Addressing Financial Heterogeneity in a Monetary Union

Cristina Badarau
*Larefi, Université Montesquieu Bordeaux IV, France*

Grégory Levieuge
*LEO, Université d’Orléans, France*

Abstract

This paper addresses the issue of macroeconomic policies in a financially heterogeneous monetary union. Optimized policy rules are used, under various scenarios of budgetary policies, in a two-country DSGE model. The results indicate that an Euro-wide monetary policy strategy based on national information does not offset the costs associated with the abandonment of national monetary policy. Decentralized budgetary policies need to be more proactive in countries which are structurally more sensitive to shocks. For independent common monetary policy, cooperation between governments is comparable to a coalition, causing losses for every member. The welfare-improvement at the union level only comes from reducing public expenditures divergences.

**JEL classification:** E44, E58, E63

**Keywords:** DSGE model, monetary union, financial heterogeneity, monetary and budgetary rules, cooperation.

* Corresponding author: Cristina Badarau; Larefi, Université Montesquieu Bordeaux IV, Avenue Léon Duguit, 33608 Pessac Cedex, France, Tel: 0033556848536, Email: florina-cristina.badarau@u-bordeaux4.fr

(Co-author: Gregory Levieuge, LEO, Université d’Orléans, Rue de Blois, BP-6739, 45160 Orléans Cedex 2, France, Email: gregory.levieuge@univ-orleans.fr.)
I. INTRODUCTION

European Monetary Union (EMU) members have not been identically affected by the subprime mortgage crisis, despite an initial common financial shock. In this respect, this crisis reasserts the structural heterogeneity of the EMU. \(^1\) Recent studies indicate that the European financial system, in particular, remains far from being integrated. The banking markets precisely appear as the most heterogeneous financing market; \(^2\) price differentials remain high compared to other monetary unions, and home biases in lending to and borrowing of small non-financial corporations and households are persistent. \(^3\) According to the seminal models provided by Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke et al. (1999), the financial accelerator can explain these price differentials. \(^4\)

However, the subprime mortgage crisis has also demonstrated that banks constitute key actors for the transmission of financial shocks. To this respect, several recent contributions\(^5\) have highlighted the importance of the *bank capital channel*: through the adjustments of their balance sheet structures, banks act as amplifiers for the transmission of shocks to the real economy. Following this literature, the question of banks’ financing is as problematic as the question of external financing for entrepreneurs. Because of an agency problem between banks and their creditors, the former bear an external financial premium which is negatively related to their capital ratio. The resulting counter-cycle external financing premium is ultimately passed on to entrepreneurs, through the credit conditions.

Considering simultaneously the main factors underlying the bank capital channel, an

---

\(^1\) The heterogeneity of the EMU is discussed in Jondeau and Sahuc (2008), MacDonald and Wojcik (2008), Sekkat and Malek Mansour (2005), Angeloni and Ehrmann (2007), Ekinci et al. (2007), Hofmann and Remsperger (2005), Lane (2006).

\(^2\) See Baele et al. (2004) and ECB (2008) for instance.

\(^3\) See Angeloni et al. (2003).

\(^4\) Empirical evaluations of the financial accelerator mechanism are provided by Gomes et al. (2003) and Christensen and Dib (2008).

empirical study by Badarau-Semenescu and Levieuge (2010) indicates that European countries are ought to be more (Germany, Italy, Netherland) or less (Finland, France, Spain) sensitive to this mechanism. The reasons rely on structural, institutional and behavioural differences. This concerns, for instance, differences about concentration in the banking market, the importance of bank loans’ substitutes, the existence (or not) of long-term relationships between firms and banks, the dependency towards banking credit, the bank capitalization and liquidity. The bank capital channel thus constitutes an interesting way to model the effects of the financial heterogeneity in the euro area. In this perspective, Badarau and Levieuge (2011) provide a DSGE model of financially-asymmetric monetary union and show that: 1) symmetric shocks induce cyclical divergences inside the union, and 2) a common monetary policy worsens the cyclical divergences.

Given the role of banks in propagating shocks (namely financial shocks which have become recurrent in the last decades), the heterogeneity of banking markets raises the question of the appropriate macroeconomic policies in such a context. Certainly, avoiding huge financial crisis requires adequate micro and macro-prudential measures. The reduction of financial heterogeneity also demands a convergence of structural policies. However, both need time and strength of will to be implemented. It is thus worth examining the suitable mixing of the two main existing EMU policy tools: the common monetary policy led by an independent central bank and the budgetary policies conducted by national governments. Since 2008, intensive debates have concerned the lack of coordination of economic stimulus plans inside the euro area, and the way the EMU-members could cooperate to help both the most affected countries and the union as a whole. Discussions also exist about the appropriate design of monetary policy.

The recent literature analyzing the mix between monetary and fiscal policy in an asymmetric monetary union is quite scarce. Van Aarle et al. (2002, 2004) study fiscal and monetary policy rules in a two-country monetary union model with standard structural asymmetries. However, their analysis is not based on a micro-founded model, and they do not address the issue of optimal monetary policy. The interactions between monetary and fiscal
policy in a monetary union have also been analyzed in dynamic stochastic general-equilibrium models. Gali and Monacelli (2008) and Ferrero (2009), for example, study the optimal policies comparative to simple policy rules in a monetary union where all the policy agents care about union wide variables. Grimm and Ried (2007) use a two-country model with a central bank maximizing union-wide welfare and two fiscal authorities minimizing comparable, but slightly different, country-wide losses. Heterogeneity is introduced by the presence of inflation and output divergences due to specific productivity shocks and to the diverging conduct of national fiscal policies (concerned with national output and inflation targets). Comparing the welfare losses in static games for the three authorities, they conclude that cooperation between all the authorities is the best-performing scenario.

While the full cooperation (between national governments and the supranational central bank) is the first best solution, the literature is quite mitigated about the situation of partial cooperation, in which there is no cooperation between monetary and fiscal policies (only between national fiscal policies). Indeed, such a cooperative game between a subset of players is comparable to a coalition, which leads to welfare losses for all the players\(^6\). In this respect, the institutional organization in the euro area rather corresponds to the monetary leadership scenario than to an overall cooperation.

The aim of this paper is to extend this literature, by analyzing the effects of various policy-mix scenarios in such an institutional context, in the presence of financial heterogeneity and financial shocks. In this respect, our work is close to Faia (2002) and Gilchrist et al. (2002). However, while they only focus on the appropriate monetary policy in a currency union, an original contribution of our paper is to further provide an evaluation of different policy-mix strategies in such a context.

To this end, we proceed to some policy experiments based on the DSGE model proposed by Badarau and Levieuge (2011), for a monetary union composed of two countries with distinct

---

banking structures and national budgetary policies. As a leader of the game, the common central bank can choose to target the average inflation rate in the union or to target the inflation divergences inside the union. As discussed in De Grauwe and Piskorski (2001), Angelini et al. (2002), Gros and Hefeker (2002), De Grauwe and Sénégas (2006), Brissimis and Skotida (2008) or Badarau-Semenescu et al. (2009), this second scenario, which corresponds to a standard national information based monetary policy strategy, can be viewed as a solution to counteract the effects of structural heterogeneity.

