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Multi-Factor Gegenbauer Processes and European Inflation Rates

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Abstract

In this paper we specify a multi-factor long-memory process that enables us to estimate the fractional differencing parameters at each frequency separately, and adopt this framework to model quarterly prices in three European countries (France, Italy and the UK). The empirical results suggest that inflation in France and Italy is nonstationary. However, while for the former country this applies both to the zero and the seasonal frequencies, in the case of Italy the nonstationarity comes exclusively from the long-run or zero frequency. In the UK, inflation seems to be stationary with a component of long memory at both the zero and the semi-annual frequencies, especially at the former. In all cases, we find evidence of mean reversion, implying that the effects of exogenous shocks on inflation are transitory and activist policies are not required in response to them. This process is slower in the case of France and Italy compared with the UK.

- **JEL Classification:** C22, O40
- **Keywords:** Fractional Integration, Long Memory, Inflation

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I. Introduction

Modelling inflation is still a controversial issue, and a consensus is yet to be reached on whether it is a stationary I(0) or a nonstationary I(1) variable. More recently, it has been suggested that it might be an I(d) process, with d lying between 0 and 1. Such processes exhibit long memory, with a pole or singularity in the spectrum at the long-run or zero frequency. This idea was introduced in the mid-1960s by Granger (1966) and Adelman (1965), who pointed out that for most aggregate economic time series the spectral density has a typical shape with a spike as the frequency approaches zero, and differencing the data frequently leads to overdifferencing at the zero frequency. However, it might be that the series is characterised by more than one pole or singularity in the spectrum, but, given the strong influence of the component at the zero frequency, these poles may not be apparent in the periodogram or in any other estimate of the spectral density function. This is particularly relevant if seasonal components are present in the data, as, for instance, in the case of quarterly or monthly data. There exist procedures for estimating the fractional differencing parameter in this context using seasonal long-memory models; however, many of them have the limitation of imposing the same degree of integration at all frequencies in the spectrum. For instance, this is the case for the Dickey-Hasza-Fuller (DHF, 1984) tests for seasonal unit roots in a non-fractional context. Hylleberg, Engle, Granger and Yoo (HEGY, 1990) present a procedure that allows one to consider unit roots at zero and each of the seasonal frequencies separately, although it focus exclusively on the I(0)/I(1) cases, not permitting fractional degrees of differentiation. Alternatively, some authors have used seasonally adjusted data, though the use of seasonally adjusted procedures have been strongly criticized by many authors.[1]

By contrast, in the present study we specify a multi-factor long-memory process that enables us to estimate the fractional differencing parameters at each frequency separately, and adopt this framework to model quarterly prices in three European countries (France, Italy and the UK). The flexibility of this approach is important in order to obtain more precise specifications for the inflation processes than those based on the standard I(0)/I(1) cases, which is relevant, for instance, for forecasting purposes. Note that our approach include the standard non-seasonal and seasonal I(0) and I(1) models as particular cases of interest being more general in the sense

[1]Subscribers to this view include Ghysels (1988), Barsky and Miron (1989), Chatterjee and Ravikumar (1992), Hansen and Sargent (1993), and Braun and Evans (1995) among others.

that it permits us to consider fractional degrees of integration not only at zero but also at the seasonal frequencies. By allowing fractional degrees of differentiation we permit a greater degree of flexibility in the dynamic specification of the series, not achieved with the classical methods based on ARMA/ARIMA representations. Thus, for example, in standard I(0)/I(1) contexts mean reversion occurs only in the I(0) (ARMA) case, while shocks are permanent in the unit root, ARIMA or I(1) models. In the context of fractional integration mean reversion occurs as long as d is smaller than 1 and lower is the value of d, faster is the convergence process. Our results indicate that the three series of prices (i.e. for France, Italy and the UK) can be well described in terms of fractional processes.

The outline of the paper is as follows: in Section II we briefly review the literature on modelling inflation, focusing particularly on long-memory models. In Section III we describe the statistical framework employed in the paper. Section IV presents the empirical results. Section V analyses the forecasting performance of the model, whilst Section VI summarises the main findings and offers some concluding remarks.

II. Literature Review

The empirical literature on inflation is vast. In the last couple of decades attention has often focused on European countries, as inflation convergence is one of the requirements for EMU membership specified in the Maastricht treaty. Several studies have carried out standard unit root tests (see, e.g., Barsky, 1987; Rose, 1988; McDonald and Murphy, 1989; Kirchgasser and Wolters, 1993), with mixed results depending on the span of data. Long-memory models have then become increasingly popular (see, e.g., Chung and Baillie, 1993, and Franses and Ooms, 1997). Much of the evidence supports the view that inflation is fractionally integrated with a differencing parameter that is significantly different from zero or unity. For instance, using US monthly data, Backus and Zin (1993) found a fractional degree of integration. They argued that aggregation across agents with heterogeneous beliefs results in long memory in the inflation process. Hassler (1993) and Delgado and Robinson (1994) provided strong evidence of long memory in the Swiss and Spanish inflation rates respectively. Baillie, Chung and Tieslau (1996) examined monthly post-World War II CPI inflation in ten countries, and found evidence of long memory with mean-reverting behaviour in all countries except Japan. Similar results were reported by Hassler and Wolters (1995) and

Baum, Barkoulas and Caglayan (1999).[2] The above papers, however, focus exclusively on the case where the singularity in the spectrum takes place at the zero frequency. In other words, seasonality is not taken into account or if so it is considered simply a short run (stationary) process.

Other studies have also attempted to take into account possible persistence and heteroscedasticity in inflation rates (see, e.g., Chambers, 1998, Bollerslev and Wright, 2000, Ferrara and Guegan, 2001a). In particular, a general model, which extends the FIGARCH, FIEGARCH and FARMA-GARCH specifications of Baillie *et al*. (1996), Bollerslev and Mikkelsen (1996) and Ling and Li (1997), has been proposed by Guégan (2000), whose framework combines long-memory behaviour with quasi-periodic behaviour in the conditional variance of the series. Guégan (2003) considers a k-factor Gegenbauer process with heteroscedastic errors. In the present paper the procedures adopted for estimating and testing the fractional differencing parameters are robust to conditionally heteroscedastic errors, and the main focus is on estimating the degree of persistence of time series through the orders of integration at specific frequencies in the spectrum.

