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South Africa's Public Debt: Long-term Dependence, Structural Breaks and Multifractality

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Abstract This paper aims to analyse the evolution of public debt in South Africa using new and original methods. The case of South Africa has been little studied in the literature on debt because the level of debt in this country remains reasonable. Moreover, the use of non-standard methods allows for a fine-grained analysis of the public debt time series and, consequently, to draw unprecedented conclusions. Using the Multifractal Detrended Fluctuation Analysis (MF-DFA) method borrowed from solid state physics and medicine, we conclude that South Africa's debt has a multifractal character, which originates from the long memory effect. Thus, South Africa's public debt is unsustainable. The inefficiency of this market has been particularly exacerbated by the various shocks throughout the sample period.

Keywords: public debt, long memory effect, multifractality, South Africa, MF-DFA, ARFIMA

JEL Classifications: C12, C22, C65, F34, H63, N17

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I. Introduction

The Covid-19 crisis, like other previous economic crises, shows us in no uncertain terms the importance of sound management of public finances. The current crisis has shown that "well managed" countries were able to cope with the pandemic more or less easily, using the leverage of public spending. Other, less well-equipped countries have seen their debt levels soar. Obviously, good management of public funds is not only justified by the likelihood of crises. The threat of a possible default due to deteriorating public finances can exacerbate economic, political and social tensions. Recent work by Reinhart and Rogoff (2010) has shown that high levels of debt can dampen economic growth, by directing resources towards debt servicing rather than productive investment. Explosive debt behaviour can lead to short-term

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difficulties, but it can also undermine future prosperity goals. The term explosion refers to a sudden, even abrupt, build-up of public debt. The level of debt would move away from the fundamental value of the debt. The aim is to question the sustainability of the fiscal policy criticised during the 1982 debt crisis. A sustainability of fiscal policy that is examined through budget deficits or debt (see, among others, Chortareas et al., 2008; Westerlund and Prohl, 2010). This situation becomes critical when interest rates rise and/or economic growth falls. This observation has led some researchers (Panizza and Presbitero, 2014) to analyse the causes and consequences of the increase in public debt, while others have instead tried to answer the question of the optimality of the public debt rate (Egert, 2015).

In general, it can be said that African countries, with a few exceptions, are experiencing a vertiginous rise in their public debt despite the debt relief programmes (HIPC and MDRI)[1] initiated by certain international organisations. This context of economic vulnerability is exacerbated by a rise in public debt, which is faster than tax and export revenues. This public debt dynamic is explained by total expenditure, the dynamics of interest rates and tax revenues, but above all by the gap between the interest rate and economic growth (Favero and Francesco, 2007).

One country stands out in this observation, namely South Africa. Our attention is particularly drawn to the economic situation of this country which, in the recent past, appeared to be an undisputed leader on the African continent. However, over the past ten years, the country has lost competitiveness mainly, but not exclusively, due to the evolution of commodity prices, rising wages in the mining sector and repeated strikes. Standard and Poor's downgraded South Africa's creditworthiness in 2017 as the credit rating was changed to Junk Status. While South Africa does not have a public debt situation comparable to other African countries, it remains vulnerable in many ways. In the aftermath of the 2007-08 global financial crisis, questions have been raised about the sustainability of public finances in general, and public debt stability in particular.

This resulted in an unprecedented economic recession in 2009. The recession resurfaced at the end of 2016, resulting in a sharp rise in public debt. It should be noted, however, that the analysis of public debt trends is difficult to grasp, as cyclical and structural factors collide. In order to disentangle these factors, we have chosen, in this article, to deliberately focus on the study of the debt time series in order to understand its behaviour and explain its trajectory.

The careful study of time series, such as debt or other macroeconomic variables, is more than necessary, as they contain hidden information that only "fine" methods can reveal. A thorough analysis of the time series would allow the detection and analysis of periods of crisis and debt explosiveness, but also and above all to detect the possible existence of fractality, non-stationarity and long memory in a context of instability.

It should be remembered from the outset that one of the important issues in the study of

1) The Heavily Indebted Poor Countries (HIPC) initiative and the Multilateral Debt Relief Initiative (MDRI).

time series is stationarity. The stationarity properties of the series will determine the type of modelling and the asymptotic properties of the test statistics. Beyond that, stationarity allows us to know whether the shocks on a given variable are permanent or transitory. This seems important for prediction, the ultimate goal of all modelling. In the same vein, the concept of long memory is important. It is materialised by the autocorrelation function which decays at a hyperbolic speed, much slower than that of the classical time series. This time series behaviour is difficult to implement by autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. With this in mind, autoregressive fractionally integrated moving average (ARFIMA) were introduced by Granger and Joyeux (1980), at the frontier of unit root models and models imposing an exponential decrease in autocorrelations. Fractional models allow for a better modelling of the long-term behaviour of the series and have the advantage of producing better long-term forecasts. However, they are dependent on the consequences of non-stationarity, which often leads to over-differentiation (Pérez and Ruiz, 2002). Conclusions may be erroneous when the stationarity assumption is removed.

In the continuation of our analysis, we propose a new method that takes into account the existence of a non-linear process, capable of detecting long-term correlations (Kantelhardt et al., 2002). The MF-DFA (Multifractal Detrended Fluctuation Analysis) method is a generalised multi-fractality method capable of measuring not only long-term correlation, but also multifractal characteristics as well as skewness (Bouoiyour et al., 2018; Bariviera et al., 2017; Alvarez-Ramirez et al., 2018). This method quantifies the multiple scaling exponents within a time series and assesses the short and long-term dependence. In the present case, this new and much more robust method outperforms the ARFIMA model which may confuse the long memory process with structural breaks (Diebold and Inoue, 2001; Granger and Hyung, 2004; Hsu and Kuan, 2000; Charfeddine and Guégan, 2006). It is also applied to verify the existence of an arduous or fragmented geometric shape representing a degree of self-similarity within the fractional dimensions of the series (Mandelbrot, 1982).

This article takes a fresh look at the dynamics of public debt by exploring its multifractal properties. Our contribution to literature is doubles. First, through the recursive and MF-DFA approach, we are moving away from standard methods that fail to analyse in detail the complexity of public debt. To our knowledge, this is the first article to deal with the issue of the multifractality of public debt. The large fluctuations in public debt are due to the long-term correlation between small and large fluctuations. Thus, an increase in the level of debt is followed by an increase, while a decrease is also followed by a decrease. Beyond economic fluctuations, countercyclical policies in the face of the financial crisis or the one against inequality contribute to characterizing the long memory of debt and the structure of the economy. The multifractal character at the end of our analysis is proven with a persistent process. Second, we are tackling a country with relatively low but equally vulnerable debt levels. Indeed, despite a relatively low debt ratio, severe shocks such as Covid-19 and the 2008 financial crisis are likely to lead to market inefficiency and, by extension, public policy inefficiency. In sum, South Africa's public debt is unsustainable.

The remainder of this paper is structured as follows. Section 2 presents the South African context, in relation to the debt issue. Section 3 presents the methodology, while Section 4 presents the results. Section 5 concludes the paper, while providing recommendations.

II. South African Context

Developing countries, particularly those in sub-Saharan Africa, have been successful in reducing their debt levels. Some through debt relief programmes, while others through the implementation of public debt stabilisation policies. With a neoliberal agenda and the election that ended apartheid in 1994, South Africa attempted to stabilise its debt levels under the banner of prosperity and inclusion. However, the global financial crisis of 2007-2008 has raised concerns about the sustainability of public finances. These concerns have been reinforced by negative GDP per capita growth and limited fiscal space due to a growing debt burden of close to 50% of GDP. Unemployment levels and poverty rates continued to rise, while corruption and political uncertainty contributed to slowing productivity growth. South Africa's economic situation was not always so uncertain. In the 1960s, South Africa's economic growth rate was among the highest in the world (Mohr and Rogers, 1996).

