of the firm less subsidy receipt, which is what the firm would get in the absence of a subsidy if it can attain a Stackelberg leader position, the two export subsidy programs are equivalent.

In proposition 1, the equivalence between the two export subsidy programs depends on the fact that there is no cost in raising revenue to finance the subsidy. We now turn to the case in which raising such a revenue is costly. To simplify the following analysis, we assume that the government can costlessly raise revenue up to an amount T and it is prohibitively costly to raise any more revenue. The amount T is binding in the sense that it is less what the government needs for an optimal export subsidy, either in a specific or an ad valorem term. In what follows we shall compare the welfare effects of these two types of export subsidies when the government is subject to the present budget constraint.

Because the budget constrain is binding, the following conditions hold if subsidies are in ad valorem or specific term, respectively:

$$T = rR(q(r, 0), q^*(r, 0)) \tag{5}$$

$$T = sq(0, s). \tag{6}$$

Equations (5) and (6) show that for any given level of T, there is a corresponding specific or ad valorem subsidy rate to satisfy the budget constraint. From (5) and (6), the change in r or s for a change in T can be

Note that $R_q = p'q + p > 0$, $R_{q^*} = p'q < 0$, and that by (5) $q_r > 0$ and $q_r^* < 0$. The expression in (7) is positive, implying that an increase in T results in an increase in the ad valorem subsidy rate. This result is also true in the case of specific subsidy.

Recall that under an ad valorem export subsidy regime (so that $s = 0$), the welfare function of the domestic economy, W^a, is defined as $W^a = \pi - rR$. The marginal effect of T on W^a is

$$\frac{dW^a}{dT} = \left(\frac{}{q} q_r + \frac{}{q^*} q_r^* - \frac{}{r}\right) r_T - r\frac{dR}{dr} r_T - r_T R.$$

Substituting $\partial\pi/\partial q = 0$, $\partial\pi/\partial q^* = (1 + r)p'q$, $\partial\pi/\partial r = R$, and (7) into the above equation and rearranging terms yield:

$$\frac{dW^a}{dT} = (R_{q^*} q_r^* - rR_q q_r) r_T. \tag{9}$$

Similarly, setting $r = 0$, we can define the welfare function of the economy under a specific export subsidy regime as $W^s = \pi - sq$ and derive the marginal effect of T on W as:

$$\frac{dW^s}{dT} = (R_{q^*} q_s^* - sq_s) s_T. \tag{10}$$

Note that expressions (9) and (10) are derived under different regimes, and in general they are not comparable. We can, however, restrict to the case in which $T = 0$ and then compare the welfare effects of small specific and ad valorem export subsidies. Substitute (7) and (8) into (9) and (10) and then set r in (9) and s in (10) to zero to obtain:

$$\left.\frac{dW^a}{} \right| = \frac{R_{q^*}}{q^* q} R_q > 0 \tag{11}$$

trade welfare level. But their welfare increases are not the same. This can be proved by subtracting (12) from (11) to yield:

$$\left.\frac{dW^a}{dT}\right|_{T=0} - \left.\frac{dW^s}{dT}\right|_{T=0} = \frac{R_{q^*}\,\pi^*_{q^*q}}{D}\left(\frac{R_q}{R} - \frac{1}{q}\right) < 0, \qquad (13)$$

which is negative because $R_{q^*} < 0$, $\pi^*_{q^*q} < 0$ and

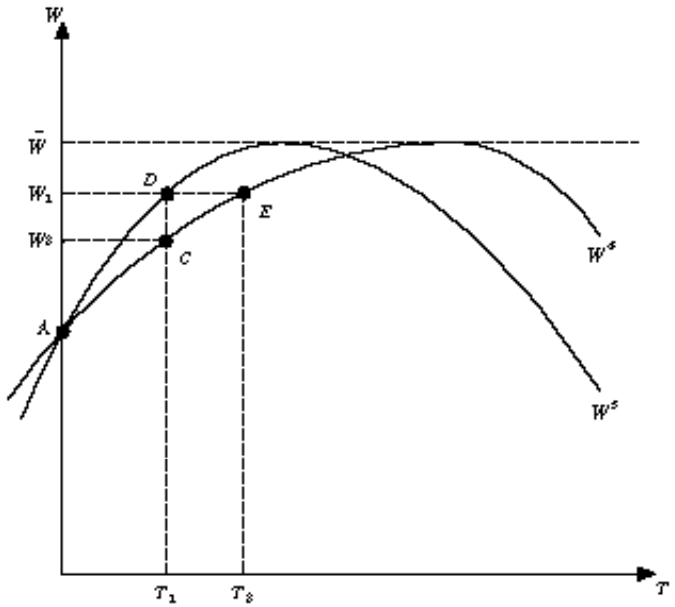
$$\frac{R_q}{R} - \frac{1}{q} = \frac{p'}{p} < 0.$$

Expression (13) shows that, at the free-trade equilibrium, the effect of a subsidy on welfare increase is higher under specific subsidy than under ad valorem subsidy. This result is shown in Figure 1 where W^a and W^s are the welfare curves under ad valorem and specific subsidies and OA is the welfare level under free trade (*i.e.*, $T = 0$). By (13), curve W^s is steeper than curve W^a at the vertical intercept. In other words, a larger subsidy expenditure is required to reach a slightly higher welfare level under an ad valorem export subsidy than under a specific subsidy.