III. The statistical framework

In this paper we consider various time series long-memory models. The first is the standard I(d) model given by

$$\begin{aligned} (1-L)^d x_t &= u_t, \quad t = 1, 2, \ldots, \\ x_t &= 0, \quad t \leq 0, \end{aligned} \tag{2}$$

where x_t is an observable time series, or alternatively the errors in a regression model of the form:

$$y_t = \beta' z_t + x_t, \qquad t = 1, 2, \ldots , \tag{3}$$

where z_t are deterministic regressors such as an intercept ($z_t = 1$) or an intercept with a linear time trend ($z_t = (1, t)^T$); L is the lag-operator ($Ly_t = y_{t-1}$), u_t is assumed to be I(0),[3] and, given the quarterly frequency of the data analysed here, to follow a

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[2]Other papers dealing with long memory in inflation rates in the context of structural breaks are Bos, Franses and Ooms (1999, 2001), Gadea, Sabate and Serrano (2004), Franses, Hyung and Penn (2006) and Gil-Alana (2008), and forecasting issues are examined in Franses and Ooms (1997) and Barkoulas and Baum (2006).

[3]An I(0) process is defined as a covariance stationary process with spectral density function that is positive and finite at any frequency. It thus includes the stationary ARMA models.

seasonal autoregressive (AR) model of the form:

$$\phi(L^{s})u_{t} = \varepsilon_{t}, \quad t = 1, 2, \dots , \tag{4}$$

where s indicates the number of time periods per year, and ϵ_t is a white noise process. This specification implies that the long-run dynamic behaviour of the series is captured by the fractional differencing parameter d only, while the seasonal structure is a purely short-run phenomenon described by the (seasonal) AR coefficients.

A second model considered in this study is the seasonal $I(d_s)$ process described by

$$(1 - L^{S})^{d_s} x_t = u_t, \qquad t = 1, 2, \dots, \tag{5}$$

where d_s once more can take a fractional value. Porter-Hudak (1990) applied a seasonally fractionally integrated model of this type to quarterly US monetary aggregates, and concluded that a fractional ARMA model was more appropriate than the usual ARIMA specification for these series. Other recent empirical papers on seasonal fractional integration using a model such as (5) for macroeconomic series are those of Gil-Alana and Robinson (2001) and Gil-Alana (2002).[4] A limitation of this approach is that it imposes the same degree of integration at the zero and seasonal frequencies. For example, in the quarterly case, i.e., s = 4, the polynomial $(1–L^4)^{ds}$ can de decomposed into $(1–L)^{d_s}(1+L)^{d_s}$ $(1+L^2)^{d_s}$ imposing the same degree of integration d_s at all frequencies: the zero, the semi-annual (π), and the annual ($\pi/2$ and $3\pi/2$) frequencies respectively.

The model in (5) can be generalised using multi-factor Gegenbauer processes. Specifically, we can consider processes of the form:

$$\prod_{u=1}^{k} (1 - 2cos_r^{(u)} L + L^2)^{d_u} x_t = u_t, \qquad t = 1, 2, \dots, \tag{6}$$

where k is a finite integer indicating the maximum number of cyclical (seasonal) structures. First we focus on the case of a single structure, i.e., k = 1,

$$(1 - 2cosw_r L + L^2)^{d_c} x_t = u_t, \qquad t = 1, 2, \dots, \tag{7}$$

where w_r and d_c are real values, and u_t is I(0). For practical purposes we define w_r

[4]Other papers dealing with seasonal fractional integration in the context of forecasting are Ray (1993) and Sutcliffe (1994).

$= 2\pi r/T$, with $r = T/j$, and thus j will indicate the number of time periods per cycle, while r refers to the frequency with a pole or singularity in the spectrum of x_t. Note that if $r = 0$ (or $j = 1$), the fractional polynomial in (7) becomes $(1 - 2L + L^2)^{d_c} = (1 - L)^{2d_c}$, which is the polynomial associated to the common case of fractional integration at the long-run or zero frequency in (2). The type of processes described in (7) was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Chung (1996a,b) and Dalla and Hidalgo (2005) among others. Gray *et al*. (1989) showed that, defining $\mu = \cos w_r$, the polynomial in (7) can be expressed for all $d_c \neq 0$ as

$$(1 - 2\mu L + L^2)^{-d_c} = \sum_{j=0}^{\infty} C_{j,d_c}(\mu) L^j,$$

where

$$C_{j,d_c}(\mu) = \sum_{k=0}^{j} \frac{(-1)^k (d_c)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d_c)_j = \frac{\Gamma(d_c + j)}{\Gamma(d_c)}$$

and $\Gamma(x)$ is the Gamma function. Alternatively, we can use the recursive formula, $C_{0,d_c}(\mu) = 1$, $C_{1,d_c}(\mu) = 2\mu d_c$, and

$$C_{j,d_c}(\mu) = 2\mu\left(\frac{d_c - 1}{j} + 1\right) C_{j-1,d_c}(\mu) - \left(2\frac{d_c - 1}{j} + 1\right) C_{j-2,d_c}(\mu), \quad j = 2, 3, \ldots,$$

(see, for instance, Magnus *et al*., 1966, or Rainville, 1960, for further details on Gegenbauer polynomials). These authors also showed that x_t in (7) is stationary if $d_c < 0.5$ for $|\mu = \cos w_r| < 1$ and if $d_c < 0.25$ for $|\mu| = 1$.[5]