The trend was reversed from the 1970s onwards with a continuous slowdown in economic growth. However, South Africa remains highly competitive regionally, if less so globally. It remains one of the most unequal economies in terms of income and wealth. Although South Africa has embarked on a process of stabilising public debt, the trajectory of public debt has increased significantly, reaching 54.6% of GDP in 2018. Since the political transition in 1994, South Africa's public debt level has been relatively sustainable with debt below 60% of GDP (Calitz et al., 2016). This reflects good fiscal measures focused on improving the primary balance in response to rising debt (Jooste et al., 2011).

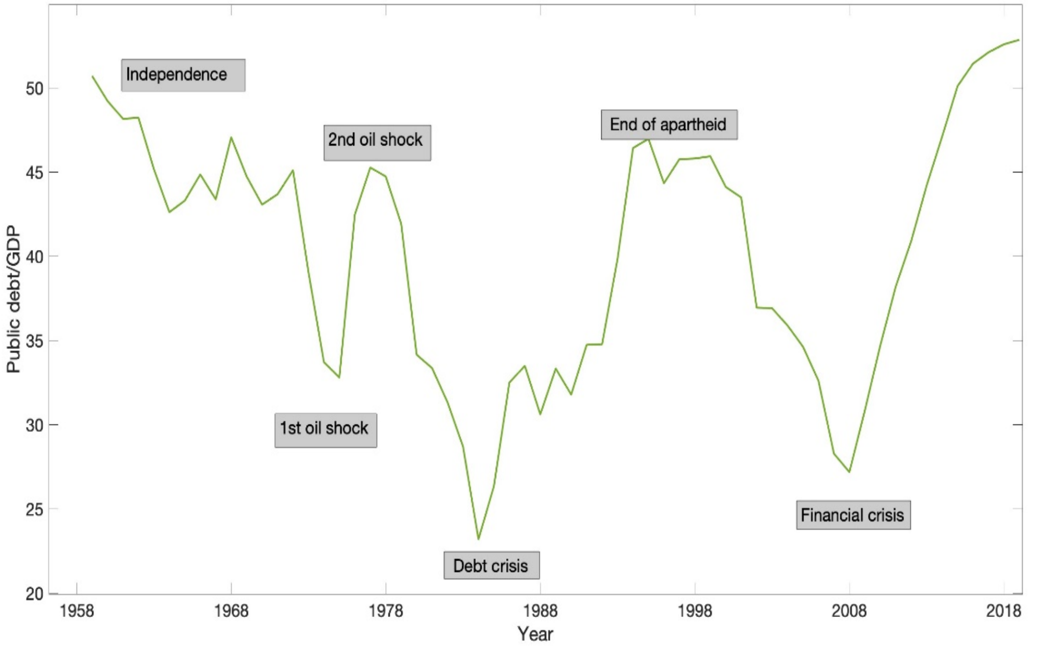
It is interesting to recall the events that led to the rise in debt (Figure 1). The advent of independence (1961) marked a break[2] with a relatively high level of debt. This was mainly due to the readjustment of public accounts which were indexed to those of the colonising country.

Following the first oil shock in 1973, the situation changed radically. Indeed, after this shock, the OPEC countries (Organisation of Petroleum Exporting Countries) lent heavily to all the countries of the South, and at low interest rates. This was the case for South Africa, which

2) South Africa had a centralised system before independence. The public accounts of the colonising country and the colony could be confused and inflate South Africa's debt levels.

was considered a creditworthy borrower because of its high export revenues. The country then saw its debt trend upwards, and in relatively large proportions with the second oil shock in 1979.

Figure 1. History of South African public debt



Already in debt, the government has attempted to rebalance its accounts by cutting social spending, as well as public investment spending. As a result of the oil counter-shock, the main feature of which is the sharp drop in the price of oil, the South African economy has seen significant improvements in its economic situation and significant reductions in its public deficits, a particularly interesting situation for this country which has seen a considerable drop in its level of indebtedness.

South Africa's cautious approach to debt in the early 1980s was intended to protect it from the debt crisis that rocked the world during that period. However, the combined effect of falling export prices and rising interest rates led to the largest debt crisis on the African continent in 1982. A general loss of confidence had set in, and private banks were the first to be affected by this phenomenon. The IMF (International Monetary Fund) and the World Bank agreed to provide new loans to repay the previous ones and avoid a succession of bank failures. In the case of South Africa, the debt burden really increased during this period, driven by the crisis and political decisions during the apartheid period.

The strengthening of democracy and the desire to stabilise the country, which was repeatedly shaken by ethnic fragmentation, led to an accumulation of debt and an exacerbation of deficits.

This was the price to pay for meeting social needs and tackling poverty and inequality.

This institutional instability has been compounded by instability in economic growth. The country grew by 3.3% in the 1970s, but this gradually declined in the 1980s to 2.2%, and then to just under 0.2% between 1990 and 1994. Interest on the public debt was then the largest budget item, not to mention inflation, which reached a record level of 14.6% (on average). This period was particularly marked by the build-up of apartheid debt obligations[3)], leading the government to take drastic initiatives to stabilise the economy[4)] and reduce the debt ratio. This was done in order to achieve independence from creditors and to improve the image of the South African economy facing international investors (Hamilton and Viegi, 2007). The aim was to reverse the trend by creating a fiscal environment characterised by responsible borrowing that would make South Africa more attractive to international investors.

It is clear that the apartheid period and the period of segregation blocked the structural transformation of the South African economy. However, they did facilitate the development of a system of cheap labour for the mines and farms. This has reinforced economic inequality, which is one of South Africa's major problems. It should be recalled that manufacturing remains the main contributor to GDP through the mining and energy sectors (Fine, 2018). At the end of apartheid, South Africa structured its economy to increase its creditworthiness, based on consolidated institutions of representative democracy. The government took strong action to further liberalise the economy with bold decisions on transparency, which resulted in reassuring markets. The country's ranking has been improved and interest rates have fallen significantly, resulting in a significant reduction in public debt.

This virtuous circle did not last long. The trend was reversed again by the global financial crisis of 2007-2008. It led to an increase in public debt due to the dramatic fall in public revenues, despite efforts to control them. The sharp decline in expenditure between 2005 and 2009 did not allow for a reduction in borrowing. Overall, political tensions, combined with the national and international recession, prevented the government from reducing spending. The economic transformation under the new democratic era mentioned earlier fell short of expectations. The prospects for economic growth remained unhappily unchanged. Tax revenues increased, but this did not prevent the government from borrowing to cover deficits. Income inequality remained virtually unchanged despite the attention of various governments.

Given the socio-economic situation, a political strategy based solely on the promotion of blacks to positions of high responsibility and on transfers of ownership of economic assets to black South Africans may not be sufficient and may ultimately undermine social and political stability. Figure 1 traces all of the above events in the South African context and the evolution of South Africa's public debt. Periods of falling or rising debt can be explained by the shocks

3) In 1993, the debt of the authorities and public enterprises amounted to 39.801 billion.

4) The objectives of fiscal policy were to reduce the budget deficit and improve the primary balance.

the country has experienced. South Africa's debt history is marked by relatively high levels of public debt mainly in 1976, 1994 and 2018. From 2008 onwards, debt levels have been on a worrying upward trend. It reached a high level comparable to that of 1994 and the 1970s.

