Partial Credibility, Information Selection and the Signalling Channel of Sterilized Interventions

Silke Fabian Reeves
The George Washington University

Abstract

The signalling channel suggests that central banks use sterilized interventions in the foreign exchange market to convey information about future monetary policy to the market. To date, this theory is not sufficiently supported by theoretical work that establishes the link between intervention signals and exchange rates. This paper develops a two country model of sterilized interventions. I argue that reputational effects cannot eliminate the credibility problem between central banks and the private sector and that agents will only partially use available information to form exchange rate expectations. Both partial credibility and non-rational expectations reduce the effectiveness of interventions. (JEL Classifications: E52, F31, F41)
I. Introduction

The signalling (or expectations-) channel suggests that sterilized interventions in the foreign exchange market can affect exchange rates by conveying information about future monetary policy. Thereby, sterilized interventions are official purchases (sales) of foreign exchange whose money market effects are immediately offset by open market sales (purchases) of domestic government securities of equal amount. According to the signalling channel, central bank purchases of foreign exchange indicate an expansionary future monetary policy consistent with the intervention. If successful, market participants revise their exchange rate expectations. This, in turn, is immediately reflected in the spot exchange rate.

Since Mussa’s [1981] initial article on this subject, research has advanced mostly along empirical lines (see Edison [1993] for a survey). On the one hand, these empirical studies confirm that sterilized interventions may have an informational role through which they can impact onto exchange rates (Dominguez [1992]; Dominguez and Frenkel [1990]; Jurgensen [1983]). Specifically, Dominguez [1990] and Kaminski and Lewis [1996] find that sterilized G-3 interventions in the US dollar market during the 1980s have caused a revision of exchange rate expectations. On the other hand, these studies also find considerable variability in the magnitude and persistence of exchange rate effects.

This paper turns to the theoretical foundations of the signalling channel. It develops a formal two-country model, where interventions are used by both nation’s central banks to signal future monetary policy intentions. Notice that signalling is only successful if interventions lead to a revision of exchange rate expectations. Firstly, this requires that interventions are observable and that the informational content of intervention signals is used in the market for forming exchange rate expectations. Signalling...
ary policy stance implied by interventions must be credible. This paper explicitly considers both of these issues.

Turning to the latter, the existence of a tradeoff between internal and external policy objectives may result in a credibility problem due to the potential incentives for the central bank to renege on monetary policy signals. In contrast to the repeated games literature, I argue here that reputational effects cannot completely eliminate this credibility problem between the central bank and the private sector. Instead, I suggest that credibility is only partial in the sense that the market attaches a probability less than one to the event that a nation’s central bank subsequently monetizes its interventions. This reduces the effectiveness of intervention operations. Regarding the former issue, evidence from both the economic and psychological sciences suggests that market participants will select only a fraction of daily information impinging on the foreign exchange market. This process of information selection is formally incorporated into the model in a theory of non-rational expectations. These also reduce the effectiveness of sterilized interventions to manipulate exchange rates. Partial credibility and non-rational expectations may thus explain the differing experiences of countries and time periods observed in the real world.

The purpose of this paper is thus twofold: First, it provides a potential explanation for some of the questions raised by the empirical findings. Secondly, it develops a complete two-country model which may then be used to address the question of optimal exchange rate management and intervention policies. To my knowledge, the author’s dissertation (Fabian [1993]) represents the only such framework, which explicitly models the process of expectation formation and the macroeconomic effects of intervention policies. The present paper is based on this work. An accompanying paper (Reeves [1997a]) discusses the application of this theoretical framework to
credibility is incorporated into the analysis. The model is further modified in Section V which considers non-rational exchange rate expectations. Section VI concludes with a summary.