If there is more than one cyclical structure, then the appropriate specification is the multi-factor Gegenbauer process described in (6), with $w_r^{(u)} = 2\pi r^{(u)}/T; r^{(u)} = T/j^{(u)}$, where $j^{(u)}$ indicates the number of time periods per cycle corresponding to the u^{th} cyclical structure. Empirical studies based on multiple cyclical structures (also named *k*-factor Gegenbauer processes) include Ferrara and Guégan (2001b), Sadek and Khotanzad (2004) and Gil-Alana (2007). In the case of quarterly time series data, we can generalise (5) by considering a model like (6) with k = 3, and

$$(1 - 2\cos w_r^{(1)} L + L^2)^{d_1} (1 - 2\cos w_r^{(2)} L + L^2)^{d_2} (1 - 2\cos w_r^{(3)} L + L^2)^{d_3} x_t = u_t, \qquad (8)$$

with $w_r^{(1)} = 0$ or 2π ($r^{(1)} = 0$, T), $w_r^{(2)} = \pi$ ($r^{(2)} = T/2$), $w_r^{(3)} = \pi/2$ ($r^{(3)} = T/4$),

[5]Estimation methods using this approach can be found in Arteche and Robinson (2000) and Arteche (2002).

implying that (8) can be written as

$$(1-L)^{2d_1}(1+L)^{2d_2}(1+L^2)^{d_3}x_t = u_t, \tag{9}$$

or, alternatively,

$$(1-L)^{d_1^*}(1+L)^{d_2^*}(1+L^2)^{d_3^*}x_t = u_t, \tag{10}$$

with $d_1^* = 2d_1$; $d_2^* = 2d_2 = 2d_2$, and $d_3^* = d_3$, which is a seasonal (quarterly) long-memory model with different orders of integration at each of the frequencies. Thus, d_1^* is the order of integration related to the long-run or zero frequency; d_2^* refers to the semi-annual frequency (π), and d_3^* is related to the annual frequency ($\pi/2$ and 3 $\pi/2$) of a 2π cycle. In some of the models, we employ the Whittle estimator in the frequency domain. Note that the Whittle function is an approximation to the likelihood function so the estimates should be close to the maximum likelihood estimates. Along with this method, we also employ different versions of Robinson's (1994) tests. Robinson (1994) proposed a very general specification that includes all the above models as particular cases of interest, with integer or fractional degrees of differentiation.[6] The functional form of the different versions of the test statistics employed here are described in the Appendix.

IV. Empirical Evidence of Long Memory in European Prices

The series analysed in this section is the logarithm of quarterly CPI in Italy, France and the UK. The sample period goes from 1957Q3 to 2007Q3 in all three countries, and the data source is the IMF's International Financial Statistics published on the IMF webpage. We use quarterly data because this is the frequency usually employed in macroeconomic studies. Moreover, the use the monthly data would make it necessary to estimate seven different orders of integration noting that (1-L^{12}) can be decomposed into (1-L)(1+L)(1+L^2)(1+L+L_2)(1-L+L^2)(1+$(3)^{1/2}$L+L^2) (1-$(3)^{1/2}$L+L^2).

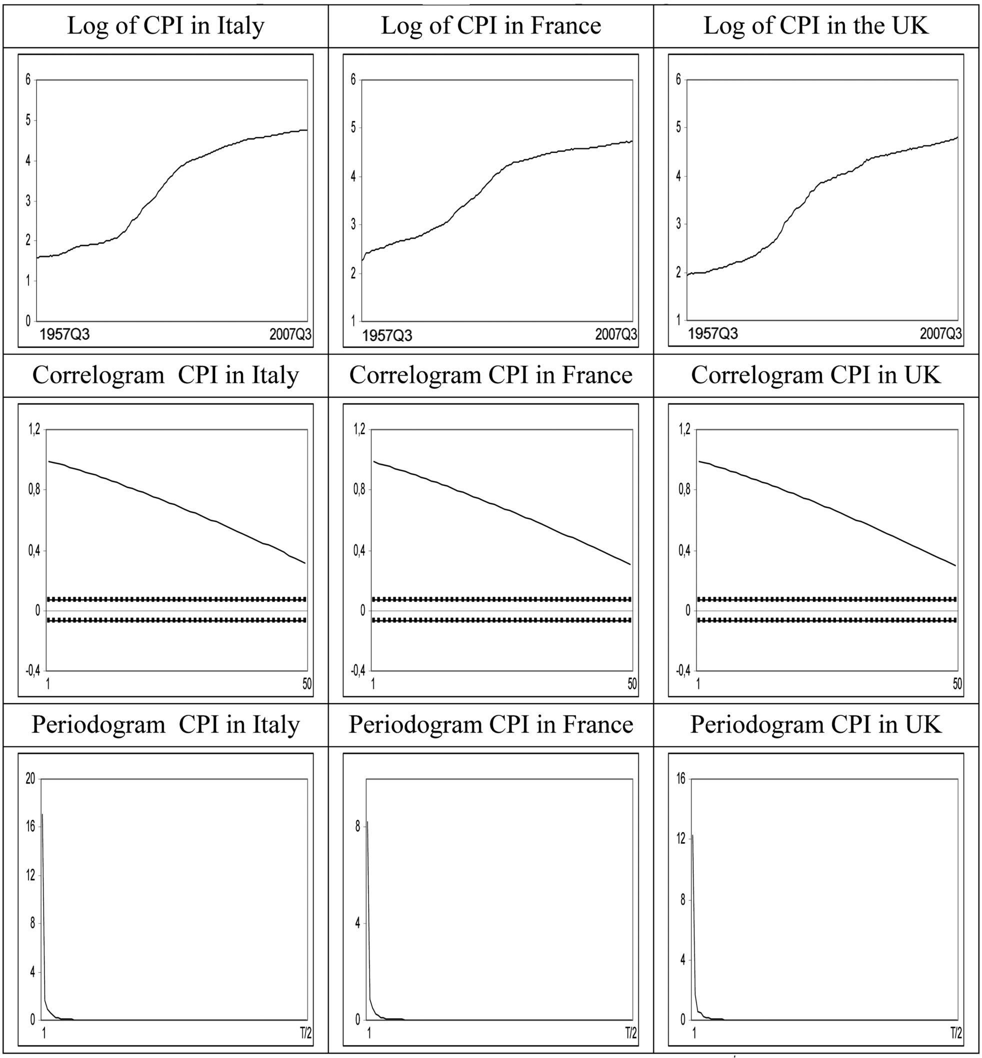
Figure 1 displays plots of the three time series, as well as the first 50 sample

[6]Another advantage of Robinson's (1994) method is that its limit distribution is standard (normal) independently of the inclusion or not of deterministic terms in (3) and the way of modelling the I(0) disturbances.

autocorrelation values, and the periodograms computed at the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, j = 1, 2, …, T/2.