Proposition 2: If the government budget is constrained, the expenditure of raising the domestic welfare (output) to a certain level is higher under an ad valorem subsidy regime than under a specific subsidy regime.

Proof: The proof consists of three steps. In the first step, we shall demonstrate that there is a one-to-one correspondence between the domestic output q and the domestic welfare. Then, we shall show that the relation between either a specific or an ad valorem subsidy rate and q is monotonic. Finally, it will be shown that the subsidy expenditure to raise the domestic output to a certain level below the optimal one is smaller if the subsidy rate is in a specific than in an ad valorem term.

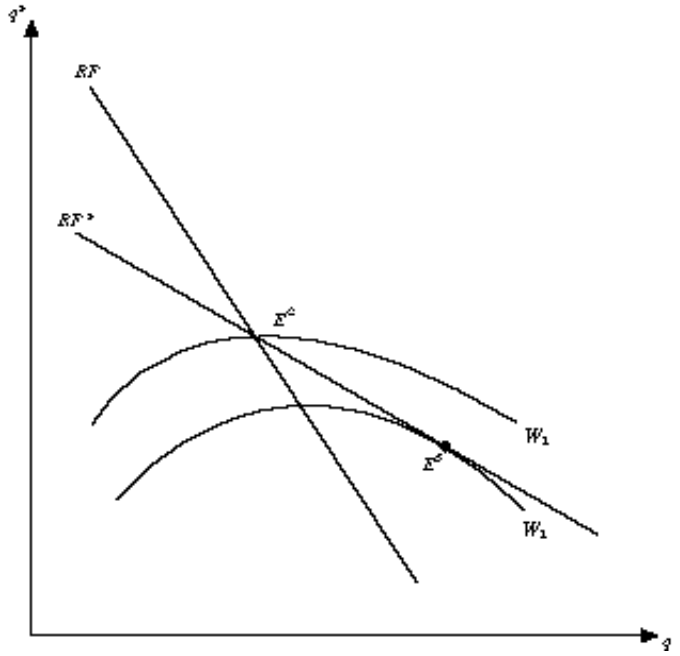
Figure 1
Welfare Effects of Specific and Ad Valorem Export Subsidies



the absence of government subsidy; it is represented by W_1. As export subsidy goes up, the reaction function of the domestic firm moves outward and the equilibrium move downward along the foreign reaction function to a higher welfare level such as W_2. Hence, if there is no subsidy budget constraint, the optimal subsidy equilibrium is at E^s. With a budget constraint the equilibrium should locate somewhere between E^c and E^s, depending on the size of the subsidy budget: The domestic welfare becomes higher as the equilibrium moves closer toward E^s. Clearly, before point E^s is reached there is a one-to-one correspondence between the domestic output and welfare.

Now let us compare the subsidy expenditures of promoting domestic

Figure 2
One-to-One Correspondence between Quantity and Welfare



Alternatively, we can analyze a specific subsidy by setting $r = 0$ in (3a) and rearranging terms to give:

$$s = C_q - R_q. \tag{15}$$

Note that because of the same output level under consideration, the marginal costs (or marginal revenues) in conditions (14) and (15) have the same value. These two conditions can be used to determine the difference in the government expenditures required to induce the domestic export to a level q under the two regimes, which is defined as

which means that to raise the output of the domestic firm to a level higher than the free-trade level below the optimal one, the subsidy expenditure under an ad valorem subsidy regime is higher than that under a specific subsidy regime. Because of the one-to-one correspondence between national welfare and domestic output, we immediately get the proposition.

The intuition for this result is as follows. Let us choose a positive specific export subsidy, s, less than the optimal level. At the Nash equilibrium, the output of the domestic firm is at the level at which marginal cost is equal to the effective marginal revenue, which is the sum of the marginal revenue from the market plus the per unit subsidy, *i.e.*, $R_q + s$. Suppose now that the government switches to an ad valorem export subsidy, with a rate equal to $r = s/p$, where p is the prevailing price. If the domestic firm does not alter its output, it can receive the same subsidy and the same profit. However, because $p > R_q$, marginal cost is greater than the new effective marginal revenue, $R_q + rR_q$, inducing the domestic firm to lower its output. If the government wants to encourage the firm to produce the same output as before, it has to raise the ad valorem subsidy rate. In other words, the government has to spend more to induce the domestic firm to produce a certain output level under an ad valorem subsidy regime than under a specific subsidy regime.

To illustrate the above analysis, we refer again to Figure 1. Suppose that when subject to a binding budget constraint the government can spend up to a subsidy expenditure of T_1 under a specific export subsidy program, the maximum welfare reached is W_1. To reach the sarne welfare level under an ad valorem scheme, more expenditure is needed, *i.e.*, T_2 as shown in the diagram, or if the government can spend only T_1, a lower welfare level W_2 is reached under this subsidy scheme.

The above result is now stated in the following proposition:

Proposition 3: When subject to a binding government budget constraint, the

ing government budget constraint. If there is no subsidy expenditure constraint, a specific export subsidy and an ad valorem export subsidy are equivalent in the sense that the same maximum welfare level can be reached: They both lead the industry equilibrium to what would, in the absence of a subsidy, the Stackelberg leader-follower position in output space with the domestic firm as a leader. However, a specific export subsidy would dominate an ad valorem export subsidy if the government is subject to a binding expenditure constraint.

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