As for budgetary policies, national governments are allowed to strongly cooperate or to conduct non-cooperative policies after committing to a simple coordination mechanism inside the union, as notified by the Treaty of Lisbon. Whatever the underlying scenario, the Central Bank is supposed to follow an optimized Taylor rule, while the national governments follow optimized balanced-budget rules allowing for temporary deficits. The novelty of our work in respect to this literature is to address the question of the optimal policy-mix in a micro-founded financially heterogeneous monetary union, by distinguishing 1) between full and partial cooperation among governments, and 2) between monetary strategies based on national or aggregate information. No previous reference combines these features simultaneously. However, they are important, on the one hand to properly characterize the context of the euro area, and on the other hand to draw plausible and robust normative conclusions for the economic policy.

We first find that an Euro-wide monetary policy strategy based on national information does not offset the costs associated with the abandonment of national monetary policy. Next, we find that decentralized budgetary policies need to be more proactive in countries that are structurally more sensitive to shocks. However, when the monetary policy is conducted independent of budgetary policies, cooperation between national governments is comparable to a coalition causing losses for every member. Such a cooperative strategy is welfare-improving at the union-wide level only in that it reduces public expenditures divergences.

The reminder of this paper is organized as follows. Section II resumes the baseline DGSE model with financial heterogeneity on which policy experiments are based. Section III is devoted
to the analysis of centralized vs national-information oriented monetary policy. Section IV comprises the analysis of cooperative vs non-cooperative budgetary policies, while section V discusses the policy-mix. The last section formulates some concluding remarks.

II. THE MODEL

The model we are using in this work is provided by Badarau and Levieuge (2011). It describes a two-country monetary union with heterogeneous national banking structures. We simply improve this model by considering that governments are active and try to stabilize national variables. This allows us to introduce the discussion on budgetary policy and to define different policy-mix scenario.

As depicted in appendix 1, six categories of national agents optimize their decisions in this model: households, entrepreneurs (wholesale producers), retailers, capital producers, banks and the government. Households supply labor and own the retail firms. They receive wages from entrepreneurs and profits from retailers, and use them for consumption and savings. Because the model consists of a two-country monetary union, domestic households simultaneously consume domestic goods and goods produced in the other country of the union. They also pay lump-sum taxes to the government, necessary to finance public expenditures. Entrepreneurs use labor and capital as input (partially financed by debt) to produce wholesale final goods, in perfectly competitive markets. Retailers buy wholesale goods from the producers. They slightly differentiate them (with no costs) and retail them in a monopolistic competition market. CES aggregates of retail products are bought by households and also by capital producers, who transform retail goods in capital (used by the entrepreneurs, in the production process). Banks collect funds from national households, to afterwards finance investment projects by lending to the national entrepreneurs. The national banking sectors have a particular place in the model as they embed the structural heterogeneity. The strength of the bank capital channel is indeed supposed to be different in the two countries.
At the union level, a common Central Bank is responsible for the conduct of monetary policy. As for the euro area, the main task of the Central Bank is to maintain price stability, while the national governments should insure the stability of national aggregates.

A. The general equilibrium

Each country is inhabited by a continuum of infinitely-lived households represented by the unit interval. These agents choose consumption \((C)\) and leisure \((L)\) and determine their working time \((H = 1 - L)\) remunerated at a real rate \(W\). Their one period utility function is:

\[
U(C_t, H_t) = \frac{\sigma_c}{\sigma_c - 1} (C_t)^{\frac{\sigma_c - 1}{\sigma_c}} - \frac{\sigma_h}{\sigma_h + 1} (H_t)^{\frac{\sigma_h + 1}{\sigma_h}}
\]

(1)

with \(\sigma_c\) the intertemporal elasticity of substitution in consumption, and \(\sigma_h\) the elasticity of the disutility associated to labor.

Consumption is a composite index which depends on the consumption of domestically produced goods and goods produced in the other country of the union. The origin of goods is indexed by 1 and 2, while \(C\) and \(C^*\) denote aggregate consumption in the first and the second country of the union, respectively. \(\gamma \in [0,1]\) represents the relative preference for consumption of domestically produced goods, in each country.

\[
C = \frac{C_1^\gamma C_2^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}; \quad C^* = \frac{(C_1^*)^{\gamma} (C_2^*)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}
\]

(2)

Price indexes for the two countries are respectively: \(P = P_1^\gamma P_2^{1-\gamma}\) and \(P^* = P_1^\gamma P_2^{1-\gamma}\), and the law of one price is supposed to hold.

Households choose a sequence of consumption, labor, bank securities \((A_t)\) remunerated with a real interest rate \(r_t^A\) and other possible financial investment \((D_t)\) at the real risk-free interest rate \(r_t^f\), which maximizes an intertemporal utility function, based on (1), subject to the following budget constraint:
\[ P_tC_t + P_tD_t + A_t \leq P_tW_t H_t + A_{t-1\alpha} R_t^\alpha + P_tD_t \rightharpoonup -T_t + \Pi_t \] (3)

In the constraint (3), \( R_t^\alpha = 1 + r_t^\alpha \) and \( R_t^\prime = 1 + r_t^\prime \) denote the gross real returns of the two alternative financial investments for households, \( T_t \) represents lump-sum taxes and \( \Pi_t \) are the dividends received from the ownership of retail firms. A symmetric constraint applies to the second country of the union. The first order conditions associated to \( C_t, D_t, A_t \) and \( H_t \) for the two countries are shown in table 1.

**Insert Table 1 - First order conditions for the households' optimization**

At the optimum, there is no preference of the households to choose one financial investment rather than another one. The labor supply is given by the last condition in table 1, and the nominal interest rate is the same inside the union (chosen by the common Central Bank):

\[
\frac{(R_{t+1}^\prime)E_t[P_{t+1}^*/P_t]}{(R_{t+1}^*)E_t[P_{t+1}^*/P_t]} = \frac{(R_{t+1}^*)E_t[P_{t+1}^*/P_t]}{(R_{t+1}^*)E_t[P_{t+1}^*/P_t]}.
\]

As in Gali and Monacelli (2009), this allows us to write:

\[
C_t = C_t^*(\Theta_t)^\sigma
\]

(4)

where \( \Theta_t = P_t^*/P_t \) is an expression of the bilateral terms of trade.

*Wholesale producers* combine capital \( (K_t) \) and labor \( (L_t) \) with a Cobb-Douglas constant return to scale technology:

\[
Y_t = a_t K_t^{\alpha - \alpha} L_t^{1-\alpha} \quad \text{and} \quad Y_t^* = a_t^*(K_t^*)^{\alpha} (L_t^*)^{1-\alpha}
\]

(5)

with \( a_t \) an exogenous productivity factor that follows a standard autoregressive process in the model: \( a_t = \rho_a a_{t-1} + \epsilon_a \), where \( \epsilon_a \) defines a productivity shock, with zero mean and unit variance. The labor input in (5) is a composite index of households labor \( (H_t) \) and entrepreneurial labor \( (H_t^E) \):

\[
L_t = H_t^\alpha (H_t^E)^{1-\alpha}. \quad \text{The entrepreneurs supplement their income by supplying their labor force, remunerated at a rate } W_t^E. \quad \text{Note that the total entrepreneurial labor is normalized to unity. In each country, the investment } (I_t) \text{ is supposed to concern domestically}
\]
produced goods. The accumulation of physical capital is introduced by the following equation, with $\delta$ the depreciation rate:

$$K_{t+1} = (1 - \delta)K_t + I_t$$  \hspace{1cm} (6)

The stock of capital is renewed each period. To produce wholesale final goods for the period $t + 1$, the entrepreneur buys, at the end of the period $t$, the capital $K_{t+1}$ at a price $Q_t$. Because he cannot entirely self-finance his project, he uses his own net wealth $(NE_t)$, and borrows the remainder $(B_t)$ from a representative bank. In turn, the representative bank uses its inside accumulated capital $(NB_t)$ and other complementary funds raised from households $(A_t)$ to lend $B_t$ to a representative firm.