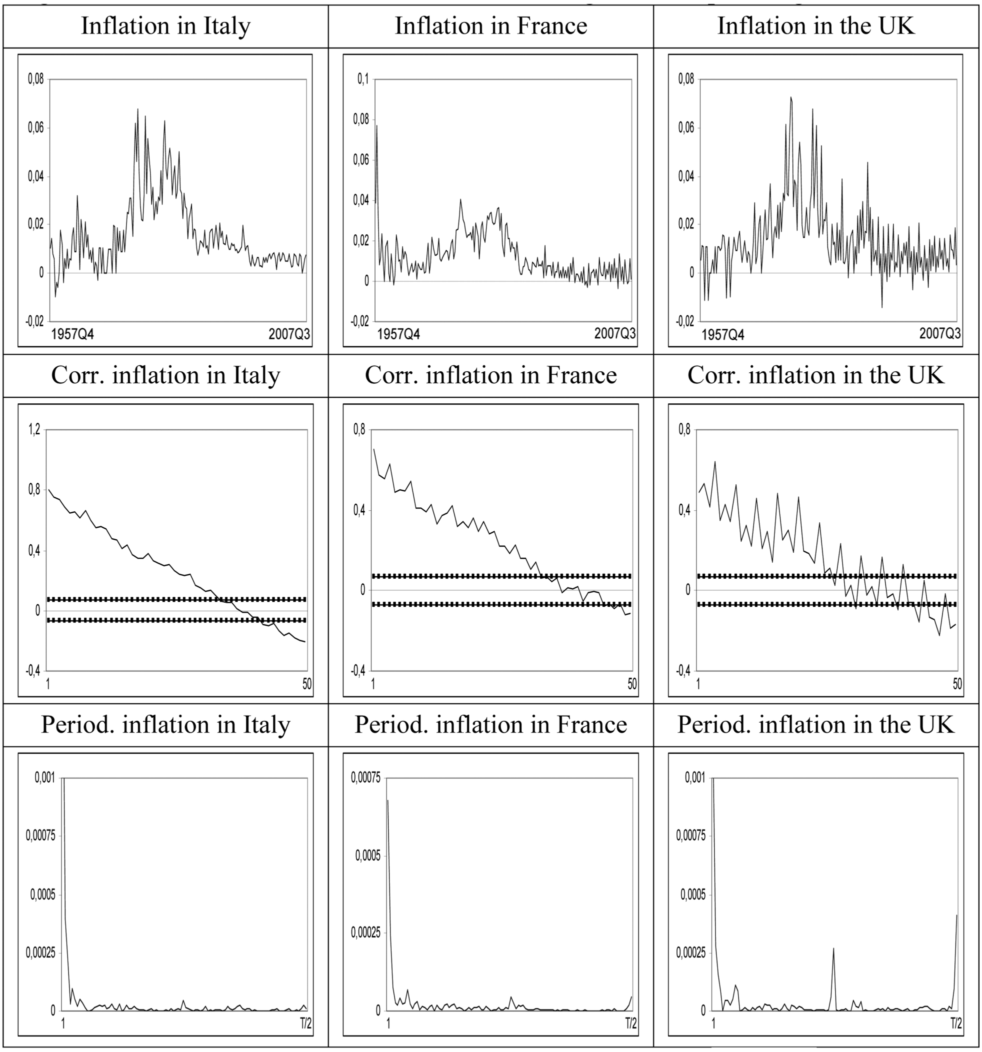
The sample autocorrelation values are all significantly positive and decay very

Figure 1. Time series plots with correlograms and periodograms



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.070 for the series used in this application. The periodograms are computed based on the discrete frequencies $\lambda_j = 2\pi j/T$.

Figure 2. First differences of the series with correlograms and periodograms



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.070 for the series used in this application. The periodograms are computed based on the discrete frequencies $\lambda j = 2\pi j/T$.

slowly. Also, the periodograms exhibit the highest values at the smallest frequency. Both features might be an indication of nonstationarity and possibly of fractional integration behaviour. Figure 2 is similar to Figure 1 but based on the first-differenced data, that is, the inflation rates in the three countries. The correlograms still suggest here that the series are nonstationary, with clear seasonal patterns, and the periodograms still present a large peak at the zero frequency (as well as other smaller peaks at the seasonal frequencies), which may suggest that the inflation rate series are fractionally integrated with d constrained between 0 and 1. Along this section we focus on the log-prices series rather than the inflation rates noting that even though there is little doubts about the existence of a unit root in the log prices, this is an assumption that should be tested, and using the fractional I(d) framework we can test the unit root hypothesis from an alternative approach to the standard methods based on AR models.

The first model we consider is the standard I(d) one with seasonal autoregressions. We allow for different seasonal AR(k), (with k = 1, 2 and 3) processes, and using the likelihood criteria (AIC and BIC) we conclude that the AR(1) model is sufficient to describe the seasonal short-run dynamics in the three series. In other words, we estimate the model,

$$y_t = \mu + x_t; \qquad (1-L)^d x_t = u_t; \qquad u_t = \rho u_{t-4} + \varepsilon_t, \qquad t = 1, 2, \ldots, \qquad \text{(M1)}$$

for the two cases of no regressors (i.e. $\mu = 0$ in (M1) and with an intercept (μ unknown in (M1).[7] Here, we employ the Whittle estimator in the frequency domain developed by Dahlhaus (1989), along with a simple version of the tests of Robinson (1994) that is suitable for this type of model (see, e.g. Gil-Alana and Robinson, 1997).[8] The results are reported in Table 1.