III. Methodology

One of the contributions of our study lies in the innovative econometric method we use. We will propose a new method called the Multifractal Detrended Fluctuation Analysis (MF-DFA). The latter is used for the determination of the scaling properties of fractals as well as for the detection of the long memory effect in a noisy environment and non-stationary time series. In the appendix, we complete the analysis using standard econometric methods (stationarity, Fractional integration, etc.). The estimates of the integration parameter (d) suggest the existence of a fractional process while the recursive approach reveals explosive periods in the case of South African debt. Despite the quality of these results, these classical methods could suffer from biases resulting from overdifferentiation and confusion between long memory and structural breaks. In addition, so-called classical methods require a more in-depth preliminary analysis of long-term stationarity, which is almost absent from the literature (Syssoyeva-Masson and Andrade, 2017). The innovative MF-DFA method overrides these methods without imposing any preliminary analysis and allows for more robust results. This method was initially used by Peng et al. (1995) to check the power law characteristics in the study of heartbeats. It has proven to be an interesting method to quantify the scaling properties of the variability of cardiac parameters. Also, the multifractal analysis is in no way altered by shocks or non-stationarity. Detrended fluctuation analysis (DFA) has been used to predict survival in heart failure by distinguishing between normal heart failure and congestive heart failure. In addition to heart rate dynamics, it has been applied to DNA sequences, neural spiking, cloud structure, geology, ethnology, meteorology, solid state physics and more recently, time series in economics.

The MF-DFA procedure consists mainly of five steps. The first three are identical to the conventual DFA procedure.

Assuming x_i is a series of length N, the support is defined as the set of indices k with non-zero values of x_i.

The "Profile" of the series x_i is determined as follows:

$$y_k = \sum_{t=1}^{k} \left[x_i - \overline{x} \right], \qquad k = 1, \ldots, N. \tag{1}$$

The subtraction of the mean is not necessary, as it will be systematically eliminated in the third step of detrending. From the above definition, we have $y_k = 0$.

The "Profile" y_k is split into N_s = int (N/s) non-overlapping segments of the same length s.

$$F^2(s,v)=\frac{1}{s}\sum_{i=1}^{s}\{Y[(v-1)s+i]-y_v(i)\}^2 \tag{2}$$

where $y_v(i)$ is the best polynomial fit of Y [(v - 1) s + i] in each of the segments v of N_s. In the DFA procedure, the fluctuations $F^2(s,v)=(Y_{vs}-Y_{(v-1)s})^2$ do not consider the trend in the original series. In the MF-DFA, the trend in the time series is destroyed by subtracting $y_v(i)$. The polynomial of order m used in the adjustment procedure can eliminate the polynomial trend of order m - 1 in the original series. Fluctuation analysis is thus available for data affected by trends or stationarity hazards.

First the local trend for each of the $2N_s$ segments is calculated using the least squares adjustment method and then the variance.

$$F^2(s,v)=\frac{1}{s}\sum_{t=1}^{s}\{Y[N-(v-N_s)s+i]-y_v(i)\}^2 \tag{3}$$

for v = N_s + 1, ... , $2N_s$.

The fluctuation function is determined from the average of all segments.

$$F_q(s)=\left\{\frac{1}{2N_s}\sum_{v=1}^{2N_S}\left[F^2(s,v)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}}. \tag{4}$$

In the case where q=2, the standard DFA procedure is followed. The scaling behaviour of the fluctuation functions is estimated by analysing the log-log of $Fq(s)$ vs. S for different values of q.

$$F_q(s)\sim s^{h(q)} \tag{5}$$

where $h(q)$ is the generalized Hurst exponent. When $h(q)$ is constant for all q, the series is considered monofractal. Otherwise, it is multifractal.

Note: For large scales, $s > N/4$, $Fq(s)$ becomes statistically unreliable as the number of segments Ns becomes very small, especially when averaging in step 4. For this reason, scales $s > N/4$ are usually excluded from the adjustment procedure for determining $h(q)$. Furthermore, systematic deviations of the scaling behaviour in equation (5) that can be corrected, appear for very small values of the scales $s \approx 10$. In general, the exponent $h(q)$ in equation (5) can be related to q. For stationary series, h is identical to the well-known Hurst exponent H.

The singularity exponent α and the corresponding multifractal spectrum $f(\alpha)$ can be evaluated using the generalized Hurst exponent spectrum as follows:

$$\alpha = \tau\ (q) \tag{6}$$

where $\alpha = h(q) + \dfrac{dh(q)}{dq}$.

The spectrum function is defined as follows:

$$f\ (\alpha) = q\ [\alpha - h\ (q)] + 1 \tag{7}$$

The generalized Hurst exponent $h(q)$ obtained from the MF-DFA procedure is related to the Reyni exponent $\tau(q)$.

$$\tau\ (q) = qh\ (q) - 1 \tag{8}$$

The Reyni exponent spectrum is also used to distinguish multifractal from monofractal signals. It also gives the calibration of the general exponent.

IV. Results

Time series are generally affected by non-stationarity due to trend and noise in the series; and some irregularity in past data. We rely on Bank for International Settlements (BIS) data on public debt as a percentage of GDP for South Africa. These data cover the period from the last quarter of 1960 to the last quarter of 2018 following the work of Avdjiev et al. (2021); and Fisera et al. (2021).

Table 1. *Descriptive Statistics and Unit Root Tests*

Descriptive statistics							
Observations	Mean	Min	Max	Standard deviation	Skewness	Kurtosis	Jarque-Bera
233	42,80	27,5	59	7,46	-0,10	1,82	13,76***
Unit root tests		KPSS		ADF		PP	
Debt		0,52**		-1,35		-2,76	
△ Debt		0,41		-35,70**		-4,01**	

Note. ***, **, * significant at 1%, 5% and 10% respectively.

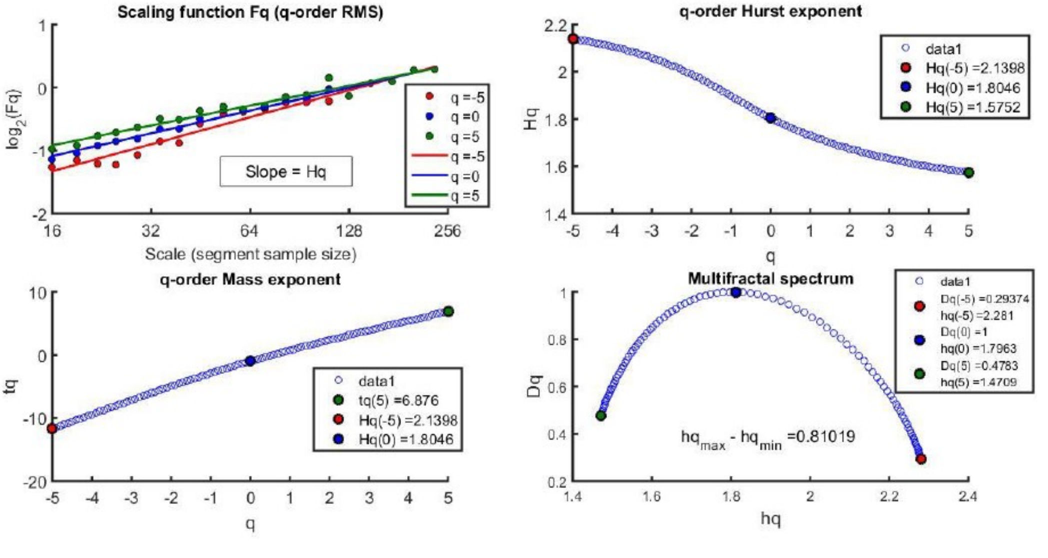
The descriptive statistics and unit root tests are presented above in Table 1. Public debt follows a leptokurtic distribution with the tail of the distribution spread to the left. This is consistent with the Jarque-Bera normality test which rejects the assumption of normality of the distribution. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests deal with a null hypothesis in favour of a unit root while the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test examines the null hypothesis of stationarity. The results of these tests provide evidence for the presence of unit roots. Overall, the ADF and PP tests do not reject the presence of a unit root while the KPSS test rejects stationarity.