II. The Two-Country Model

The two open economies are described by the flexible price monetary model (see, for instance, Frenkel [1978]; Stevenson, Muscatelli and Gregory [1988], pp. 265-74):

\[
M_t - P_t = \alpha_1 Y_t - \alpha_2 r_t \tag{1}
\]

\[
M_t^* - P_t^* = \alpha_1 Y_t^* - \alpha_2 r_t^* \tag{1}'
\]

\[
r_t = r_t^* + S_{t+1}^e - S_t \tag{2}
\]

\[
P_t = P_t^* + S_t \tag{3}
\]

\(M_t\) is the money supply, \(P_t\) the price level, \(Y\) exogenous and constant real income and \(r_t\) the real interest rate of the home country in period \(t\). Starred variables refer to the foreign country. \(S_t\) denotes the current nominal exchange rate expressed as the home currency price for one unit of foreign currency. The exchange rate is flexible, and \(S_{t+1}^e\) is its expected future value. All variables, except interest rates, are expressed in logarithms. It is assumed that each country’s residents only hold their own respective currency, and that interest and income elasticities in the standard money demand functions (1) and (1)' are identical. Thus exchange rate effects will not be due to country differences. Uncovered interest parity given by equation (2) implies that domestic and foreign assets are perfect substitutes. This rules out a portfolio balance effect.\(^2\) Turning to (3), the assumption of purchasing power parity throughout assures an immediate price adjustment
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limited to signals of future monetary policy. In particular, sterilized interventions have no real effects.

If expectations of the future exchange rate are formed rationally,

\[ S_t = E_t S_{t+1}, \]  

where \( E_t X_{t+k} \) is the k-period-ahead mathematical expectation of the random variable \( X \) based on all information available at time \( t \), including the past history of all relevant variables and the model structure. Ruling out the presence of bubbles, the forward-looking solution for \( S_t \) is given by

\[ S_t = \frac{1}{1+2} \sum_{k=0}^{\infty} \left( \frac{2}{1+2} \right)^k E_t (M_{t+k} - M_{t+k}^*) - 1(Y - Y^*). \]

This highlights the role of expected future monetary policies for the determination of exchange rates. Let agents initially have static expectations. Thus the money supplies are expected to be permanently fixed at \( M_0 \) and \( M_0^* \), and the (initial) equilibrium in period \( t = 0 \) is

\[ S_0 = (M_0 - M_0^*) + 1(Y^* - Y). \]  

III. Perfect Credibility and Rational Expectations

Assume now that the monetary authorities can perfectly monitor money supplies in each period. The key idea for the signalling model developed here is that central banks use sterilized interventions in period 1 of volumes \( I \) and \( I^* \) to signal changes in monetary policy for period 2. Thereby, positive (negative) values of \( I \) and \( I^* \) indicate purchases (sales) of foreign exchange. Formally, I capture the monetary policy signals embodied in interventions as an announcement of permanent changes in the money supplies to
policy change will be carried out with certainty.\(^5\) Anticipating policy changes, market participants revise their expectations about future money growth. Thus, the current exchange rate responds immediately by incorporating the new policy stance. Note that

\[
S_2 = (M_2 - M_2^*) + \gamma (Y^* - Y),
\]

if \(E_2M_{2+k} = M_2\) and \(E_2M_{2+k}^* = M_2^*\) for all \(k > 0\).

Using (5) and (6), the expected long run exchange rate is given by

\[
E_1S_2 = S_0 + \gamma (1 - I^*). \tag{7}
\]

Using (1) - (3) and (7) yields the spot exchange rate (short run equilibrium) as

\[
S_1 = S_0 + \frac{2}{1+2} (1 - I^*).
\]

Summarizing, even though no “fundamentals” are changed in period \(t = 1\), sterilized interventions cause an immediate jump in the short run exchange rate. This is exclusively due to the revision of exchange rate expectations induced by the information about future monetary policy change. For instance, purchases of the foreign currency by both central banks (i.e. \(I > 0\) and \(I^* < 0\)) immediately result in a depreciation of the home currency. This result has also been obtained by studies of announcement effects in rational expectations models of exchange rate determination (Edwards [1983]; Engel and Frankel [1984]). Note that the short run exchange rate moves in the same direction as the long run rate but falls short of the entire long run adjustment. Price flexibility in every period dampens the exchange rate movement.\(^6\) Within the economic constraints postulated by equations (1) – (4), however, the exchange rate effects achieved by any given volume of interventions are maximized.
This depends on the deterministic nature of the model studied so far: The commitment technology makes intervention signals perfectly credible. The entire certain sequence of future money supplies is determined and known in period 1. Signals and future monetary policy are inextricably linked. Section IV incorporates credibility problems into the analysis. Section V extends the model to take account of uncertainty. Then even a perfectly credible signal may have only limited impact on the exchange rate.