It is also assumed that there are some internal capital-adjustment costs $\Phi(\cdot)$ borne by the capital producers, who buy $I_t$ units of final goods and transform them into physical capital which they afterwards sell to entrepreneurs.

$$\Phi(I_t, K_t) = \frac{\phi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \text{ for } \phi > 0$$  \hspace{1cm} (7)

Denoting by $\rho_i = P_{i,1}^g / P_{i,1}$ the relative price of wholesale goods produced in country 1 and by $Q_i$ the Lagrange multiplier associated with the process of capital accumulation, and given the term of trade $P_1 / P_2 = P_1^t / P_i = \Theta_i$, the profit maximization program of domestic entrepreneurs gives the first order conditions (relative to $H_i, H_i^E, I_i$ and $K_{t+1}$ respectively), reported in table 2.

**Insert Table 2 - First order conditions for wholesale producers’ optimization**

As in Levieuge (2009) the profit maximization of capital producers is internalized in this program. The first two conditions define the demand for labor. The third one gives the Tobin’s Q ratio. The last relation represents the expected gross return on holding an unit of capital from $t$ to
at the optimum, the entrepreneurs’ demand for capital insures the equality between the expected marginal return on capital and the expected marginal cost for the external financing. This cost is derived from the external financial premium the firms have to bear as a consequence of an asymmetric information situation between them and their bank creditors. Moreover, not only entrepreneurs have private information about the risk and the return of their projects, but banks also have private information about the risk and the realized return of their activity. It is then analogously assumed that debt contracts between banks and households (as ultimate creditors) occur in an asymmetric information context. Following the demonstration by Badarau and Levieuge (2011), the expressions for the external financial premium bear by banks on the one hand, and by firms on the other hand, are given in table 3.

*Insert Table 3 - Financial market equilibrium in the member countries*

The external finance premium for banks \( S^B_t \) is, in logarithmic form of, is the difference between the non-default net return on the bank’s loans portfolio required by the household \( r^{B}_t = \beta^{B}_t - 1 \) and the risk-free interest rate \( r^{f}_t = \beta^{f}_t - 1 \). As described in table 3, \( S^B_t \) only depends on the banks’ financial leverage, defined by the ratio of loans (\( B_t \)) over accumulated inside capital (\( NB_t \)).

For entrepreneurs, the external finance premium \( S^E_t \) is, in logarithmic form, the difference between the net return on the entrepreneur’s physical capital required by the bank \( r^K_t = \beta^K_t - 1 \) and the risk-free rate \( r^{f}_t = \beta^{f}_t - 1 \). As we can see in table 3, it depends not only on the entrepreneur’s accumulated net wealth (\( NE_t \)), but also on the accumulated inside capital of the bank (\( NB_t \)). Thus, the lending interest rate required by a bad-capitalized bank ought to be higher than that charged by a healthier one. This implies that entrepreneurs internalize the banks’ external financing costs. In line with the bank capital channel mechanism, a deterioration of the banks’ balance sheet implies a tightening of the lending conditions to entrepreneurs.
The entrepreneur’s net worth \( (NE_i) \) comes from its accumulated benefits \( (VE_i) \) and, to a lesser extent, from the wage he perceives \( (W_i^E) \) in offering its labor force\(^7\). The coefficient \( \gamma^E \) in table 3 corresponds to the survival probability of the entrepreneur. The remaining net wealth of the constant proportion \( (1 - \gamma^E) \) of entrepreneurs leaving the market each period is entirely used to consume final goods \( (CE_i) \): 

\[
CE_i = (1 - \gamma^E)\left[VE_i + WE_i\right] = \frac{1 - \gamma^E}{\gamma^E} NE_i .
\]

Finally, the value of the firm \( (VE_i = Q_i^E K_i^E S_i^E R_i^E B_i^E) \) is given by the gross return on capital, after the repayment of the debt and of the associated interest.

In a similar way, the bank inside capital \( (NB_i) \) comes mainly from the accumulated benefits of the intermediation activity \( (VB_i = R_i^K B_{i-1}^E - S_{i-1}^E R_i^E A_{i-1}^E) \). Furthermore, it is assumed that a proportion \( (1 - \gamma^B) \) of banks leaves the market each period, and then transfers a small part \( (\ell^B) \) of their inside capital to new banks\(^8\) (for an aggregated amount \( T_i^B \)). Their remaining capital is devoted to final goods consumption: 

\[
CB_i = (1 - \gamma^B)(1 - t^B)VB_i = \frac{(1 - \gamma^E)(1 - t^E)}{\gamma^E (1 - t^E) + t^B} NB_i .
\]

Retailers are represented by firms, held by households, that purchase wholesale goods and retail differentiated final goods. Following Calvo (1983), it is assumed that a retailer changes his price each period with probability \( 1 - \varsigma \). Subsequently, the retailer pricing behavior leads to the following 'new Phillips curves' in the two countries of the union:

\[
\hat{\pi}_{1,t} = \beta \hat{\pi}_{1,t+1} + \kappa \hat{\pi}_t \quad \text{and} \quad \hat{\pi}_{2,t} = \beta \hat{\pi}_{2,t+1} + \kappa \hat{\pi}_t^* ,
\]

where \( \pi_{1,t} = \log \left( P_{1,t} / P_{1,t-1} \right) \) and \( \pi_{2,t} = \log \left( P_{2,t} / P_{2,t-1} \right) \) give the inflation rates calculated for the domestically priced goods for the two countries, \( \kappa = \frac{\left(1 - \varsigma\right)(1 - \varsigma \beta)}{\varsigma} \) and \( \rho_i, \rho_i^* \) are respectively

\(^7\)This assumption just allows the entrepreneurs to borrow immediately; otherwise, they should face an unrealistic high external finance premium.

\(^8\)In line with other financial accelerator models, this assumption gives the possibility to new banks to benefit from initial capital, which is essential for the access to external financing. Without initial wealth, newcomers would suffer prohibitive external financial premium.
the real marginal cost for a representative retailer in each country. $\hat{x}_i$ defines, for all $x_i$, the deviation of a variable $x_i$ from its steady-state value.