The rejection of the stationarity hypothesis implies that a positive shock is permanent and will not be cancelled out on average. The differentiation of the series led to the absence of a unit root.

A. Multifractality result

We use a polynomial of degree three to calculate the best polynomial fit of the profile in each segment, which can eliminate the second-order polynomial trend in the original series (Kantelhardt et al., 2003; Zhao et al., 2017). Figure 2 (q-order RMS), top left, shows the fluctuation function of public debt based on the s-scale for various values of q. The public debt has experienced relatively large fluctuations. These fluctuations increase with the scale and reveal the existence of a long-run correlation of small and large fluctuations in public debt. The structure of the series reveals that it is not limited to a scale exponent over time. An increase in the level of debt is followed by an increase, while a decrease is also followed by a decrease.

Figure 2. Multifractal analysis using the MF-DFA method



In the figure (q-order Hurst exponent), top right, the Hurst exponents are presented as a function of time scales. The Hurst exponent plot is a downward sloping and somewhat non-linear with increasing values of q. The crossover point reveals a change in the properties of the series at different time or space scales. When q varies from -5 to 5, Hq decreases from 2.13 to 1.57. The lack of constancy leads to the conclusion of multifractality in the time series independently of their short and long term components. Moreover, H(q) is greater than 0.5 for all values of q. This finding reveals the presence of long-term memory in the public debt which gradually decreases with respect to the highest lags. We thus have a persistent process.

As shown in the q-order Mass exponent figure (bottom left), $\tau(q)$ follows an upward trajectory with respect to the values taken by q. The temporal structure then plays an important role in their multifractality. According to the Hurst exponent, a multifractal time series will fluctuate much more over time. The multifractal series by definition has a regular interval structure with a Hurst exponent greater than 1. The robustness of this assertion and the role of the time structure will be verified by the width and shape of the multifractal spectrum proportional to the time variations of the Hurst exponent.

The spectrum, in the figure Multifractal spectrum (bottom right), in the form of an inverted parabola, conventionally quantifies the degree of multifractality.[5] The larger the width of the spectrum, the more uneven the distribution of the debt series, and thus the stronger the multifractal character. The measure of the multifractal strength D(h) is obtained by the difference between the h(q) max and h(q) min as mentioned in the graph. The multifractal spectrum D(h), which varies with the order h(q) and t(q) which increases with q, can be strongly considered as a good indicator of multifractality. We conclude that the public debt significantly meets the multifractal characteristics. We conclude that there is a long and strong memory. This conclusion testifies to the ineffectiveness of public policies and market. This situation can be related to market imperfections such as illiquidity, the existence of risk or even some market speculation. On the other hand, the absence of memory would be synonymous with efficient behavior.

B. Source of multifractality

In this section, we investigate the source[6] of the multifractality of South Africa's public debt. The first source is the long memory effect of small and large fluctuations and the second is the wide tail of distributions (Matia, Ashkenazy and Stanley, 2003). There are two procedures for detecting sources of multifractality, namely Shuffling and Surrogate (Benbachir and El Alaoui, 2011).

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5) The degree of multifractality which is the width of the singularity spectrum represents the difference between the Max and Min probability.

6) This section also serves as a robustness test of the results of the previous section.

Shuffling consists of randomly swapping the series to weaken the long memory effect by performing the following steps:

- generate pairs (p, q) of random numbers (with p, q $\leq$ N) where N is the total length of the series to be shuffled;
- swap the inputs p and q;
- repeat the first two steps for 20 N times (this step ensures that the order of the series is switched completely).

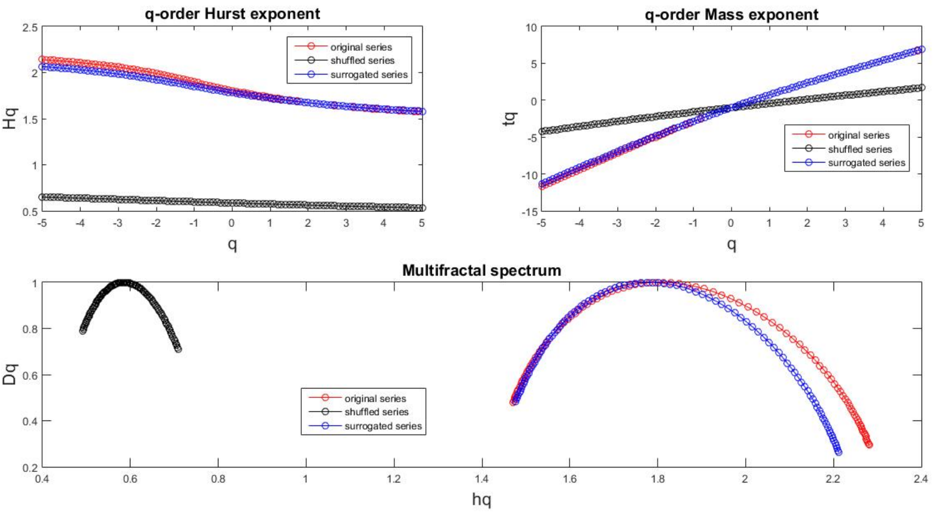
At the surrogate level, we want to weaken the leptokurtic distribution that may characterise the series. This involves randomising the Fourier phases of an original time series in order to destroy the non-linearities stored in the phases. To do this, we used the iterated amplitude adjusted Fourier transform (iAAFT) method.[7] We compare the results obtained with the original series. Figure 3 thus illustrates the original series, the shuffled series and the surrogate series. If the multifractality were caused by a long memory effect (a leptokurtic distribution), we would see a potential hurst exponent h(q) of the shuffled (surrogated) series independent of q. In our case, we find that the Hurst exponent is greater than 0.5 for both the surrogated and the shuffled series. The dependence of the long-term correlations has a larger contribution than that of the broad tail of the distribution, as shown in the figure (q-order Hurst exponent), top right.

The Reyni exponent in the figure (q-order Mass exponent), top right, shows the degree of nonlinearity. The surrogate series and the original series show the strongest non-linearity and thus the highest multifractality. The shuffled series shows the least non-linearity.

To better identify the contribution of each source of multifractality, we will look at the width of the multifractality spectrum of the original series, the shuffled series and the surrogated series. The singularity spectrum in the figure Multifractal spectrum (bottom of Figure 3), relating to the original series, the shuffled series and the surrogate series shows that the multifractality of the original series is the largest with a spectrum width of 0.81 while the shuffled series has a narrower width of 0.21. The spectrum width of the surrogate series is equal to 0.73. The long-run dependence and the leptokurticity phenomenon contribute to the determination of the multi-scale characteristics of public debt. The widths of the singularity spectrum provide information on the extension of the memory effect which is much more important than the leptokurticity effect in multifractality. The multifractality of debt is the product of the operating structure of the state and the short-term effect of the level of activity linked to the cycle. Policymakers should implement new reforms and regulations (i.e., prudential standards) to promote market efficiency and increase investor confidence.