IV. Credibility and Central Bank Reputation

A. Partial Credibility

In reality, a commitment technology will not be available to central banks. In the strategic game that central banks play against the private sector, the monetary authorities could renege on intervention signals, if there exist incentives to do so. This will be anticipated by rational market participants. Consider the case where the home central bank simultaneously pursues an exchange rate target, $S_\ast$, and a money supply target, $M_\ast$. Without loss of generality (wlog), let $M_\ast = M_0$. Thus the monetary policy change associated with interventions may be costly. Reputational problems exist as agents perfectly understand the central bank’s temptation not to change the money supply in period 2: $E_1 M_2 = M_0$. Thus, no intervention signal is credible.

Repeated interactions with the public could, however, allow central banks to build a reputation for forthright policy making. Thus, at any point in time, central banks will face a reputation determined by the past history of intervention policy. Let $p$ and $p^\ast$ denote the probabilities which market participants attach to the events that the domestic and foreign central banks, respectively, implement the monetary policy implied by their intervention
Thereby, $R$ and $R^*$ denote the events of the home and foreign central banks reneging on intervention signals. $R_-$ and $R_+^*$ are their complements and indicate forthright policies. $E_t(X | Y)$ is the time $t$ conditional expectation of variable $X$ given $Y$.

Taking expectations in (6) and using (9) yields

$$E_t S_2 = (M_0 + 1p) - (M_0^* + 1^* p^*) + 1(Y - Y).$$

Together with (1) - (3) and (5) this gives the period 1 exchange rate

$$S_1 = S_0 + \frac{2}{1+2} \{pI - p^*I^*\}.$$

Thus partial credibility reduces the effectiveness of sterilized interventions. The following sections focus on the determinants of central bank credibility.

B. The Repeated Signalling Game

To determine the size of $p$ (similarly $p^*$), consider the following policy problem for the home central bank:

$$\min_{M_2} (S_2 - S)^2 + (1 - ) (M_2 - M^2), 0 < < 1$$

where $S_1 = h(E_1 M_2)$, $M_\leq = M_0$, and $h$ is a real-valued function. This problem exemplifies a typical tradeoff between internal and external policy objectives. Interpreting expectations $E_1 M_2$ as the private sector’s “strategy”, let a solution to the above game be denoted by a strategy pair $(M_2, E_1 M_2)$.

In a commitment regime the solution is given by $(\hat{M}_2, \hat{M}_2)$, where $\hat{M}_2$ denotes the money supply that minimizes (10). If central bank objectives are unknown to the private sector, the central bank may make $\hat{M}_2$ known through interventions in the foreign exchange market. It is this informational asymmetry that gives rise to the signalling effect. In the absence of a commitment technology, monetary policy is set discretionarily, and the pri-
ivate sector expects $E_1 M_2 = M_0 = M_\bot$, regardless of central bank interventions (or announcements) in period 1. Thus, in the one shot game, monetary authorities have no means of affecting the current exchange rate, because there exists no time-consistent future monetary policy except for $M_2 = M_\bot$.