The national goods and labor markets equilibrium conditions imply: 9

$$Y_i = \Theta_i^{(1-\gamma)/(2\gamma-1)} C_i \left[ \gamma + (1-\gamma) \Theta_i^{1-\sigma_i} \right] + CE_i + CB_i + I_i + G_i$$ (9)

$$Y_i^* = \left( \Theta_i \right)^{\gamma/(1-2\gamma)+\sigma_i} C_i^{\gamma} \left[ (1-\gamma) + \Theta_i^{1-\sigma_i} \gamma \right] + CE_i^* + CB_i^* + I_i^* + G_i^*$$ (9')

and respectively:

$$\left( H_i \right)^{\sigma_0+1}_{\sigma_0} = \left( C_i \right)_{\sigma_i} \rho_i \left( \Theta_i \right)^{-\gamma} \Omega \left( 1-\alpha \right) Y_i$$ (10)

$$\left( H_i^* \right)^{\sigma_0+1}_{\sigma_0} = \left( C_i^* \right)_{\sigma_i} \rho_i^* \left( \Theta_i \right)^{-\gamma} \Omega \left( 1-\alpha \right) Y_i^*$$ (10')

Finally, in addition to technological shocks, already considered in equations (5), financial shocks are also introduced in the model. In previous equations, $Q_i$ represents the fundamental value of the physical capital, given by the present value of dividends to be obtained by the wholesale firms' shareholders. We now allow for the possibility that the market value of capital, denoted hereafter by $Q_i^m$, differs temporarily from its fundamental value $Q_i$, because of a temporary financial shock ($\epsilon_{q_i}$)

$$Q_i^m = Q_i + \epsilon_{q_i}$$ (11)

with $\epsilon_{q_i}$ a random variable of zero average and 0.2 standard deviation. If the shock arises in $t$, it affects the market value $Q_i^m$ of the capital only at this period; afterwards, the equality between $Q_i^m$ and $Q_i$ still holds10.

---

9 Total consumption in the economy includes households’ consumption and the consumption of the entrepreneurs and banks that have failed in the previous period (CE and CB respectively).

10 Then, the financial shock corresponds to a one-period shock, whereas Bernanke and Gertler (1999) and Leveque (2009) simulate an exogenous multi-period bubble. The aim here is not to reproduce the effects of a long-lasting financial bubble, but simply to adequately insert financial shocks in the model.
Hence, in case of a financial shock, the fundamental return on the physical capital given in Table 2 becomes an abnormal return on capital given by:

$$R_{t}^{km} = \left[ \rho_{t}(\Theta_{t})^{\gamma_{t}} \alpha \frac{Y_{t}}{K_{t}} - \frac{\phi}{2} \left( \delta^{2} - \left( \frac{I_{t}}{K_{t}} \right)^{2} \right) + (1 - \delta)Q_{t}^{m} \right] / Q_{t-1}$$

(12)

Therefore, $Q_{t}^{m}$ replaces $Q_{t}$ in the equations in Table 3, defining the dynamics of entrepreneurs’ net worth, banks’ net worth, and the subsequent external finance premiums, respectively. So, when $Q_{t}^{m} > Q_{t}$, the entrepreneurs’ and banks’ net values increase without any fundamental justification. The seeming improvement of their balance sheet allows them to obtain better conditions for external financing, stimulating the national investment and output (and inversely in case of adverse financial shock).

The model is closed by monetary and budgetary rules that are discussed in details in the following sections.

B. The model parameterization

The calibration for the parameters and the main macroeconomic ratios at their steady-state is detailed in Appendix 2, and is made according to the references found in the literature for the euro area. Ratios such as capital/GDP, investment/GDP or total consumption/GDP are all compatible with the estimations revealed by Fagan et al. (2001). Moreover, it is realistically supposed that banks have a lower default probability than entrepreneurs, and that the ratio $B / NB$ belongs to the interval $[5,10]$. Finally, we realistically obtain that, in the steady state, the probability for a bank to leave the credit market is lower than that of entrepreneurs. Additionally, the audit is more costly for households than for banks, which justifies the presence of the latter in the economy. The banking system embeds the structural financial heterogeneity, as it is assumed that the two countries differ in two aspects: (i) in the ratio of loans over inside capital for banks at the steady-state, and (ii) in the sensitivity coefficient of the banks’ external 11 Numerical values are in line with those used by Sunirand (2003) and Levieuge (2009) for the euro area.
finance premium to their financial leverage. Considering both criteria, country 1 becomes more sensitive to shocks than country 2.

Badarau and Levieuge (2011) analyze in details how such financial heterogeneity accentuates cyclical divergences. This raises the question of how monetary, budgetary and policy-mix can mitigate the asymmetric effects of common shocks. To this end, alternative strategies for the Central Bank and for the governments will be analyzed, through the optimization of their respective policy rules, in the next sections.

III. CENTRALIZED VS NATIONAL INFORMATION ORIENTED MONETARY POLICY

We consider an independent common Central Bank, like the European Central Bank, whose policy is responsible for the union-wide price stability and which does not cooperate with the national governments (in accordance with the Article 130 of the Treaty on the European Union). This common Central Bank is supposed to conduct its monetary policy following an interest rate rule given by:

$$\hat{r}_t^n = \beta_0 \hat{r}_{t-1}^n + (1 - \beta_0) \beta_1 \hat{\pi}_{t}^{MU} + \epsilon_n$$

where $\hat{\pi}_{t}^{MU} = \frac{1}{2} (\hat{\pi}_t + \hat{\pi}_t^*)$ is its union-wide inflation deviation from the target, and $\beta_1$ its corresponding reaction coefficient. $\beta_0$ is the smoothing coefficient of the nominal interest rate. $\epsilon_n$ represents a monetary policy shock of zero average and standard deviation equal to 1.

As a common monetary policy induces a stabilization bias in a heterogeneous monetary union, worsening cyclical divergences (see Badarau and Levieuge, 2011), two configurations are alternatively considered for the optimization of $\beta_1$ and $\beta_0$. First, in the centralized strategy, the
Central Bank just stabilizes the average inflation for the union, and is not concerned by national divergences. The loss function to be minimized is:\(^\text{12}\)

\[
L^{\text{CB}} = \text{var}(\hat{x}^\text{MU}) + \lambda \text{var}(\Delta \hat{x}^n)
\]

where \(\text{var}(\hat{x})\) defines the second order moment for the variable \(\hat{x}\), and \(\Delta \hat{x}^n = \hat{x}_i^n - \hat{x}_{i-1}^n\). \(\lambda\) is the relative importance given by the monetary authority to the interest rate smoothing.

Second, a monetary strategy based on national information responds to the situation in which the Central Bank is simultaneously concerned by the union-wide inflation stabilization and by the stabilization of the inflation differentials inside the union (see Badarau-Semenescu et al., 2009). It thus becomes an inflation-divergences oriented monetary strategy. The loss function of the Central Bank becomes:

\[
L^{\text{CB}} = \text{var}(\hat{x}^\text{MU}) + \text{var}(\hat{x}^\text{MU}) + \lambda \text{var}(\Delta \hat{x}^n), \text{ for } \hat{x}^\text{MU} = \frac{\hat{x}_i - \hat{x}_i^*}{2}
\]

Monetary decisions are supposed to be independent of governments’ behavior. Optimization is made in the presence of stochastic and symmetric technological and financial shocks, whose variances are given in section II.