7) The AAFT (amplitude adjusted Fourier transform) and STAP (Statically Transformed Autogressive Progress) algorithms also make it possible to maintain exactly the same distribution and address the surrogate problem.

Figure 3. Source of multifractality



V. Conclusion

This paper investigates the dynamic properties of public debt in South Africa using the MF-DFA method. Testing the memory property has important policy implications. Most studies, focusing on the issues of stationarity and long-run dependence, have simply applied the ARFIMA model. This model involves the estimation of the memory parameter (d) and the study of long memory behaviour. The classical methods are very sensitive to stationarity issues and do not take into account the assumed nonlinearity of the process, whereas the new method we have proposed here (MF-DFA) provides conclusive and robust results.

Our results suggest that fiscal austerity takes place before the crisis or recession period. The good times would create room for dealing with public deficits during the recession. Working simultaneously to reduce debt and increase the growth rate would contribute to economic stability in times of expansion. The experience of the United States in the aftermath of the Second World War, with strong and sustained economic growth, has shown that it is quite possible to improve living standards and reduce public debt levels at the same time. Public debt to improve growth and job creation should be directed towards upgrading infrastructure, improving the skills of workers of all ages, especially technical skills, and building on scientific research. Regular investment in this direction would lead to increased productivity. It is therefore essential to reconsider the debt problem, define strategies for action and review the debt-based economic model. While South Africa has remained competitive regionally, it is less competitive internationally. Despite a relatively low interest rate and a public debt level of around 60%

of GDP, South Africa's vulnerability could be exposed to an unanticipated shock.

The fact that South Africa's public debt series shows characteristics of long memory means that the country needs to implement important reforms, not only by prioritising certain sectors of the economy, but also and above all the structure of its debt. The conclusion of this work lies in the fact that despite a relatively low debt ratio, a country cannot avoid reforming its debt. In other words, it is not the amount of debt that is important, but its structure.

The fragility of the South African economy stems from its dependence on the mining and energy sectors. South Africa would therefore benefit from reducing its dependence on natural resources and diversifying its economy. The long memory characteristic of debt implies that debt generates debt and the economy enters a vicious circle. There is thus a self-similarity in the series as a function of shocks or economic policies put in place, hence its multifractal character. Public debt movements are correlated with small and large fluctuations at different scales. The debt burden then becomes a structural component of the economy.

From an economic point of view, fractality and long memory indicate the conjunction of several phenomena. First, the existence of significant extreme values. In other words, the debt does not follow a Gaussian process. Second, the presence of a gap between the observed value of the debt and its fundamental value. The greater this difference, the slower the adjustment period for this debt. Finally, public debt is unsustainable. The role of the authorities will in this case be to avoid upheavals due to unexpected events, in relation to external shocks or inappropriate fiscal policies.

The case of South Africa reveals that the concept of unsustainability does not refer to a high level of public debt. This unsustainability is further exacerbated by unforeseen events such as crises or government measures.

The analysis of the dynamic behaviour of the public debt required the application of non-standard methods to analyse in detail the complexity of the series, while trying to extract hidden features, which the classical methods are unable to detect. Using these new methods, we conclude that South Africa's debt has a multifractal character, the origin of which is the long memory effect.

Conflict of interest

There are no conflicts of interest.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Appendix

Appendix A. Classical method

Appendix A1. Fractional integration

The concept of long memory or persistence appears on autocorrelation samples of certain time series that converge at a slow rate to zero. These are at the frontier[8)] of the ARMA and ARIMA models. The long memory models based on the presence of a random walk and a unit root are based on the ARFIMA process (Granger and Joyeux, 1980; Hosking, 1981). They generalise the ARIMA process in which the differentiation exponent (d) is an integer. The particularity of these models is that they take into account the long-term correlations of the data and the real value of d. This model is at the limit of the extreme cases of unit root models and stationary models that impose an exponential decrease of the autocorrelation and therefore a spectrum limited to the zero frequency (Pérez and Ruiz, 2002). The integrated fractional autoregressive moving average time series process denoted ARFIMA (p, d, q) is described by:

$$\Phi(L)(1-L)^d y_t = \theta(L)\epsilon_t,\ \epsilon_t \backsim i.i.d.\left(0, \sigma^2_{\in}\right) \tag{A.1}$$

where y_t is the first difference in public debt, L is the lag operator, $\varphi(L) = 1\varphi_1 L$ - ...$\varphi_p L^p$ and $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ are respectively the autoregressive and moving average polynomials of the delay operator L, and $(1\text{-}L)^d$ is the fractional differentiation operator defined by :

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(-d)L^k}{\Gamma(-d)\Gamma(k+1)} \tag{A.2}$$

with Γ(.) denoting the gamma function (Abramowitz and Stegun, 1972). Assuming that d ϵ [0 ; 0,5], the ARFIMA process confirms the presence of a long memory or positive long-term dependency. It is a stationary process. The autocorrelations are positive and decrease hyperbolically towards zero as the delay increases. The spectral density is concentrated around low frequencies and tends to infinity when the frequency tends to zero. The opposite situation, with negative long-term dependence or anti-persistence (intermediate memory), assumes that d is between 0 and -0.5. The autocorrelations decrease hyperbolically towards zero and the spectral density is

8) Conventional modelling does not take into account intermediate cases with a fractional differentiation parameter "d". ARFIMA is a direct extension of the usual ARIMA processes allowing the integer integration order to take fractional values.

dominated by the high frequency components (it tends towards zero when the frequency tends towards zero). The process is said to be invertible if d > -0.5 and stationary when d < 0.5.

Appendix A2. Structural breaks

In general, the econometric literature has difficulty identifying a bubble in real time. Standard unit root tests fail to detect explosive behaviour in the presence of recurrent explosive bubbles (Evans, 1991). Phillips et al (2015) develop a test to capture mildly explosive alternatives. Yoon (2012) does not obtain conclusive results with the Phillips et al. (2011) test designed to detect a single episode of explosive behaviour. This test may lack power if there are shorter recurring episodes of explosive behaviour in a time series.

Following Phillips et al (2015), assume the following random walk process with fully negligible drift:

$$y_t = dT^{-\eta} + \theta y_{t-1} + e_t,\ e_t \sim N(0, \tau^2),\ \theta = 1 \tag{A.3}$$

where d is a constant, η is a location coefficient that controls the amount of drift when the sample size, T, approaches infinity and e_t is the error term. The specification of model (A.3) is usually complemented by transient dynamics in order to perform exuberance tests, just as in standard augmented Dickey-Fuller unit root (ADF) tests against stationarity. The recursive approach with the rolling window proposed by Phillips et al. 2015 can then be written:

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p} \varnothing_i \Delta y_{t-i} + \epsilon_t \tag{A.4}$$

where $\epsilon_t \sim iid(0, \sigma^2)$ is individually and independently distributed, y_t is the variable tested for explosiveness and therefore public debt, μ the constant, ρ the autoregressive coefficient, p the maximum number of lags, Δ is the difference operator and $\varnothing_i$ the coefficients of the lagged first differences. In the classic bubble test, we rely on a right-hand side variation of the standard ADF unit root test where the null hypothesis is a unit root and the alternative hypothesis is a slightly explosive autoregressive coefficient. The aim is to test:

$H_0 : \rho = 1$
$Ha : \rho > 1.$

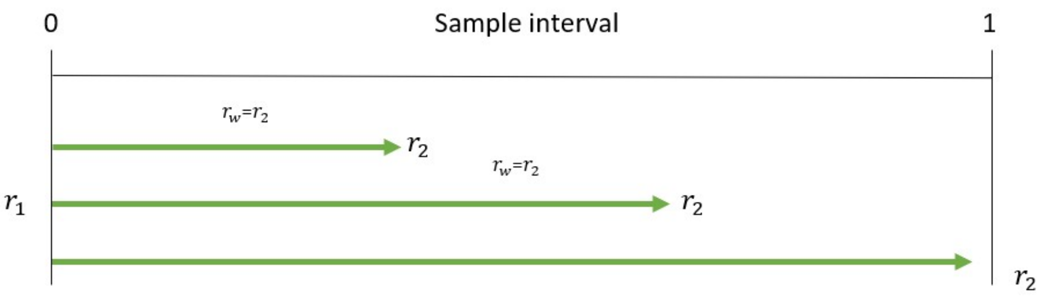
To simplify the presentation, we normalise the sample size by T corresponding to [0, 1]. Let us note by ρ_{r_1, r_2} and ADF_{r_1, r_2} the coefficient estimated by equation (A.4) and the ADF statistics normalised to $[r_1, r_2]$. Let us also note r_w the size of the regression window, defined by $r_w = r_2 - r_1$ et r_0 the initial window size (fixed). The difference between the tests lies in the way r_2 and r_1 are defined.

Supremum ADF test. The test developed by Phillips et al (2011) is very sensitive to unit root changes. As bubbles can collapse periodically, conventional unit root tests have limited power to detect bubbles. The supremum ADF (SADF) test was applied by Yoon (2012) to test the explosive nature of US debt. The method is defined as a recursive calculation with a fixed starting point and an expansion window. The initial window size is defined randomly. The first observation $r_1 = 0$ and the end point r_2 are defined according to a minimum choice of the window such that $r_w = r_2$ and r_w is the fractional window size of the regression by normalising the initial sample [0,1]. The test is performed sequentially on different subsamples (Figure A1). The first subsample contains the observations of the initial sample; and is then extended until all observations of the full sample are included in the tests. Each estimate yields an ADF statistic called ADF_{r_2}, the SADF statistic is the upper bound value of the sequence ADF_{r_2}. This statistic can be denoted as:

$$SADF(r_0) = Sup_{r_2 \epsilon [r_1, 1]} \left\{ ADF_{r_2} \right\} \tag{A.5}$$

This technique is used to identify periods when the explosive property of the bubble becomes dominant in the public debt process. The beginning of the bubble is estimated as the first date at which the ADF statistic is greater than the corresponding critical value of the unit root test. The end of the bubble will be determined as the first period when the ADF statistic is below the critical value. The limitation of the SADF test lies in its inability to detect a second bubble when the first dominates. The test is also limited by the size of the initial window which is fixed.

Figure A1. Illustration of the SADF procedure

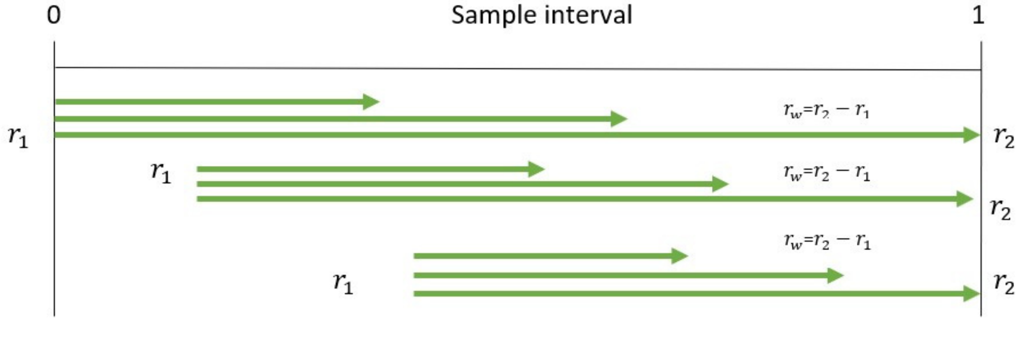


Generalized SADF test. Building on the previous method, the generalized SADF (GSADF) test uses a more flexible window and a starting point that varies in the range [0, $r_2 - r_1$]. This is a recursive ADF regression test via a sliding window procedure. Instead of fixing the starting point of the sample, the GSADF test changes the starting and ending points of the sample over a possible range of windows. This moving sample test takes into account several structural breaks at unknown dates, allowing for consistent bubble detection (Baldacci et al., 2015). The flexibility of the window allows for gains in efficiency and power compared to previous tests. The test progressively shifts the window frame towards the end of the sample and significantly improves the discriminative power of multiple bubble detection. This is an answer to the limitation of the GSADF test. The GSADF statistic is expressed as follows:

$$GSADF(r_0) = Sup_{r_1 \epsilon [r_2 - r_0]}^{r_2 \epsilon [r_1 - 1]} \left\{ ADF_{r_1}^{r_2} \right\} \tag{A.6}$$

The main reason for using this statistic is that bubbles can collapse temporarily and therefore standard unit root tests can limit the period of bubble capture (Caspi, 2016). Homm and Breitung (2012) deduced that this test is suitable for detecting bubbles, especially when one or two bubble episodes are involved. Based on this method, bubbles are consistently detected even with smaller samples (Caspi, 2013; Caspi, 2016).

Figure A2. Illustration of the GSADF procedure



When the null hypothesis of no bubble is rejected, there are explosive periods that are also dated with a starting point and an end date of the explosive process. The starting point of a bubble is the date, expressed as T_{re}, at which the sequence ADF_{r_2} crosses the critical value from below. Similarly, the end point of a bubble is also defined as the date, expressed as T_{rf}, at which the sequence ADF_{r_2} crosses the critical value, but from above. Ultimately, based on the GSADF, explosive periods can be referred to as:

$$\hat{r_e} = Inf_{r_2 \epsilon [r_0, 1]} \left\{ r_2 : \; ADF_{r_2} \rangle cv_{r_2}^{\beta_T} \right\} \tag{A.7}$$

$$\hat{r_f} = Inf_{r_2 \epsilon [\hat{r_e}, 1]} \left\{ r_2 : \; ADF_{r_2} \langle cv_{r_2}^{\beta_T} \right\} \tag{A.8}$$

where $cv_{r_2}^{\beta_T}$ est 100 (1 - β_T) % critical value of the standard ADF statistic based on the window observations. Similarly, estimates of the bubble period based on the GSADF are given by:

$$\hat{r_e} = Inf_{r_2 \epsilon [r_0, 1]} \left\{ r_2 : \; BSADF_{r_2}(r_0) \rangle cv_{r_2}^{\beta_T} \right\} \tag{A.9}$$

$$\hat{r_f} = Inf_{r_2 \epsilon [\hat{r_e}, 1]} \left\{ r_2 : \; BSADF_{r_2}(r_0) \langle cv_{r_2}^{\beta_T} \right\} \tag{A.10}$$

Where $cv_{r_2}^{\beta_T}$ est 100 (1 - β_T) % critical value of the standard ADF statistic based on the window observations. $BSADF_{r_0}$, for r_2 $\epsilon[r_0$,1], is the backward-looking ADF statistic that relates to the GSADF statistic by the following relationship:

$$GSADF(r_0) = \sup_{r_2 \in [r_2, 1]} \left\{ BSADF_{r_2}(r_0) \right\} \tag{A.11}$$

In each subsample, the length of the offset in the ADF equation (A.4) is selected using the Akaike Information Criterion (AIC).