It can be seen that the results vary substantially depending on the inclusion or

Table 1. Estimates of the parameters in model (M1): I(d) with seasonal AR(1)

	No regressors (μ = 0)		An intercept (μ unkown)		
	d	Seas. AR	D	Intercept	Seas. AR
FRANCE	0.966 (0.707, 1.131)	0.166	**1.549 (1.438, 1.707)**	**2.2384 (354.58)**	**0.331**
ITALY	0.957 (0.768, 1.100)	0.115	**1.552 (1.487, 1.632)**	**1.5802 (224.38)**	**0.024**
U.K.	0.968 (0.723, 1.103)	0.016	**1.307 (1.225, 1.410)**	**1.9468 (175.72)**	**0.482**

In bold, the significant models according to likelihood criteria. In parenthesis (in the 2nd and 4th columns) the 95% confidence bands for the values of d. In the 5th column, they are t-values.

[7]We also allowed for a linear time trend in the undifferenced regression model in (M1). However, the coefficient for the time trend was found to be statistically insignificant in all cases.

[8]For the estimates with the Whittle function, the method was performed on the first-differenced series (i.e. the inflation rates), then adding one to the estimated values of d. The tests of Robinson (1994) were performed on the original data since this method is valid for any real value of d, thus including nonstationary hypotheses (i.e. $d \geq 0.5$). The results were identical in the two cases.

not of an intercept in the model. If $\mu = 0$, the estimated values of d are slightly below 1 in the three cases and the unit root null hypothesis cannot be rejected in any of the three countries. However, if an intercept is included, d is found to be significantly above 1 in all cases, being equal to 1.549 for France, 1.552 for Italy, and 1.307 for the UK. Moreover, the intercept is statistically significant in all three countries, suggesting that it should be included in the model. Thus, according to this specification, inflation may be well described in terms of a long-memory I(d) process with d ranging between 0 and 1, and being nonstationary (d > 0.5) in the case of France and Italy.[9]

In the second specification we assume that the seasonal structure can be described in terms of a long-memory process and consider a model of the form:

$$y_t = \mu + x_t; \quad (1 - L^S)^{d_s} x_t = u_t; \quad u_t \approx I(0), \quad t = 1, 2, \ldots, \tag{M2}$$

again for the two cases of no intercept ($\mu = 0$) and an intercept, and we assume

Table 2. Estimates of the parameters in model (M2): Seasonal I(d_s)

i) White noise disturbances					
	No regressors ($\mu = 0$)		An intercept (μ unkown)		
	d	AR coeff.	d	Intercept	Seas. AR
FRANCE	0.914 (0.798, 1.040)	xxxxx	**1.652 (1.580, 1.739)**	**2.28902 (214.82)**	xxxxx
ITALY	0.849 (0.724, 0.992)	xxxxx	**1.806 (1.737, 1.887)**	**1.59599 (144.22)**	xxxxx
U.K.	0.852 (0.704, 0.996)	xxxxx	**1.723 (1.641, 1.825)**	**1.95494 (162.45)**	xxxxx
ii) AR(1) disturbances					
	No regressors ($\mu = 0$)		An intercept (μ unkown)		
	d	AR coeff.	d	Intercept	Seas. AR
FRANCE	0.197 (0.190, 0.206)	0.986	-0.150 (-0.388, 0.061)	3.74244 (423.98)	0.999
ITALY	0.295 (0.285, 0.308)	0.991	-0.182 (-0.367, 0.068)	3.26953 (297.55)	0.999
U.K.	0.243 (0.234, 0.255)	0.988	-0.200 (-0.358, 0.064)	3.46846 (368.19)	0.999

In bold, the significant models according to likelihood criteria. In parenthesis (in the 2nd and 4th columns) the 95% confidence bands for the values of d. In the 5th column, they are t-values.

[9]We performed diagnostic tests on the estimated residuals (in particular, no serial correlation: Durbin, 1970; Godfrey, 1978a,b; homoscedasticity: Koenker, 1981; and functional form: Ramsey, 1989) and could not reject the null at the 5% level in any of the three series.

now that the I(0) disturbances u_t are white noise and AR(1). The results, which are now based on another version of Robinson's (1994) tests (see, e.g., Gil-Alana, 2002) are displayed in Table 2.

As in the previous table, the results are very sensitive to the inclusion or not of an intercept. Specifically, in the uncorrelated case, the estimated values of d are smaller than 1 without intercepts, while they are substantially above if an intercept is included in the regression model. If we allow for autocorrelation in the error term in the form of an AR(1) process, the orders of integration are much smaller than in the uncorrelated case, being even negative if an intercept is included. These results are highly influenced by the AR coefficient that is in the three cases very close to 1.[10] This clearly indicates that the component at the zero-frequency plays a very important role when modelling these series. When performing Likelihood Ratio (LR) tests to determine if there is a weak dependence (AR) structure, the results support the white noise specification in the three countries.

Finally, we consider a 3-factor Gegenbauer process of form as in (10) for the

Table 3. Estimates of the parameters in model (M3): 3-factor Gegenbauer I(d)

	i) White noise disturbances						
	No regressors ($\mu = 0$)			An intercept (μ unkown)			
	d_1	d_2	d_3	d_1	d_2	d_3	Interc.
FRANCE	0.331 (0.291, 0.406)	0.077 (0.055, 0.108)	0.170 (0.125, 0.227)	**0.314** **(0.287, 0.348)**	**0.081** **(0.058, 0.112)**	**0.168** **(0.123, 0.226)**	**-.0142** **(-2.402)**
ITALY	0.272 (0.252, 0.296)	0.030 (0.008, 0.059)	0.000 (-.058, 0.039)	**0.271** **(0.253, 0.294)**	**0.029** **(0.009, 0.054)**	**0.000** **(-.080, 0.053)**	**0.0091** **(1.182)**
U.K.	0.179 (0.156, 0.211)	0.038 (0.012, 0.066)	0.000 (-0.89, 0.073)	**0.178** **(0.155, 0.191)**	**0.037** **(0.011, 0.059)**	**0.000** **(-0.61, 0.046)**	**0.0122** **(2.848)**
	ii) AR(1) disturbances						
	No regressors ($\mu = 0$)			An intercept (μ unkown)			
	d_1	d_2	d_3	d_1	d_2	d_3	Interc.
FRANCE	0.135 (0.068, 0.267)	0.156 (0.104, 0.491)	0.197 (0.139, 0.283)	0.020 (-.063, 0.131)	0.305 (0.127, 0.589)	0.204 (0.135, 0.311)	0.0119 (32.501)
ITALY	0.140 (0.069, 0.255)	0.083 (0.048, 0.131)	0.000 (-.091, 0.071)	0.137 (0.068, 0.248)	0.081 (0.050, 0.129)	0.000 (-.097, 0.058)	0.0139 (6.894)
U.K.	0.054 (0.006, 0.132)	0.078 (0.055, 0.104)	0.000 (-.047, 0.037)	0.018 (-.086, 0.149)	0.030 (0.007, 0.102)	0.000 (-.049, 0.057)	0.0141 (14.720)