The results reveal that the coefficient \(\beta_i\) is optimally higher in the centralized strategy \((\beta_i = 1.45704)\) compared to the national information based strategy\(^\text{13}\) \((\beta_i = 1.43749)\). As expected, the centralized monetary policy is more reactive to symmetric shocks than a policy taking the specific situation of each member country into account.\(^\text{14}\) Therefore, faced with

---

\(^{12}\) The central bank loss function could be derived from the intertemporal utility function of the representative agent, as in Woodford (2003). Nevertheless, this is in fact not a result, but a hypothesis. The representative agent cannot be the central banker once the central bank is independent. Moreover, a vast and persuasive literature indicates that the central bankers’ preferences depend on institutional and political matters, and not only on structural ones. See for instance the survey by Hayo and Hefeker (2008). It is not less rigorous to directly infer from the actual conduct of the ECB its preferences. From this viewpoint, it can reasonably be asserted (de facto and de jure) that inflation stability is its single objective.

\(^{13}\) In line with Sauer and Sturm (2007), Fourçans and Vranceanu (2007) and Licheron (2009), \(\beta_i\) is equal to 0.96.

\(^{14}\) For asymmetric shocks, the situation reverses. As found in Badarau-Semenescu et al. (2009) for instance, shocks are better stabilized under a monetary policy that tries to reduce inflation divergences, than under a centralized monetary policy.
symmetric shocks, a centralized monetary strategy dominates the strategy based on national information, being able to insure a better macroeconomic stability in the union. This is clearly supported by the computation and comparison of expected social losses (see table 7), in the section V below.

IV. COOPERATIVE VS NON-COOPERATIVE BUDGETARY POLICIES

Decentralized budgetary policies are conducted by the national governments. They use lump-sum taxes to finance public expenditures and they respect a balanced-budget condition allowing for temporary deficits or surpluses, in the spirit of Annicchiarico et al. (2006). Governments are concerned with national output and inflation stabilization, whereas they are not directly concerned with output growth and price changes in other parts of the union unless they decide to cooperate. In order to balance the diverging effects of the common monetary policy, the output target for the budgetary policy corresponds, as usual, to the national potential output, while the inflation target is the one announced by the common central bank, corresponding to its steady state value. To insure the stability of national aggregates, governments use conventional balanced-budget rules extended to incorporate cyclical components (see, for instance, Muscatelli et al., 2004, Annicchiarico et al., 2006 and Grimm and Ried, 2007).

\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + \rho_y \hat{y}_t + \epsilon_{g_t},
\]

\[
\hat{g}_t^* = \rho_g^* \hat{g}_{t-1}^* + \rho_y^* \hat{y}_t^* + \epsilon_{g_t}^*,
\]

where \(\hat{g}_t, \hat{g}_t^*\) denote the deviation of the fiscal balance from its steady state in each country, a positive value corresponding to a fiscal deficit, and \(\rho_g, \rho_g^*\) are coefficients for the budgetary policy inertia. \(\rho_y, \rho_y^*\) represent the reaction coefficients of the budgetary policy to national

---

15 In fact, taxes are maintained at the steady state level, in our model (which corresponds to a balanced budget in normal periods). The cyclical changes in governments’ spending explain why temporary deficits or surpluses are possible.
inflation deviation from the steady-state. $\rho_{s}, \rho_{y}^{*}$ stand for the reaction to the output-gap ($\hat{y}$ is the log-deviation of the output from the steady-state). $\varepsilon_{g}, \varepsilon_{y}^{*}$ are random shocks with zero mean and unit standard deviation.

If Ballabriga and Martinez-Mongay (2002), for instance, consider inertia as a constraint stemming mainly from the political difficulty of changing past spending commitments, we follow Vogel et al. (2006) and consider here $\rho_{g}, \rho_{y}^{*}$ as control variables, in the sense that annual budget laws define policy guidelines that are gradually implemented. Thus, the coefficients $\rho_{g}, \rho_{y}$ and $\rho_{s}$ have to be optimally found so as to minimize national loss functions.

Two configurations are considered. In the non-cooperative budgetary policy regime, which refers to an autonomous conduct of national policies, each government optimize a national loss function (17), considering as exogenous the public expenditures of the other country:

$$L^{G} = \lambda_{x}^{G} \text{var}(\hat{\pi}) + \lambda_{y}^{G} \text{var}(\hat{y}) + \lambda_{s}^{G} \text{var}(\hat{g})$$

(17)

$\lambda_{x}^{G}, \lambda_{y}^{G}$ and $\lambda_{s}^{G}$ define the national preferences for inflation, output and public expenditures stabilization, respectively.

In the cooperative budgetary policy regime, both governments are endowed with a unique cooperative loss function, calculated as the average of the national loss functions:

$$L^{\text{Coop}} = \frac{1}{2} \left( L^{G} + L^{G^{*}} \right)$$

(18)

Note that, according to the new Treaty of Lisbon, national governments benefit from autonomy in the conduct of their budgetary policies. However, they are supposed to respect a global orientation for the budgetary policy defined at the union-wide level. Such global orientation, which is still unclear in the treaty, is interpreted hereafter as a commitment of national governments to follow symmetric stabilization objectives for their budgetary policy.

This implies in our model that $\lambda^G_x$, $\lambda^G_y$, and $\lambda^G_g$ in (17) are identical for the two countries. This institutional framework can be seen as a *particular coordination mechanism* that covers not only the cooperative, but also the non-cooperative budgetary regime.

Finally, the following sequential game applies for the monetary and budgetary policies design. Central Bank chooses first its strategy by taking as given the governments’ policy.\(^{17}\) National governments observe the monetary policy orientation and define afterwards their policies.\(^{18}\)

Under the *centralized optimal monetary policy*, the optimization of the budgetary rules is summarized in table 4 for the non-cooperative budgetary regime and in Table 5 for a cooperative budgetary regime.\(^{19}\) The robustness of the results is controlled by considering in each case three sets of governmental loss functions parameters ($\lambda^G_x$, $\lambda^G_y$, $\lambda^G_g$).

*Insert Table 4 - Non-cooperative budgetary rules coefficients with centralized monetary policy*

For the non-cooperative regime (table 4), the corresponding Cournot-Nash solutions clearly indicate that, whatever the governmental loss functions parameters are, the coefficients for inflation and output stabilization are, as expected, negative in the optimal budgetary rules.\(^{20}\) Moreover, in absolute value, these coefficients are systematically lower in country 2 than in country 1. As country 1 is more sensitive to shocks, it needs more stabilization by the budgetary policy. This means that, with a simple non-cooperative budgetary regime, national governments could play an active role in mitigating asymmetries in the transmission of shocks due to the structural heterogeneity.

\(^{17}\) See also Benassy (2003).

\(^{18}\) This sequential solving is usual and logical in the euro area context. See Andersen (2005) for instance.

\(^{19}\) Results with an inflation-divergences oriented monetary policy are qualitatively similar. The tables of results corresponding to this scenario are available on request.

\(^{20}\) Indeed, when the output gap is positive and/or the inflation is higher than its steady-state value, governments should be more restrictive, namely should reduce public expenditures, in order to stabilize national variables. The opposite policy is expected when the output gap is negative or inflation is too low.
In contrast with these results, optimal cooperative budgetary rules are not consistent with the stabilization needs of member countries as it can be seen in table 5. For example, the coefficient associated to the inflation gap in the budgetary rule of country 2 is positive, corresponding to a definitely destabilizing policy.