Appendix B. Results of ARFIMA and MF-DFA

Appendix B1. Long-term dependence

Table B1 presents the results of ARFIMA on the differentiated South African debt series. Recall that the particularity of the ARFIMA process is that it takes into account both the long-term behaviour and the short-term dynamics. The autoregressive and moving average parameters account for short-term behaviour while the fractional integration parameter accounts for long-term behaviour. The estimates of the integration parameter (d) suggest the existence of a fractional process in the case of South African debt. The coefficient on the memory parameter (d) is positive and significant at the 5% level. We can then reject the random walk process. There is a long memory or persistence effect. The results assume a long memory because d is between 0 and 0.5. In other words, the long memory parameter indicates the presence of a long-term dependence (Hamilton, 1994). The autocorrelations are positive and decrease hyperbolically towards 0 as the lag increases. The auto-regression coefficient φ is negative and significant while the moving average parameters θ are all positive and significant except for one parameter. This results in an ARFIMA (1, 0.44, 4) model, i.e. with both short and long term modelling. Thus, the ARFIMA model consists of an AR process of order 1, a fractional integration (0.44) and an MA process of order 4. Figure B2 shows that this is indeed a stationary and invertible process.

Table B1. *ARFIMA*

	Memory parameter	Autoregressive parameter	Moving average parameter			
	d	φ	θ_1	θ_2	θ_3	θ_4
South Africa	0,44*** (0,10)	-0,47*** (0,08)	1,14*** (0,31)	0,94** (0,31)	0,97** (0,31)	0,28 (0,31)

Note. (*), (**), (***) indicate significance at 10%, 5% and 1% respectively.
Estimated standard deviation of residuals: 2.98; loglikehood: 4.25.

The validity of the results should be checked by analysing the residuals. Figure B3 shows that it is indeed white noise at the 5% significance level. The analysis of the autocorrelation function (ACF) of the residuals confirms a general mean-reverting process. We can therefore conclude that this is an ARFIMA (1, 0.44, 4) model for South Africa's debt. The reaction functions in Figure B4 corroborate the above results, i.e. a shock to South African debt will indeed have permanent effects on the series. The reaction functions from the asymptotic method and the counterparty method converge and show an inflection around the value 10. This behaviour marked by a hyperbolic decrease in autocorrelations is atypical of the ARFIMA model as reported by Hassler and Kokoszka (2010), Kokoszka and Taqqu (1995), and Hosking (1981). Finally, the cumulative periodogram plot (Figure B5) shows that there is good evidence on the quality of the model especially with statistically independent residuals. The long memory could be due to structural breaks in the series (Granger and Hyung, 1999; Diebold and Inoue, 1999). In the following subsection we explore the presence of structural breaks in time.

Appendix B2. Explosivity

The results of the explosivity tests are reported in Table B2 showing evidence of explosive debt behaviour in South Africa. We used the SADF and GSADF tests to check whether an explosive bubble exists in the long run. The critical values of our samples are obtained via Monte Carlo[9] simulation with 2000 replications.

9) The Monte Carlo simulations show that the bootstrap methodology works well and allows us to identify explosive processes and periods of explosive collapse.

Table B2. *Results of SADF and GSADF Tests*

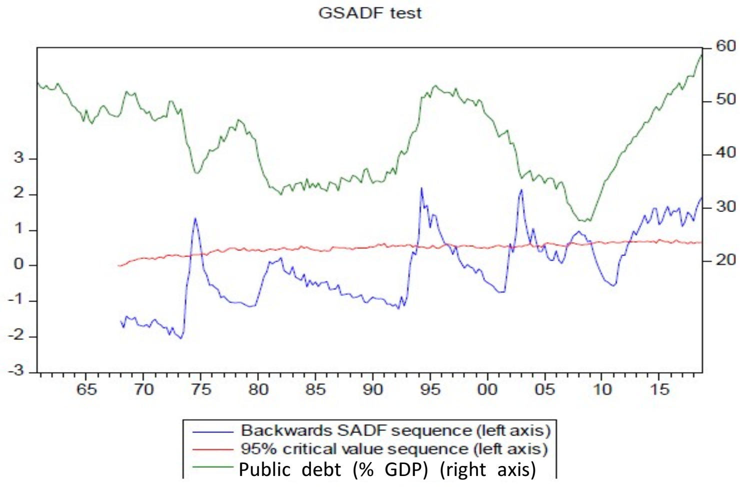
Country	GSADF	SADF
South Africa	2,19**	0,65

Note. (*), (**), (***) indicate significance at 10%, 5% and 1% respectively.

We use the GSADF method which is an improved version of the SADF method, powerful enough to detect periodic bubble collapse (Hou, 2010; Jianglin, 2010; Wu, Gyourko and Deng, 2012; Shen et al., 2005; Bystrov and Mackewicz, 2016). Consistent with the work of Phillips et al. (2015), the GSADF mobile sample diagnostic outperforms the SADF test. Based on an increasing sample size for the detection of explosive behaviour over several periods, it rarely generates false positives, even for relatively small sample sizes. As the GSADF test covers more subsamples of data, it allows for more flexibility and calculates the number of explosive periods as well as the start and end date.

The hypothesis of debt explosiveness is strongly accepted in South Africa. The initial window size was set at 0.12. Figure B1 shows the public debt series, the curve of the BSADF statistic and the corresponding critical values. When the critical value is below the BSADF statistic, it marks the beginning of an explosive period and when the critical value rises above it, it marks the end of that explosive period. Table B3 summarises the five explosive periods in South African public debt between 1960 and 2018. South Africa's debt experienced its first boom in the early 1970s when economic growth slowed. The country lost its global competitiveness in a context of income and wealth inequality.

Figure B1. Timestamp bubble periods



However, this result should be qualified by noting that, unlike most African countries, South

Africa's debt ratio has not exceeded 60% of GDP since 1960. The second explosive period (1993Q4 - 1996Q3)[10] that followed was due to the end of apartheid. It should be recalled that during apartheid South Africa went through a process of massive borrowing to stimulate economic growth. The third boom period (2002Q2 - 2004Q2) is explained by the combination of inflation[11] targeting monetary policy and strong growth. Of course, this combination led to higher interest rates and an explosion of public debt.

Irresponsible borrowing and over-reliance on domestic capital was the legacy of apartheid, a period particularly marked by apartheid debt obligations, forcing the government to opt for stabilisation of the economy by reducing public debt through an austere fiscal programme. Between 2006 and 2009, the dramatic fall in government revenue led to an explosion in debt (2006Q4 - 2009Q2). This period was marked by political tensions, combined with the national and international recession, which prevented the government from reducing spending. Since the political transition in 1994, South Africa's public debt level has been relatively sustainable with debt below 60% of GDP (Calitz et al., 2016). This reflects sound fiscal measures focused on improving the primary balance in response to rising debt (Jooste et al., 2011).

Table B3. *Explosive Periods*

Country	Period 1	Period 2	Period 3	Period 4	Period 5
South Africa	1974Q2 - 1975Q1	1993Q4 - 1996Q3	2002Q2 - 2004Q2	2006Q4 - 2009Q2	2012Q2 - 2018Q3

Note. Q1, Q2, Q3 and Q4 represent quarter 1, 2, 3 and 4 respectively.

In the aftermath of the 2008 global financial crisis, South Africa opted for a countercyclical policy. It should be recalled that southern African countries generally failed to meet their growth targets in the 2008-2009 recession. There was a contractionary effect on global demand, particularly on the region's exports. This has limited incomes and reduced employment in the mining sector. The consequence has been an increase in debt. The weakening of the economy has reduced revenue collection and the increase in the wage bill has in turn led to an increase in public expenditure. The public sector wage bill is the largest in the emerging countries because of the high remuneration of public sector employees.