Games of this type have been studied in the macroeconomics literature (Barro and Gordon [1983]; see Pearsson and Tabellini [1990] and Rogoff [1987] for surveys). Reputational models show that the existence of some prior beliefs about the central bank being a commitment type, who always honors policy signals, together with assumptions pertaining to the duration of play and the policymaker’s discount rate establish $(\hat{M}_2, \hat{M}_2)$ as the equilibrium of the repeated two-period stage game given by (10) (Backus and Drif- fill [1985], Kreps and Wilson [1982], and Milgrom and Roberts [1982]). The key idea is that the monetary authorities obtain credibility by mimicking the commitment type. Once the market observes intervention operations to be subsequently monetized, this represents (conclusive) evidence for forthright policy making in the future. The private sector chooses the pure strategy $M_2$, and intervention signals are perfectly credible with $p=1$, where time subscripts are omitted for convenience. This eliminates the credibility problem. Notice, however, that perfect credibility cannot explain the use of sterilized interventions. Mere policy announcements would be sufficient.

Instead I argue here that the credibility of policy signals is only partial with $p, p^* < 1$. With partial credibility sterilized interventions may be used to enhance the (imperfect) credibility of policy statements. This function of the intervention instrument has been suggested by Obstfeld [1990]. Formally, if there exists more than one alternative type who is forthright with some positive probability, then partial credibility, rather than perfect credibility, is the outcome of the repeated policy game (Reeves [1997b]).

Despite forthright
central bank intentions and policies, the credibility problem between the central bank and the private sector is not eliminated. The repeated three-player game with partial credibility is modeled in Appendix 2.

Other arguments in support of $p$ and $p^*$ being strictly less than one are the potential existence of random economic shocks as well as coordination failures. Both of these factors may cause a deviation from policies $(\hat{M}_2, \hat{M}_2)$. In the former case, central bank policy intentions are unobservable ex post, and the public can thus not distinguish between forthright and reneging policies (Fudenberg and Levine [1987]). In the latter case, the multiplicity of potential equilibria in the repeated game creates strategic uncertainty (Rogoff [1987]). This may, in particular, lead the market to not anticipate forthright intervention- and monetary policies and the reputational equilibrium.

C. Further Determinants of Credibility

Beyond the reputational factors discussed within the previous section and modeled in Appendix 2, additional determinants of credibility exist. These depend on (i) the individual central bank’s loss functions and (ii) the potential repeated interaction between the central banks.

(i) Welfare Costs and Available Instruments. Interventions have a potential role of backing central bank statements and thus enhancing credibility, allowing them to differ from mere policy announcements. In this context, note that if the monetary authorities renegade on intervention signals, $S_2 = S_0$. These “costs of dishonesty” arise, since the exchange rate will return to its original level if the money supply is not changed in period 2. The resultant costs of policy $M_0$, $C_{M_0}$, will be higher, the larger the volume of interventions, $I$, and also the larger the announced policy change, i.e. the size of the signal, $Z$. Costs $C_{M_0}$ may be a further mechanism that lends credibility to
The costs incurred when carrying out monetary policy changes, $C_M$, are the foregone internal policy objectives. These will be higher, the larger the signal. Summarizing,

$$p = \tilde{f}[\text{Rep}, C_M(I, Z)],$$

where $\text{Rep}$ is the reputation determined by past play. This formulation requires that the central bank’s opportunity set includes the independent choice of $I$, $Z$, and $M_2$. We will not attempt to solve, nor specify, this complex decision problem here. Instead, let

$$p = \begin{cases} f(\text{Rep}) & \text{for } Z \leq I \\ 0 & \text{for } Z > I \end{cases}$$

with $f : \mathbb{R}^3 \to [0, 1]$. In addition let $\gamma$ be fixed exogenously. A (minimal) intervention volume of $I$ is thus required to ensure $p$ for the signal $\gamma I$. Notice that interventions have no role in enhancing credibility beyond $p$, but that mere announcements of policy change do also not suffice. This reduces the arsenal of instruments to the optimal policy setting of $Z = \gamma I$. Similarly for the foreign central bank.

(ii) Repeated Interactions of Central Banks. If central banks set intervention policies unilaterally, $p$ and $p^*$ are likely to be independent. However, with policy coordination and if central banks repeatedly interact, central bank reputations will be jointly determined in the three player game between both central banks and the private sector.