*Insert Table 5 - Cooperative budgetary rules coefficients with centralized monetary policy*

At first glance this result seems counter-intuitive. However, in line with De Grauwe et Senegas (2004) and Badarau-Semenescu et al. (2009), the cooperative loss function $L_{\text{Coop}}$ can be alternatively written:

\[
L_{\text{Coop}} = \lambda_x^G \text{var}(\pi_{UM}) + \lambda_y^G \text{var}(y_{UM}) + \lambda_z^G \text{var}(z_{UM}) + \lambda_x^G \text{var}(\pi_{UM}) + \lambda_y^G \text{var}(y_{UM}) + \lambda_x^G \text{var}(\pi_{UM}) \tag{19}
\]

In other words, the cooperative loss function implicitly leads the governments to comply with centralized objectives of stabilization, but also to fight national divergences. One way to fulfill the objective of cyclical convergence consists for each country making an effort to reach the average performance of the union. This explains the positive sign of the national inflation stabilization in the budgetary rule of country 2 (which is supposed to be less affected by shocks). Most important, the need for responding to divergences in inflation, output and public expenditures finally makes the individual stabilizations less satisfying than in the non-cooperative regime. Figure 1 illustrates this point, in case of restrictive and symmetric monetary shock.

*Insert Figure 1 - National responses to a restrictive monetary shock (for $\lambda_x = 1; \lambda_y = 1.5; \lambda_z = 0.5$)*

As the government in country 2 takes care of macroeconomic divergences in the union, its policy is as a whole not expansionist enough to duly stabilize its national output (divergences are otherwise exacerbated). More precisely, the reaction to inflation divergences implies a
reduction of public expenditures in country 2, whereas public expenditures increase in country 1 (cf. $\rho_x$ and $\rho_x^*$ coefficients in table 5). In the same way, the reduction of output divergences implies a lower increase of public spending in country 2 compared to country 1 (see $\rho_y$ and $\rho_y^*$ coefficients in table 5). Certainly, given the accommodative budgetary policy of country 1, the reduction of government spending divergences would require an increase in public expenditure in country 2. However, this objective is offset by the need to reduce inflation and output divergences.

Consequently, the global effect of these mixed assignments leads to (excessively) moderate public expenditures in country 2. Country 1, in turn, cannot implement a stimulus scheme as ambitious as it would in the non-cooperative regime. It would otherwise be penalized by a growing public expenditures gap. This situation is unsatisfactory for all the members of the monetary union. Further demonstration is provided in table 6, where the stabilization performances of a cooperative vs non-cooperative regime are compared, following negative monetary and financial shocks respectively. It is clear that compared to the non-cooperative regime, the cooperative regime allows for a better stabilization of the divergences between member countries, but to the detriment of the stabilization of national variables (the decline in output is particularly higher).

This situation of counterproductive cooperation echoes some results found in the literature\textsuperscript{21} and is reaffirmed here in a specific context of heterogeneous monetary union with a fully independent central bank and a coordination framework. This situation typically matches the Euro Area. In a situation in which the central bank conducts a monetary policy regardless of national budgetary policies, the cooperation between a subset of players (i.e. the governments) is comparable to a coalition in the global economy, which leads to welfare losses for all the players.

*Insert Table 6 - Stabilization performance of a cooperative/non-cooperative regime after shocks*

V. POLICY-MIX ANALYSIS

In this section, we analyze the qualitative properties of four alternative policy-mixes (centralized / inflation-divergences oriented monetary policy with cooperative / non-cooperative budgetary policies), evaluated with respect to an usual union-wide social loss function, computed as the average of national social loss functions:

\[
EL_s = \frac{1}{2} \left[ \hat{\lambda}_y \text{var}(\hat{y}) + \hat{\lambda}_\pi \text{var}(\hat{\pi}) + \hat{\lambda}_g \text{var}(\hat{g}) + \hat{\lambda}_x \text{var}(\hat{x}) + \hat{\lambda}_s \text{var}(\hat{s}) + \hat{\lambda}_\lambda \text{var}(\hat{\lambda}) \right]
\]

(20)

\(\hat{\lambda}_y, \hat{\lambda}_\pi, \hat{\lambda}_g, \hat{\lambda}_x, \hat{\lambda}_s\) are symmetric preferences for the stabilization of output, inflation and public expenditures in the national social loss functions.

Two cases are considered. First, it is assumed that governments, which represent the major view of their respective society, share the same preferences as the agents for inflation and output stabilization: \(\hat{\lambda}_x = \hat{\lambda}^G_x\), for \(x \in \{y, \pi\}\). Second, we make the assumption that the society is essentially concerned about inflation and output stabilization and less concerned about the public expenditures stabilization (see, in extremis, \(\hat{\lambda}_s = 0\)). This latter assumption is motivated by the need of the demonstration: as we shall see, comparing the two cases will allow a better understanding of the results.

The evaluations for these different policy-mixes are reported in table 7. The first column reports the coefficients of the social loss function.\(^{\text{22}}\) The second column compares the corresponding expected losses derived from alternative budgetary regimes. These results are independent of the monetary policy design. The third column compares the expected losses issued from alternative monetary strategies for the central bank, independently of the budgetary regime. The last column clearly shows that, whichever the social and governments’ stabilization preferences, the expected loss induced by a centralized monetary policy is systematically lower

\(^{\text{22}}\) The results are robust and qualitatively similar for any set of preference parameters.
than the loss resulting from a monetary policy based on national targets and then designated to fight inflation divergences in the union.

*Insert Table 7 - Expected social loss comparison for alternative policy-mixes*

The results are more balanced concerning budgetary regimes. Under the assumption of identical social and governmental stabilizing preferences in the union, the results favour the cooperative regime over the non-cooperative one. However, the computation of alternative social loss functions solely defined in terms of inflation and output stabilization ($\lambda^s = 0$) reasserts the superiority of the non-cooperative regime, as obtained in the previous section. Thus, comparing the cases in which $\lambda^s \neq 0$ and $\lambda^s = 0$, it appears that the relative benefit of the cooperative regime only comes from the stabilization of public expenditures divergences inside the union.

**VI. CONCLUDING REMARKS**

This paper addresses the issue of policy-mix in a heterogeneous monetary union hit by financial shocks. The analysis relies on a two-country DSGE model with financial heterogeneity, where a country is more sensitive to adverse financial shocks than the other one. This model shows how financial heterogeneity can accentuate the cyclical divergences inside a monetary union, and indicates that the conduct of a common monetary policy worsens national divergences. This motivates the investigation of several budgetary and monetary policies' scenarios. Several conclusions can be drawn from simulations based on optimized policy rules.

First, it appears that a centralized monetary policy, seeking to stabilize the union-wide inflation rate, dominates a strategy that is concerned with the stabilization of inflation divergences inside the union. This conclusion holds whatever is the budgetary regime (cooperative or non-cooperative), thus supporting for instance the current orientation of the European Central Bank (ECB) policy.
Second, decentralized budgetary policies need to be more proactive in countries that are structurally more sensitive to shocks (those where the bank capital channel is stronger). In this case, budgetary policies can contribute to mitigate the effects of adverse shocks. A solution could come from a non-cooperative budgetary regime in the union, endowed with an implicit coordination mechanism that implies similar objective functions in the member countries (which is an interpretation of the global common orientation promoted by the new Treaty of Lisbon).