The last explosive period (2012Q2 - 2018Q3) appears in a context of fighting inequalities through the implementation of a programme to reduce poverty and social wages. This programme has required large sums of money to fight inequality, hence the use of public debt. Inequality increased by high corruption (Bhorat et. al., 2017). The deepening culture of corruption within the state has halted the transformation agenda and refocused efforts on private accumulation. Thus, the explosiveness of debt has interesting economic implications, as the timing of structural changes

10) Q1, Q2, Q3 and Q4 represent quarter 1, 2, 3 and 4 respectively.

11) The South African Reserve Bank (SARB) started inflation targeting in the 2000s.

are not properly taken into account, debt forecasts and policy decisions could be misguided.

Following the detection of structural breaks, the long memory behaviour observed by the ARFIMA process could be confused with it. Diebold and Inoue (2001), Granger and Hyung (2004); and Hyung et al. (2006) have shown that neglecting structural breaks in time series can bias the estimates of fractional differentiation parameters. In the light of this finding, we direct our analysis towards a new and much more robust method that avoids the issues of stationarity and confounding.

Figure B2. Inverse AR and MA roots

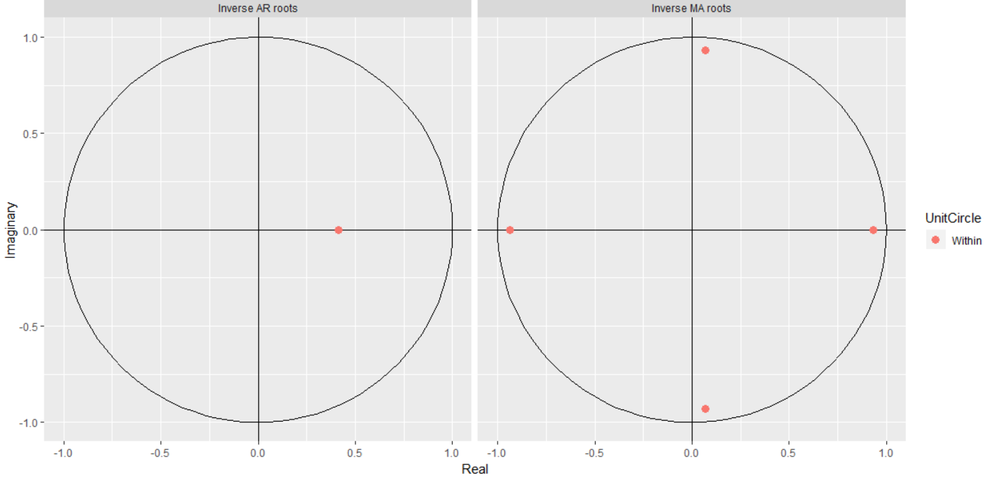


Figure B3. ARFIMA model residual (1, d, 4)

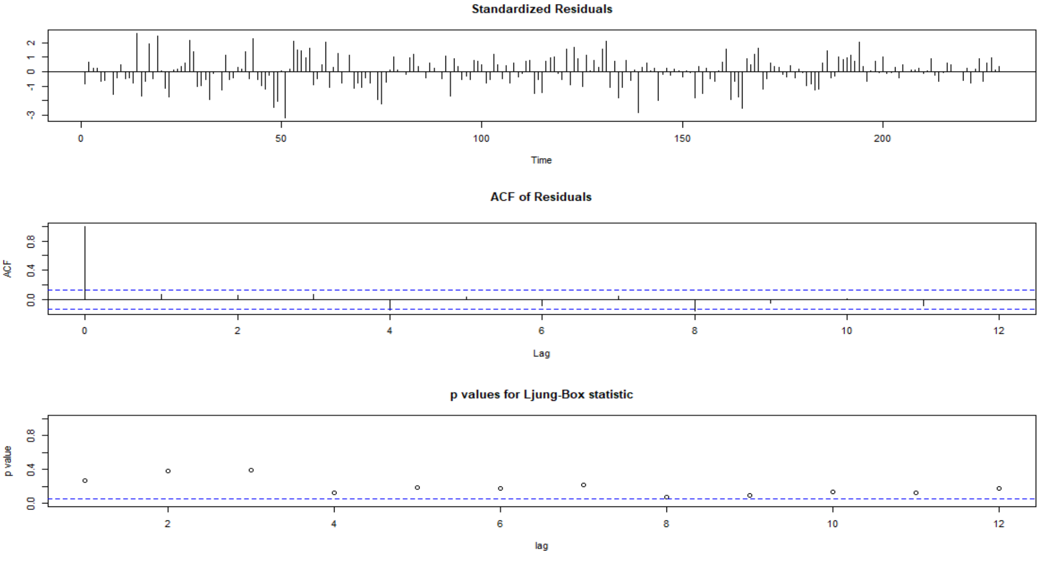


Figure B4. Reaction function

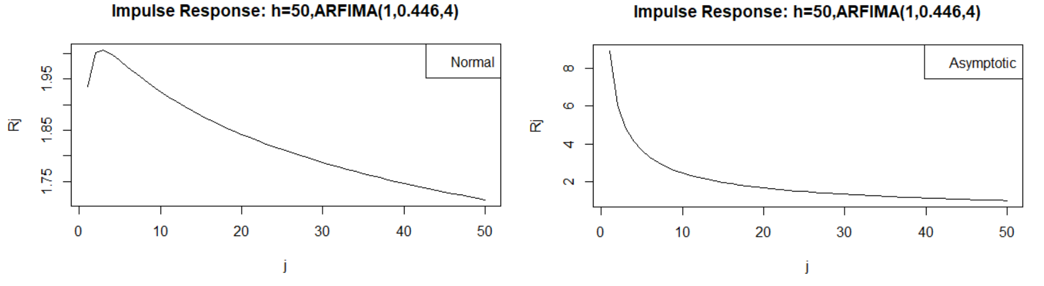


Figure B5. Periodogram

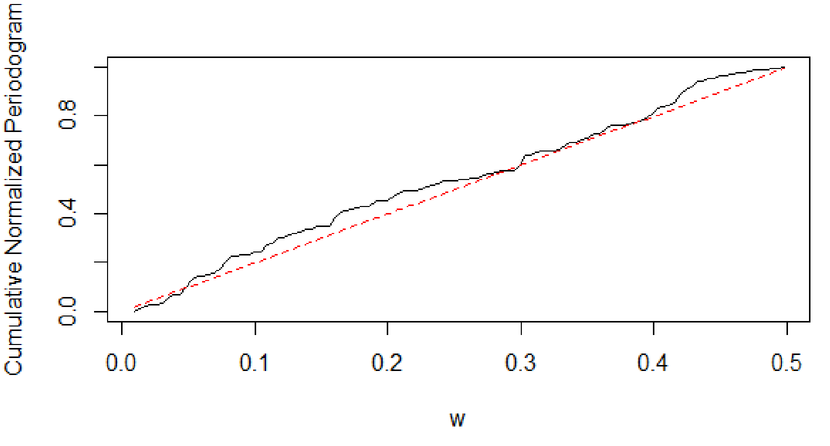


Table B4. *Generalized Hurst Exponent*

q	-5	-4	-3	-2	-1	0	1	2	3	4	5
Hq	2,13	2,10	2,05	1,98	1,89	1,80	1,73	1,67	1,63	1,59	1,57