If central banks coordinate their intervention operations by jointly choos-

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12. Bordo and Schwartz [1991] raise the interesting question in how far other sterilized money market operations might also be used to signal future changes in the money supply. Foreign exchange interventions may be superior to any other operation which changes the composition of central bank assets, due to the enhanced credibili-
ing intervention volumes as well as the money supplies implied by the signals of these policies, several factors will affect the credibility parameters $p$ and $p^*$.

a) If the private sector continues to form expectations about the domestic (foreign) money supply based solely on observed behavior of the home (foreign) central bank, both central bank’s reputations may continue to differ. However, for each central bank the costs of reneging on intervention signals increase if an international reputation is at stake as well. Not following through with intervention signals breaks the commitment vis-a-vis the private sector and the international cooperative agreement. As reputational costs rise with central bank cooperation, credibility $p$ and $p^*$ is enhanced.

b) Notice that exchange rate effects only depend on the expected changes in relative money supplies. It is thus conceivable that market participants view policy coordination as a regime shift, regarding the intervening coalition of central banks as one player with one level of credibility $p = p = p^*$ in (9) such that

$$S_1 = S_0 + \frac{2}{1 + \bar{p}(1 - l^*)}.$$  

Credibility $p_\text{rises}$ if the private sector observes $\Delta M_2 - \Delta M_2^* = \gamma(l - l^*)$. If $|p(l - l^*)| > |p - p^*|$, credibility is improved, and cooperation increases the effectiveness of sterilized interventions. Notice that this allows cooperating nations to distribute intervention obligations independently from future money supply changes.

Insofar as these distributions are not publicly announced, this creates incentives for central banks to rely on the other country’s monetary policy and free-ride on its reputation (see Dominguez [1990], p. 128). This effect tends to lower credibility. If however, a central bank is able to enforce forth-
V. Model Uncertainty and Information Selection

Notice that (9) also implies that, contingent on a central bank being forthright, intervention signals translate completely into changes in exchange rate expectations. Practical intervention experience is, however, characterized by a much lesser degree of predictability: Even if a central bank has credibility, interventions are not always successful. And despite identical economic circumstances (including reputation), interventions are at times more effective than at others.

These findings cannot be explained by the credibility issues introduced so far. In the model above, any credible intervention signal conveys a conclusive message about the future money supply. Given credibility, the exchange rate can be perfectly controlled by the monetary authorities. This is known to market participants and reflected in (conditional) expectations (8). This section now considers uncertainty in the sense that the future exchange rate cannot be inferred from true central bank signals. Real world experience may then be explained by the non-rationality of exchange rate expectations.

A. The Evidence on Rational Expectations

There is by now extensive econometric evidence against the rationality of exchange rate expectations (for a survey see Lewis [1995]). Recent studies have used survey data to directly test the conditions for rationality, namely that all information is used (orthogonality) and that forecasts are unbiased. While the evidence for unbiasedness is ambiguous, most authors reject orthogonality (Cavaglia, Verschoor and Wolff [1993]; Ito [1990]; see Takagi [1991] for a survey).16
Decisionmakers are neither able to acquire complete knowledge of the true model nor to translate the flood of informative signals into objective probability estimates for all states of the world. The situation is one of true uncertainty, where probability estimates are subjective and thus heterogeneous among individuals.

The overall evidence thus suggests that agents make only partial use of the available information. The information which is used appears to generate unbiased, efficient predictions. The next section incorporates this into the intervention model.

**B. Exchange Rate Expectations**

Uncertainty may now formally be introduced into the signalling model. The particular form of uncertainty chosen here is the randomness of the money supply. Instead of the money supply, central bank interventions contain information about the future monetary base. This is motivated by the fact that the monetary base contains only items of the central bank’s balance sheet and is thus much nearer to controllability than any other monetary aggregate. Model uncertainty is captured by the assumption that the structural relationships determining the money supplies and the corresponding probability distributions are unknown. Even though correlated, there is no one-to-one relationship between the monetary base (policy instrument) and the money supply. When observing foreign exchange interventions, traders assess the credibility of the implied monetary base signal.