Third, a cooperative budgetary regime can be counterproductive when monetary policy is conducted regardless of governments’ behavior. Indeed, the cooperation between a subset of players (i.e. the governments) is comparable to a coalition, which leads to welfare losses for all the players. So cooperation between the sole governments of a monetary union is not a panacea in case of structural heterogeneity.

Finally, at the union level, the cooperative regime seems to be preferable only because it allows a better stabilization of public deficits divergences. This result has at least two consequences. First, if the agents in a monetary union take care of inflation and output divergences, but are not concerned by public spending divergences, then a non-cooperative regime is preferable. Second, in case of cooperation, a virtuous country is sufficient to impose budgetary discipline in the union.
Appendices

Appendix 1: The main structure of the model for each member country
## Appendix 2: Baseline calibration of the DSGE model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value country 1</th>
<th>Value country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\sigma_c$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Elasticity of labor disutility</td>
<td>$\sigma_h$</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Part of retailers with unchanged prices on the period</td>
<td>$\zeta$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Capital contribution to GDP</td>
<td>$\alpha$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Part of entrepreneurial labor in total labor</td>
<td>$1 - \Omega$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Part of households labor in total labor</td>
<td>$\Omega$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Depreciation rate for capital</td>
<td>$\delta$</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Internal capital adjustment costs parameter</td>
<td>$\phi$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Part of inside capital transfers to survival banks</td>
<td>$l^B$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Banks external finance premium elasticity</td>
<td>$\psi_{\beta}$</td>
<td><strong>0.002</strong></td>
<td><strong>0.001</strong></td>
</tr>
<tr>
<td>Entrepreneurs external finance premium elasticity</td>
<td>$\psi_{\gamma}$</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Part of foreign goods in national consumption</td>
<td>$1 - \gamma$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

| Smoothing coefficient in the monetary rule     | $\beta_0$ | 0.9             | Microfunds      |
| Inflation stabilizing coefficient in the monetary rule | $\beta_1$ | 1.1             | 0.07            |

### Steady State: Exogenous fixed values

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real marginal cost</td>
<td>$\tilde{c}$</td>
<td>1/1.1</td>
</tr>
<tr>
<td>Banks loans/inside capital ratio</td>
<td>$B / NB$</td>
<td><strong>6.67</strong></td>
</tr>
<tr>
<td>Entrepreneurs net wealth/capital ratio</td>
<td>$NE / Y$</td>
<td>0.4</td>
</tr>
<tr>
<td>Public expenditures/GDP ratio</td>
<td>$G / Y$</td>
<td>0.16</td>
</tr>
<tr>
<td>Entrepreneurs probability of default</td>
<td>$F(\omega_E)$</td>
<td>0.03</td>
</tr>
<tr>
<td>Banks probability of default</td>
<td>$F(\omega^B)$</td>
<td>0.007</td>
</tr>
<tr>
<td>Average external finance premium for entrepreneurs</td>
<td>$e^K - r^f$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

### Steady State: Calculated values

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auditing cost for banks</td>
<td>$\mu^B$</td>
<td>0.018</td>
</tr>
<tr>
<td>Auditing cost for households</td>
<td>$\mu_H$</td>
<td>0.807</td>
</tr>
<tr>
<td>Variance for the $\omega$-distribution</td>
<td>$\sigma$</td>
<td>0.2531</td>
</tr>
<tr>
<td>$\omega$ threshold value for banks</td>
<td>$\omega^B$</td>
<td>0.52</td>
</tr>
<tr>
<td>$\omega$ threshold value for entrepreneurs</td>
<td>$\omega_E$</td>
<td>0.6016</td>
</tr>
<tr>
<td>Banks probability to leave the market</td>
<td>$1 - \gamma^B$</td>
<td>0.01</td>
</tr>
<tr>
<td>Entrepreneurs probability to leave the market</td>
<td>$1 - \gamma_E$</td>
<td>0.017</td>
</tr>
<tr>
<td>Capital/GDP ratio</td>
<td>$K / Y$</td>
<td>7.0549</td>
</tr>
<tr>
<td>Investment/GDP ratio</td>
<td>$I / Y$</td>
<td>0.2116</td>
</tr>
<tr>
<td>Banks consumption expenses/GDP</td>
<td>$CB / Y$</td>
<td>0.006</td>
</tr>
<tr>
<td>Entrepreneurs consumption expenses/GDP</td>
<td>$CE / Y$</td>
<td>0.048</td>
</tr>
<tr>
<td>Households consumption expenses/GDP</td>
<td>$C / Y$</td>
<td>0.5735</td>
</tr>
<tr>
<td>Total consumption expenses/GDP</td>
<td>$(C + CE + CB) / Y$</td>
<td>0.628</td>
</tr>
</tbody>
</table>
REFERENCES


Table 1. First order conditions for the households’ optimization

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_i = P_i^{-1} C_i^{-1} )</td>
<td>( \lambda_i^* = \left( P_i^* \right)^{-1} \left( C_i^* \right)^{-1} )</td>
</tr>
<tr>
<td>( 0 = \lambda_i - \beta R_i^{t+1} E_{t+1} \left[ \lambda_{t+1} \right] E_t \left[ P_{t+1} / P_t \right] )</td>
<td>( 0 = \lambda_i^* - \beta R_{i+1}^* E_{t+1} \left[ \lambda_{t+1}^* \right] E_t \left[ P_{t+1}^* / P_t^* \right] )</td>
</tr>
<tr>
<td>( 0 = \lambda_i - \beta R_i^{t+1} E_{t+1} \left[ \lambda_{t+1} \right] )</td>
<td>( 0 = \lambda_i^* - \beta R_{i+1}^* E_{t+1} \left[ \lambda_{t+1}^* \right] )</td>
</tr>
<tr>
<td>( H_t = \left( \lambda_i P_t W_t \right)^{\eta} )</td>
<td>( H_t^* = \left( \lambda_i^* P_t^* W_t^* \right)^{\eta} )</td>
</tr>
</tbody>
</table>

Note: \( \lambda_i \) and \( \lambda_i^* \) respectively denote the Lagrange multipliers associated to the budget constraint (3) in the two countries of the union.

Table 2. First order conditions for wholesale producers’ optimization

<table>
<thead>
<tr>
<th>Country 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_i \left( \Theta_i \right)^{\gamma} \Omega_i \left( 1 - \alpha \right) \frac{Y_i}{H_i} = W_t ) ; ( \rho_i \left( \Theta_i \right)^{\gamma-1} \left( 1 - \Omega_i \right) \left( 1 - \alpha \right) \frac{Y_i}{H_i^E} = W_t^E ) ; ( Q_i = 1 + \frac{\partial \Phi(\hat{\gamma})}{\partial I_i} );</td>
</tr>
<tr>
<td>( E_t \left[ R_{t+1}^E \right] = \frac{1}{Q_i} E_t \left[ \rho_i \left( \Theta_i \right)^{\gamma} \alpha \frac{Y_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left( \delta^2 - \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 \right) + \left( 1 - \delta \right) Q_{t+1} \right] )</td>
</tr>
</tbody>
</table>

Note: For the second country of the union the first order conditions are symmetric, except for the exponent of \( \Theta_i \), which becomes \( \left( \gamma - 1 \right) \) instead of \( \left( 1 - \gamma \right) \).