In bold, the significant models according to likelihood criteria.

[10]Note that the polynomial $(1-L^s)$ includes the zero-frequency since $(1-L^s)$ can be decomposed into (1-L) and $S(L) = (1+L+\ldots+L^{s-1})$ implying the existence of a unit root at zero. Therefore, there might exist a competition between the seasonal fractional differencing parameter d_s and the AR coefficient in describing nonstationarity at such a frequency.

three inflation rate series.[11] We focus on the inflation rates based on the evidence of orders of integration around 1 or above 1 at the long run or zero frequency. In Table 3 again we display the results for the two cases of white noise and AR(1) u_t, based on Robinson's (1994) parametric tests. Starting with the uncorrelated case (in Table 3(i)), it can be seen that the results are now very similar in the two cases of $\mu = 0$ and μ unknown. For France, the three orders of integration are about 0.32, 0.08 and 0.17 respectively for the 0, π and $\pi/2$ frequencies. In the case of Italy these values are 0.27, 0.03 and 0, and for the UK they are about 0.18, 0.04 and 0. In the case of AR(1) disturbances, there are some differences: the order of integration at the zero frequency is smaller for all three series than in the previous case of white noise u_t, probably owing to the competition with the AR coefficient in describing the nonstationarity at the zero frequency, and, in the case of France, the orders of integration at the seasonal frequencies are now higher. For Italy and the UK, we still find a value of 0 at the semi-annual frequency ($\pi/2$), suggesting that there is no long-memory component at this frequency in these two countries. Thus, in Table 4 we assume a 2-factor Gegenbauer process for these two countries.[12]

Assuming that the disturbances are white noise the results are the same with or

Table 4. Estimates of the parameters in model (M3): 2-factor Gegenbauer I(d)

	i) White noise disturbances				
	No regressors ($\mu = 0$)		An intercept (μ unkown)		
	d_1	d_2	d_1	d_2	Intercept
ITALY	0.275 (0.250, 0.306)	0.033 (0.006, 0.069)	0.275 (0.208, 0.307)	0.032 (0.005, 0.067)	0.00898 (1.9473)
U.K.	0.179 (0.157, 0.208)	0.030 (0.010, 0.055)	0.179 (0.156, 0.208)	0.029 (0.009, 0.054)	0.01221 (2.7631)
	i) AR(1) disturbances				
	No regressors ($\mu = 0$)		An intercept (μ unkown)		
	d_1	d_2	d_1	d_2	Intercept
ITALY	-0.552 (-.703, -.441)	-0.281 (-.381, -.183)	-1.000 (-1.537, -.960)	-0.480 (-.543, -.207)	0.01683 (438.73)
U.K.	-0.663 (-.841, -.477)	-0.153 (-.317, -.031)	-0.952 (-1.217, -.883)	-0.237 (-.251, -.217)	0.01551 (253.91)

In bold, the significant models according to likelihood criteria.

[11]Since we are now modelling the inflation rate we take first differences of the log CPI series.
[12]Note that d_2 (the order of integration at the semi-annual frequency), though small in magnitude, is statistically significant in all cases.

without intercepts. For Italy, the orders of integration are 0.275 and 0.032 respectively for the zero and the seasonal (π) frequencies, and for UK the corresponding values are 0.179 and 0.029. In all cases, the estimates are significantly different from zero. When imposing AR disturbances (in Table 4(ii)), the estimates are all negative, once more probably owing to the competition with the AR parameters in describing time dependence.[13]

In summary, having considered the three models described above, the preferred specifications for each country are the following. For France:

$$\underset{(354.58)}{y_t = 2.2384} + x_t; \qquad (1-L)^{1.549} x_t = u_t; \qquad u_t = 0.331 u_{t-4} + \varepsilon_t, \tag{M1-F}$$

$$\underset{(214.82)}{y_t = 2.2890} + x_t; \qquad (1-L^4)^{1.652} x_t = \varepsilon_t, \tag{M2-F}$$

and

$$y_t = y_{t-1} + \pi_t; \qquad \pi_t = \underset{(-2.402)}{-0.0142} + x_t, \tag{M3-F}$$

$$(1-2\cos w_r^{(1)} L + L^2)^{0.314} (1-2\cos w_r^{(2)} L + L^2)^{0.081} (1-2\cos w_r^{(3)} L + L^2)^{0.168} x_t = \varepsilon_t,$$

t-values in parenthesis and, given that $w_r^{(1)} = 0$, $w_r^{(2)} = \pi$, $w_r^{(3)} = \pi/2$, the second equation in (M3-F) can be expressed as:

$$(1-L)^{0.628} (1+L)^{0.162} (1+L^2)^{0.168} x_t = \varepsilon_t. \tag{10}$$

For Italy, the models are

$$\underset{(224.38)}{y_t = 1.5802} + x_t; \qquad (1-L)^{1.552} x_t = u_t; \qquad u_t = 0.024 u_{t-4} + \varepsilon_t, \tag{M1-I}$$

$$\underset{(144.22)}{y_t = 1.5960} + x_t; \qquad (1-L^4)^{1.806} x_t = \varepsilon_t, \tag{M2-I}$$

and

[13] Though not reported, the AR coefficients were once more very close to 1 in all cases.