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18. Other forms of uncertainty are possible as well. But choosing a “minimal amount” of uncertainty allows us to derive strong results by retaining most of the economic relations of the model.
They then revise their exchange rate expectations contingent on forthright central bank signals. Consider first the situation of perfect credibility. In this case, intervention signals by the home and foreign monetary authorities represent conclusive messages about the future domestic and foreign monetary bases, respectively. But the direct link of these instruments and the exchange rate is broken. Conditional on forthright central bank policies, expectations may be represented as

$$E_t(X) = g(\Omega_t)$$

where $\Omega_t$ is the information set publicly available in period $t$. $E_t(X)$ continues to denote the period $t$ expectation of variable $X$, but no longer represents the mathematical expectations operator. The function $g$ incorporates the cognitive processes of information selection by weighting initial beliefs by $(1 - \theta)$, new information by $\theta$:

$$E_t(S_2 | \bar{R}, \bar{R}^*) = (1 - \theta)E_0S_1 + \theta(S_0 + (1 - I^*))$$

(11)

Following Festinger [1970] and Katona [1975, 1980], the parameter $\theta$ and thus the effectiveness of intervention signals will be lower, the less overall uncertainty prevails and the more confident prior market beliefs. Also, $\theta$ will be smaller, the more consistent and thus informative previously received news items, and the stronger the resistance to unfitting signals. This may be viewed as the analogue to statistical decision theory’s distinction between “hard” and “soft” prior beliefs. This is confirmed by actual intervention experience. For instance, the German Bundesbank evaluates the “firmness” of exchange rate expectations prior to intervening in the foreign exchange market. It has been found that interventions have only limited chances for success if market confidence (in beliefs) is high.20 As long as $\theta < 1$, expectations will fail to be orthogonal. Thus agents only partially use available information.21 Moreover, if information about $R_2$ and $R_2^*$ is used, beliefs are
the non-rationality of expectations excludes a complete assessment of probability distributions over competing models of the economy. Consequently, the process governing the revision of beliefs cannot be determined from the model and thus represents an exogenous assumption. The formulation given by (11) is most natural to the previous model structure. It is consistent with the expectations about future money supplies

\[ E_1(M_2 | \bar{R}) = (M_0 + I) + (1 - \gamma)M_0, \]

\[ E_1(M_2 | \bar{R}^*) = (M_0^* + I^*) + (1 - \gamma)M_0^* \]

(12)

where, again, new information is weighted by \( \theta \). Using (6) gives

\[ E_1(S_2 | \bar{R}, \bar{R}^*) = \left\{ E_1(M_2 | \bar{R}) - E_1(M_2 | \bar{R}^*) \right\} + (1 - \gamma)(Y - Y^*). \]

(13)

With \( E_0S_1 = S_0 \) and combining (5) and (12), \( E_1(S_2|\bar{R}, \bar{R}^*) = S_0 + \theta \gamma (1 - I^*) \), and equation (11) follows. Expectations thus fully exploit the limited model structure of equations (1)-(3). Given the subjective money supply estimates (12), the model is solved by using mathematical expectations, and no additional bias is introduced. In the terminology of Persson and Svensson [1983, p. 2], this may be called a “weak form of rational expectations.”

Considering now the general case where one or both central banks may renege on intervention signals gives period 2’s expected exchange rate as

\[ E_1S_2 = p^*E_1(S_2 | \bar{R}, \bar{R}^*) + (1 - p^*)E_1(S_2 | \bar{R}, \bar{R}^*) \]

\[ +(1 - p)p^*E_1(S_2 | R, \bar{R}^*) + (1 - p)(1 - p^*)E_1(S_2 | R, R^*), \]

where

\[ E_1(S_2|\bar{R}, \bar{R}^*) = \theta(S_0 + \gamma I) + (1 - \theta)S_0, E_1(S_2|R, \bar{R}^*) = \theta(S_0 - \gamma I^*) + (1 - \theta)S_0 \]

and \( E_1(S_2|R, R^*) = S_0 \). Thus, \( E_1S_2 = S_0 + \theta \gamma \{ pI - p^*I^* \} \). The spot exchange rate is given by
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Information will equally limit the success of both nation’s intervention operations.