Table 3. Financial market equilibrium in the member countries

<table>
<thead>
<tr>
<th>Country 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_i^B = \Psi_B \left[ k_{t+1}^B \right] ), where ( S_i^B = \frac{R_i^B}{R_{t+1}^f} ), ( \frac{\partial \Psi_B(\hat{\gamma})}{\partial k_{t+1}^B} &gt; 0 ) and with ( k_{t+1}^B = \frac{B_i}{NB_i} )</td>
</tr>
<tr>
<td>( S_i^E = \Psi_E \left[ k_{t+1}^E \right] ), where ( S_i^E = E_t \left[ \frac{R_{t+1}^K}{R_{t+1}^f} \right] ), ( \frac{\partial \Psi_E(\hat{\gamma})}{\partial k_{t+1}^E} &gt; 0 ) and ( k_{t+1}^E = \frac{Q_i K_{t+1}}{NE_i + NB_i} )</td>
</tr>
<tr>
<td>( NE_i = \gamma^E \left[ VE_i + W_t^E \right] ), with ( VE_i = Q_{t+1} R_{t+1}^E K_i - S_{i+1}^E R_{t+1}^f B_i )</td>
</tr>
<tr>
<td>( NB_i = \gamma^BKB_i + T_i^B ), with ( VB_i = R_{t+1}^E B_i - S_{i+1}^E R_{t+1}^f A_i )</td>
</tr>
</tbody>
</table>

Note: All relations are identical for the second country of the union.
Table 4. Non-cooperative budgetary rules coefficients with centralized monetary policy

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1.5; \lambda_g^G = 0.5$</td>
<td>$\rho_g = 0.2189$</td>
<td>$\rho_g^* = 0.1477$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.2022$</td>
<td>$\rho_y^* = -0.1727$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.0861$</td>
<td>$\rho_\pi^* = -0.7125$</td>
</tr>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1; \lambda_g^G = 0.5$</td>
<td>$\rho_g = 0.2368$</td>
<td>$\rho_g^* = 0.1720$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1355$</td>
<td>$\rho_y^* = -0.1155$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.7648$</td>
<td>$\rho_\pi^* = -0.5162$</td>
</tr>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1; \lambda_g^G = 0.1$</td>
<td>$\rho_g = 0.2175$</td>
<td>$\rho_g^* = 0.1623$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.6526$</td>
<td>$\rho_y^* = -0.5476$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -3.6283$</td>
<td>$\rho_\pi^* = -2.3157$</td>
</tr>
</tbody>
</table>

Table 5. Cooperative budgetary rules coefficients with centralized monetary policy

<table>
<thead>
<tr>
<th>Governmental loss functions coefficients</th>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1.5; \lambda_g^G = 0.5$</td>
<td>$\rho_g = 0.1779$</td>
<td>$\rho_g^* = 0.6051$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1901$</td>
<td>$\rho_y^* = -0.0632$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.5985$</td>
<td>$\rho_\pi^* = 0.2576$</td>
</tr>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1; \lambda_g^G = 0.5$</td>
<td>$\rho_g = 0.2079$</td>
<td>$\rho_g^* = 0.5625$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.1237$</td>
<td>$\rho_y^* = -0.0459$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -0.4442$</td>
<td>$\rho_\pi^* = 0.2049$</td>
</tr>
<tr>
<td>$\lambda_\pi^G = 1; \lambda_y^G = 1; \lambda_g^G = 0.1$</td>
<td>$\rho_g = 0.1929$</td>
<td>$\rho_g^* = 0.9225$</td>
</tr>
<tr>
<td></td>
<td>$\rho_y = -0.5955$</td>
<td>$\rho_y^* = -0.0445$</td>
</tr>
<tr>
<td></td>
<td>$\rho_\pi = -1.6366$</td>
<td>$\rho_\pi^* = 0.048$</td>
</tr>
</tbody>
</table>
Table 6. Stabilization performance of cooperative/non-cooperative regime with shocks

<table>
<thead>
<tr>
<th>Financial shock</th>
<th>Monetary shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Country 1:</strong></td>
<td></td>
</tr>
<tr>
<td>$y_1 : 1.02$</td>
<td>$\pi_1 : 1.02$</td>
</tr>
<tr>
<td>$\pi_1 : 1.02$</td>
<td></td>
</tr>
<tr>
<td><strong>Country 2:</strong></td>
<td></td>
</tr>
<tr>
<td>$y_2 : 1.06$</td>
<td>$\pi_2 : 1.05$</td>
</tr>
<tr>
<td>$\pi_2 : 1.07$</td>
<td></td>
</tr>
</tbody>
</table>

**Inflation and output differentials:**

$\bar{y}^{UM} : 0.98$  
$\bar{\pi}^{UM} : 0.98$

Note: The numerical values give the ratios between the standard deviation of the variable $x$ in the cooperative compared to the non-cooperative regime during the first 5 periods following the shock: $\sigma^x_{Coop} / \sigma^x_{NCoop}$, for $x \in \{y_1, y_2, \pi_1, \pi_2, \bar{y}^{UM}, \bar{\pi}^{UM}\}$.

Table 7. Expected social loss comparison for alternative policy-mixes

<table>
<thead>
<tr>
<th>Loss functions coefficients</th>
<th>$EL_S^{NCoop} / EL_S^{Coop}$ (whatever the monetary regime)</th>
<th>$EL_S^{C} / EL_S^{C+Div}$ (whatever the budgetary regime)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_S = \lambda_G$</td>
<td>$\lambda^G_Y = 1; \lambda^G_y = 1.5; \lambda^G_g = 0.5$</td>
<td>$EL_S^{NCoop} = 1.054 EL_S^{Coop}$</td>
</tr>
<tr>
<td>$\lambda_S = \lambda_G$</td>
<td>$\lambda^G_Y = 1; \lambda^G_y = 1; \lambda^G_g = 0.5$</td>
<td>$EL_S^{NCoop} = 1.039 EL_S^{Coop}$</td>
</tr>
<tr>
<td>$\lambda_S = \lambda_G$</td>
<td>$\lambda^G_Y = 1; \lambda^G_y = 1; \lambda^G_g = 0.1$</td>
<td>$EL_S^{NCoop} = 1.12 EL_S^{Coop}$</td>
</tr>
</tbody>
</table>

Note: $EL_S^{C} = $ expected social loss with a centralized monetary policy; $EL_S^{C+Div} = $ expected social loss with monetary policy based on national information; $EL_S^{NCoop} = $ expected social loss in a non-cooperative budgetary regime; $EL_S^{Coop} = $ expected social loss in a cooperative budgetary regime.
Figure 1. Macroeconomic divergences with common vs national monetary policies

**Common monetary policy**
- Country 2: weak bank capital channel
- Country 1: high bank capital channel

**National monetary policy**
- Country 2: weak bank capital channel
- Country 1: high bank capital channel
Figure 2. National responses to a restrictive monetary shock \( (for \lambda_\pi = 1; \lambda_y = 1.5; \lambda_g = 0.5) \)

- **Output**: Cooperative budgetary regime -17.31, Non-cooperative budgetary regime -17.16
- **Inflation**: Cooperative budgetary regime -13.9, Non-cooperative budgetary regime -13.8

Country 1

Country 2