$$y_t = y_{t-1} + \pi_t; \qquad \pi_t = \underset{(1.947)}{0.0098} + x_t, \tag{M3-I}$$

$$(1 - 2\cos w_1 L + L^2)^{0.275}(1 - 2\cos w_2 L + L^2)^{0.032} x_t = \varepsilon_t,$$

or, alternatively for the last equation in (M3-I),

$$(1 - L)^{0.550}(1 + L)^{0.064} x_t = \varepsilon_t, \tag{11}$$

and finally, for the UK,

$$y_t = \underset{(175.72)}{1.9468} + x_t; \qquad (1 - L)^{1.307} x_t = u_t; \qquad u_t = 0.482 u_{t-4} + \varepsilon_t, \tag{M1-UK}$$

$$y_t = \underset{(162.45)}{1.9549} + x_t; \qquad (1 - L^4)^{1.723} x_t = \varepsilon_t, \tag{M2-UK}$$

and

$$y_t = y_{t-1} + \pi_t; \qquad \pi_t = \underset{(2.763)}{0.0122} + x_t, \tag{M3-UK}$$

$$(1 - 2\cos w_1 L + L^2)^{0.179}(1 - 2\cos w_2 L + L^2)^{0.029} x_t = \varepsilon_t,$$

or, alternatively for the last equation in (M3-UK),

$$(1 - L)^{0.358}(1 + L)^{0.538} x_t = \varepsilon_t. \tag{12}$$

We finally performed further serial correlation tests (Box-Pierce and Ljung-Box-Pierce) on the estimated residuals for all nine models and could not find evidence of any extra-(weak) autocorrelation in any of the models.

V. Forecasting Comparisons

In this section, we use forecasting performance criteria to select the best specification among the three models for each country.[14] Specifically, we use the last 20 observations for an in-sample forecasting experiment.[15] Standard measures

[14]Note that we do not directly compare our selected models with other specifications based on AR(I)MA or seasonal AR(I)MA models since such models are not supported by the results presented in Tables 1-4.

of forecast accuracy are the following: Theil's U, the mean absolute percentage error (MAPE), the mean-squared error (MSE), the root-mean-squared error (RMSE), the root-mean-percentage-squared error (RMPSE) and the mean absolute deviation (MAD) (Witt and Witt, 1992). Apart from these measures there exist several statistical tests for comparing different forecasting models. One of these tests, widely employed in the time series literature, is the asymptotic test for a zero expected loss differential of Diebold and Mariano (1995).[16] However, Harvey, Leybourne and Newbold (1997) note that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon increases, and therefore provide a modified Diebold-Mariano test statistic given by:

$$M - DM = DM\sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}},$$

where DM is the original Diebold-Mariano statistic, h is the prediction horizon and n is the time span for the predictions. Harvey *et al*. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic, and also that the power of the test is improved when p-values are computed with a Student distribution.[17]

Using the M-DM test statistic (and based on the RMSEs), we further evaluate the relative forecast performance of the different models by making pairwise comparisons. We consider 5- and 10-period ahead forecasts on a 20-period horizon. The results are displayed in Table 5.[18]

We indicate in bold in this table, for each prediction-horizon and each country, the rejections of the null hypothesis that the forecast performance of model (Mi) and model (Mj) is equal in favour of the one-sided alternative that model (Mi)'s performance is superior at the 5% significance level. We note that the results are similar for the two time horizons, though they vary across countries. In all three countries (M2) and (M3) outperform (M1), implying that a model with a long-memory component exclusively affecting the long-run or zero frequency is

[15]Note that, although model (M3) is estimated in first differences, the forecasting results are calculated for the log-prices series, i.e., y_t.

[16]An alternative approach is the bootstrap-based test of Ashley (1998), though this method is computationally more intensive.

[17]As argued by several authors (e.g. West, 2006) the nesting relationships among the models may further complicate the forecasting assessments of the results.

[18]For the 15 (and higher period)-period forecasts there is not found superiority of one model over the others.

Table 5. M-DM statistics with h = 5 and h = 10

	h = 5			h = 10	
FRANCE	M1-F	M2-F	**FRANCE**	M1-F	M2-F
M2-F	**4.272 (M2-F)**	XXXXX	M2-F	**2.892 (M2-F)**	XXXXX
M3-F	**4.172 (M3-F)**	**-7.805 (M2-F)**	M3-F	**2.812 (M3-F)**	**-5.284 (M2-F)**
ITALY	M1-I	M2-I	**ITALY**	M1-I	M2-I
M2-I	**4.370 (M2-I)**	XXXXX	M2-I	**2.958 (M2-I)**	XXXXX
M3-I	**4.291 (M3-I)**	1.349 (----)	M3-I	**2.905 (M3-I)**	0.913 (----)
UK	M1-UK	M2-UK	**UK**	M1-UK	M2-UK
M2-UK	**4.117 (M2-UK)**	XXXXX	M2-UK	**2.787 (M2-UK)**	XXXXX
M3-UK	**4.191 (M3-UK)**	**4.161 (M3-UK)**	M3-UK	**2.837 (M3-UK)**	**2.817 (M3-UK)**

In bold, the cases where one of the models outperforms the other at the 5% level. The critical value at the 5% level with 19 degrees of freedom is 1.729.

inappropriate in all cases. However, when comparing (M2) with (M3), the results are radically different from one country to another: in the case of France (M2) outperforms (M3); for Italy, it cannot be established whether (M2) is superior to (M3) or vice versa, while for the UK (M3) produces significant better statistical results than (M2).