Notice that incompatible interventions, for instance matching purchases and sales of the foreign currency by both central banks (i.e. $I = I^*$), will not be successful: while both central banks spend intervention resources and encounter a potential conflict with the attainment of their internal policy objective in the future, only the central bank with the higher reputation will achieve any gains in its exchange rate target. With $p = p^*$, there is no impact on the exchange rate as agents anticipate that relative money supplies will remain unchanged. Thus, a role of international policy coordination, then, may be to agree on compatible exchange rate targets. More generally, the reduced form equation (14) may be applied to the study of unilateral and cooperative intervention policies (see Fabian [1993]; Reeves [1997a]).

VI. Conclusions

This paper has developed a two-country model where sterilized interventions by both central banks may signal future changes in monetary policy. The analysis has incorporated two key factors that limit the ability of sterilized interventions in affecting the spot exchange rate: partial credibility and non-rational expectations. First, the potential tradeoff between internal and external policy objectives faced by the monetary authorities introduces a credibility problem between each nation’s central bank and the private sector which lends only partial credibility to intervention signals. Secondly, empirical evidence indicates that agents do not make use of all publicly available information. The cognitive processes of information selection have been formally incorporated into a theory of non-rational expectations. The next step would be an empirical testing of the hypotheses underlying the
References


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Appendix

A. Presence of a Risk Premium

If uncovered interest parity does not hold, (2) and (4) are replaced by
\[ r_t = r_t^* + E_t S_{t+1} - S_t = R P_t, \]
where \( R P_t > 0 \) denotes the risk premium on foreign-currency denominated assets. Define \( S_0' = (M_0 - M_0^*) + \alpha_2(Y^* - Y) \). Then with no expectations of policy changes, the initial equilibrium exchange rate is given by
\[ S_0 = S_0' - \alpha_2 R P_0. \]

After observing central bank interventions, \( I \) and \( I^* \), the spot rate changes to
\[ S_1 = S_0 + \frac{2}{1+\gamma} (I - I^*) + \gamma (R P_1 + R P_0). \]

If there are sterilized domestic central bank purchases of foreign exchange, agents expect an expansion of the domestic money supply to \( M_2 = M_0 + \gamma I \) (signalling effect). Also, the stock of foreign, relative to domestic, assets held by the public falls (portfolio effect). The first effect causes an expected depreciation of the home currency, the latter is likely to lower the risk premium on foreign-currency denominated assets. Comparing the solution for \( S_1 \) with the one obtained under uncovered interest parity on page 113 shows that the risk premium effect reinforces the signalling effect, in leading to a larger depreciation of the domestic currency on the spot market.

B. The Repeated Game and Partial Credibility

This appendix models the repeated policy game and shows that the credibility of intervention signals is only partial. One simple extension of the
\[ r_{1t} = r_{1t}^* + E_{1t} S_{2t} - S_{1t} \]  \hspace{2cm} (A2)

\[ r_{2t} = r_{2t}^* + E_{2t} S_{1,t-1} - S_{2t} \]  \hspace{2cm} (A2')

\[ P_{kt} = \pi_{kt}^* + S_{kt} \]  \hspace{2cm} (A3)

\[ S_{0t} = S_{2,t-1} + \nu_{t} \]  \hspace{2cm} (A4)

The time subscript \( t \) now refers to a two-period interval. Random shocks \( \nu_{t} \) are white noise. Their presence ensures a need for intervention policies at every \( t \). Notice that every second period in each time interval remains a long-run equilibrium. Thus, the current spot rate, \( S_{1t} \), is determined by \( S_{0t} \), interventions \( I_{t} \) and \( I_{t}^* \) and credibility \( \pi_{t} \) and \( \pi_{t}^* \) only, not by expected future monetary- and intervention policies in periods \( j > t \).

The home central bank minimizes the discounted present value of each period's loss function

\[ L = \sum_{t=1}^{T} t^{-1} L_{t} \text{ with } L_{t} = (S_{1t} - \bar{S})^2 + (1 - \bar{S}) (M_{2t} - \bar{M}_{t})^2, \]  \hspace{2cm} (A7)

with a money supply target \( M_{t} = M_{0t} = M_{2,t-1} \) and where \( T \) denotes the duration of play. \( T \) may be infinite. Similarly the foreign central bank with monetary target \( M_{t}^* = M_{2,t-1} \). Let both monetary authority's true types be private information to the central banks, and let the private sector assess some positive probabilities \( \pi_{0}(T_{C}) \) and \( \pi_{0}^*(T_{C}) \) for the central banks being a commitment type \( (T_{C}) \), who always sets \( M_{2t} = M_{0t} + \gamma I_{t} \) or \( M_{2t}^* = M_{0t}^* + \gamma I_{t}^* \), respectively. Let there also be positive prior beliefs \( \pi_{0}(T_{R}) \) and \( \pi_{0}^*(T_{R}) \) for a randomizing type who is forthright only with probability \( q \) such that \( M_{2t} = M_{0t} + q \gamma I_{t} \) and \( M_{2t}^* = M_{0t}^* + q \gamma I_{t}^* \). The market believes that domestic central bank behavior is governed by (A7) with probability \( \pi_{0}(T_{0}) = 1 - \pi_{0}(T_{R}) - \pi_{0}(T_{C}). \)  \hspace{2cm} (23) Similarly, \( \pi_{0}^*(T_{0}) \).

In each period \( t \geq 1 \), after observing both interventions and subsequent
\( p(T_C) = \text{Prob}(T_C | M_{2t}) \)

\[
= \frac{p_{-1}(T_C)\text{Prob}(M_{2t} | T_C)}{p_{-1}(T_C)\text{Prob}(M_{2t} | T_C) + p_{-1}(T_0)\text{Prob}(M_{2t} | T_0) + p_{-1}(T_R)q}
\]

\( \text{Prob}(\cdot) \) denotes a probability. The right hand side uses the market’s prior beliefs, \( p_{-1}(\cdot) \), about the home central bank’s type. These are last period’s posterior beliefs. Similarly, beliefs are updated for the other types and for the foreign central bank.

If the discount factors are not too large, and the duration of play is long enough, central banks will value a reputation and the increased effectiveness of sterilized interventions associated with it, choosing a forthright policy.\(^{24}\) Then, \( M_{2t} = M_{0t} + \gamma I_t \) and \( M_{2t}^* = M_{0t}^* + \gamma I_t^* \). Posterior beliefs are thus given as

\[ p(T_C) = \text{Prob}(T_C | M_{2t} = M_{0t} + I_t) \]

\[
= \frac{p_{-1}(T_C)}{p_{-1}(T_C) + p_{-1}(T_0)\text{Prob}(M_{2t} = M_{0t} + I_t | T_0) + p_{-1}(T_R)q}.
\]

Notice that credibility here depends only on whether the monetary policy changes implied by interventions have been carried out, but not on the size of the intervention volume. Depending on the particular informational asymmetry and game structure assumed, \( p_t \) and \( p_t^* \) could also depend on intervention volumes (see Section IV. C).

Central bank reputations may be derived as

\[ p = \text{Prob}(M_{2t} = M_{0t} + I_t) \]

\[ = p_{-1}(T_C) + p_{-1}(T_0)\text{Prob}(M_{2t} = M_{0t} + I_t | T_0) + p_{-1}(T_R)q < 1. \]

If the value of reputation is common knowledge, \( \text{Prob}(M_{2t} = M_{0t} + \gamma I_t | T_0) = 1 \). Otherwise, \( p(T_C) = 0 \) so that credibility rises with forthright policy making...