On the basis of these results, model (M2-F) is the preferred specification for France, implying the existence of a seasonal long-memory component, with equal order of integration at zero and the seasonal frequencies (this order of integration being equal to 1.652). In other words, inflation in France is a nonstationary seasonal long-memory process, with an order of integration of about 0.652. For Italy, models (M2-I) and (M3-I) have a comparable forecasting performance, but given the higher flexibility allowed by (M3-I) we choose this specification for this country. In this case, inflation is also nonstationary with a large component of long memory at the zero frequency and a smaller one at the semi-annual frequency (see equation (11)). Finally, for the UK, the best specification seems to be (M3-UK), namely a two-factor Gegenbauer process, one factor corresponding to the zero frequency ($d_1 = 0.358$) and the other one to the semi-annual frequency ($d_2 = 0.058$)

(equation (12)). According to this specification, UK inflation is a stationary long-memory process.

VI. Conclusions

This paper has analysed the stochastic behaviour of inflation in three European countries (France, Italy and the UK) using a general framework, namely a multi-factor long-memory process that allows for different fractional differencing parameters at the zero and the seasonal frequencies. The flexibility of the model, based on Gegenbauer processes, is a very desirable feature compared with more restrictive approaches previously used in the literature on inflation which impose the same degree of integration at all frequencies in the spectrum. (see, e.g., Backus and Zin, 1993, and Hassler and Wolters, 1995). Our results can be summarised as follows. Inflation in France and Italy is nonstationary, but in the former country this applies to both the long-run and the seasonal frequencies, whilst for the latter the nonstationarity concerns exclusively the long-run or zero frequency, and the contribution of the long-range dependence in the seasonal structure is relatively small. For the UK, inflation seems to be stationary, though with a large component of long-memory behaviour, especially at the zero frequency.

Our results indicate that inflation is a very persistent phenomenon, at least for the three countries examined here. The fact that the I(1) hypothesis is decisively rejected in all three cases implies that the series are mean-reverting, with shocks disappearing in the long run but very slowly, especially in France, and to a lesser extent in Italy and the UK. Moreover, we have shown that seasonality matters, with a positive though small degree of long-range dependence.

The analysis carried out in this study highlights the country-specific nature of the processes examined. It could be extended to other countries and also taking into account the possibility of structural breaks, stochastic volatility or non-linearities. These are clearly important issues, whose linkages with fractional processes have hardly been investigated until now, although they have already attracted the attention of some researchers. Future work will focus on them.

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Appendix

Assuming that y_t is described by equation (3), the regression errors, x_t take the form:

$$\rho(L;\theta)x_t = u_t, \qquad t = 1, 2, \ldots, \tag{A1}$$

where ρ is a scalar function that depends on L and the unknown parameter θ that will take different forms as shown below, and u_t is I(0). The function ρ is specified in such a way that all its roots should be on the unit circle in the complex plane,

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}(1-L^s)^{d_s+\theta_s}\prod_{j=2}^{p-1}(1-2\cos wL+L^2)^{d_j+\theta_j}, \tag{A2}$$

for real given numbers d_1, d^s, d_2, … d_{p-1} and integer p. Note that the second polynomial in (A2) refers to the case of seasonality (i.e. s = 4 in case of quarterly data, and s = 12 with monthly observations). Under the null hypothesis, defined by:

$$H_o: \theta = 0 \tag{A3}$$

(A2) becomes:

$$\rho(L;\theta=0) = \rho(L) = (1-L)^{d_1}(1-L^s)^{d_s}\prod_{j=2}^{p-1}(1-2\cos wL+L^2)^{d_j}. \tag{A4}$$

including thus all the specifications employed in the paper.

Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis (A3) in a model given by (3) and (A1-A2). Based on H_o, the estimated β and residuals are:

$$\hat{u}_t = \rho(L)y_t - \hat{\beta}'w_t, \qquad w_t = \rho(L)z_t; \qquad \hat{\beta} = \left(\sum_{t=1}^{T} w_t w_t'\right)^{-1}\sum_{t=1}^{T} w_t\rho(L)y_t.$$

The functional form of the test statistic is then given by:

$$\hat{R} = \hat{r}'\hat{r}; \qquad \hat{r} = \left(\frac{\sqrt{T}}{\hat{\sigma}^2}\right)\hat{A}^{-1/2}\hat{a},$$

where T is the sample size, and

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{*}\psi(\lambda_j)^2 - \sum_{j=1}^{*}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{*}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{*}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{*}\psi(\lambda_j)g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j);$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \qquad \lambda_j = \frac{2\pi j}{T}; \qquad \hat{\tau} = \arg\min_{\tau\in T^*}\sigma^2(\tau),$$

and the sums over * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l = 1, 2, \dots, s\}$ such that ρ_l, $l = 1, 2, \dots, s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$. Also,

$$\psi(\lambda_j) \;=\; Re\left[\log\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda_j};\theta)\right)\right]\theta = 0\,, \tag{A5}$$

and $I(\lambda_j)$ is the periodogram of u_t evaluated under the null. Thus, in model 1 (M1) it becomes:

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right|.$$

In case of model 2 (M2) it is:

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right| + \log\left(2\,cos\frac{\lambda_j}{2}\right) + \log\left|2\cos\lambda_j\right|,$$

and it is a (3x1) vector of form

$$\psi(\lambda_j) \;=\; \left(\log\left|2\sin\frac{\lambda_j}{2}\right|,\ \log\left(2\,cos\frac{\lambda_j}{2}\right),\ \log\left|2\cos\lambda_j\right|\right)^T,$$

in case of model 3.

The function g above is a known function coming from the spectral density of u_t,

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi}g(\lambda;\tau), \qquad -\pi<\lambda\le\pi\,.$$

Note that these tests are purely parametric, and, therefore, they require specific modelling assumptions about the short-memory specification of u_t. Thus, if u_t is a white noise, then $g \equiv 1$, (and therefore $\hat{\varepsilon}(\lambda_j) \;=\; 0$), and if it is an AR process of the form $\phi(L)u_t = \varepsilon_t$, then, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

Based on H_o (A3), Robinson (1994) established that, under certain regularity conditions

$$\hat{R} \rightarrow_d \; \chi^2_P, \qquad as \quad T\rightarrow\infty\,,$$

where p is the dimension of θ. Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution could have been calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions: a test of (A3) will reject H_o against the alternative H_a: $\theta \neq 0$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob $(\chi^2_{2,\alpha} > \chi^2_2) = \alpha$.