Privatization in Emerging Markets

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Abstract

This paper shows two examples where privatization may lead to large efficiency gains by changing the menu of taxes. First, social security privatization increases the equity position of the middle class, inducing the median voter to internalize a higher fraction of the costs of high taxes on capital, thereby reducing the capital tax rate. Second, reducing the public sector involvement in import competing activities is shown to lower the public sector’s benefits from protection, reducing thereby the equilibrium tariff rate. These indirect effects of privatization described in the paper are external to the privatized activity.

(JEL Classifications: F13, H21) <Key Words: Imperfect capital mobility, Social security privatization, Import competition, Public enterprises>

I. Introduction

One of the remarkable developments of recent years has been the privatization effort in countries characterized by the deep involvement of the State.1 Privatization has been frequently part of a broader attempt to liberalize economies characterized by stagnation.2 The purpose of this paper is to

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investigate the implications of privatization on the design of taxes. As is well recognized by the literature, privatization leads to direct economic effects due to the ownership change of productive assets. What is less appreciated, however, is that privatization may lead to other subtle effects by changing the menu of policies supported by the median voter. For example, in various countries the State was (and in some still is) directly involved in the production of import competing consumer goods and services, as well as in the production of key inputs. The alliance between the State and the import competing sector tilts the balance in favor of protective policies, as the branch of the state running the industry frequently enjoys direct access to the political process. Privatization of such industries would reduce their bargaining power in the political process, ultimately reducing protection, and enhancing efficiency. Ignoring these indirect gains may lead one to underestimate the importance of privatization. A less obvious example along this line is social security privatization, where the state has been heavily involved in designing and running the pay-as-you-go system.

The purpose of this paper is to show that in an economy where policies are determined by the median voter, privatization would lead to efficiency gains by changing the menu of taxes, in addition to all the other effects recognized by earlier literature. To illustrate this point we discuss in section II the political economy implications of social security privatization in an emerging market economy. Section III studies privatization of import competing activities, and section IV concludes.

2. For example, the Economist commented on December 6, 1997 "The third big change in Latin America's business environment, along with the taming of inflation and the liberalization of trade, has been the dismantling of the proprietorial state. Latin America has privatized with remarkable speed and on a vast scale. By one count, over the past ten years 279 companies have been sold for a total of $90 billion in the seven largest Latin American countries. Between 1990 and 1995, Latin America accounted for more than half (by value) of all privatizations in the developing world, including Central and Eastern Europe."

3. For example, Lance Taylor recently wrote "Privatization brings no obvious productivity gains, and if done in a slapdash fashion it can adversely perturb savings, investment, and financial flows." See Blecker, ed. [1996].
II. Social Security Privatization in Emerging Markets

– Labor and Capital Taxes

The aging of the baby-boomers and the backlash against the welfare state has focused attention on the future of social security. One of the innovative schemes dealing with it is the privatization of social security funds. This reform has been frequently advocated by economists, and a version of it was adopted in 1981 by Chile, followed recently by other emerging markets (including Mexico and Bolivia in 1997). Most of the analysis assessing these developments focused on the impact of the reform on saving. This paper argues that in an economy where the menu of taxes is the outcome of a political economy perspective, social security privatization would lead to efficiency gains by changing the menu of taxes. To illustrate this issue, consider an economy where efficiency considerations call for a relatively low tax rate on capital income. If the share of capital owned by the middle class is small, the median voter would impose a tax on capital income that exceeds the efficient tax by a large margin, reflecting the “beggar my (capitalist) neighbor” attitude. Granting a greater equity position to the middle class would induce it to internalize a higher fraction of the costs of high taxes on capital, reducing thereby the capital tax rate supported by the median voter.

The purpose of this paper is to illustrate rigorously this argument for an emerging market, where agents can engage in capital flight and black market investment to shelter their investment income. We consider an economy composed of three classes, where the median voter belongs to the middle class. The existing system finances a given transfer scheme with a mixture of capital and income tax. Agents have access to a “gray capital market,” sheltering their investment in tax free opportunities. The return to “gray capital” diminishes with the volume invested, as is the case when evading

4. For an overview of social security reforms see Diamond [1996], and the papers in Feldstein [1996]. For a review of the Chilean reform, see Diamond [1993] and Edwards [1996].

5. This would be the case if the supply of labor were relatively inelastic, whereas the supply of taxable capital were relatively elastic. The concern regarding saving’s “double taxation” may provide another rationale for a low tax on capital income.
taxes is harder for larger transactions. The "gray capital market" has several interpretations, like capital flight to overseas markets in developing countries, as well as domestic black market activities. Social security privatization is viewed as a policy that transfers equity from the high class to the other classes. \textit{i.e.}, prior to privatization, some of the equity of the high class provided the tax income redistributed \textit{via} social security to other classes. After social security privatization, some of this equity is effectively transferred from the high class to the other classes. This amounts to a drop both in equity position and in the tax collected from the high class.

\textbf{A. The Model}

We consider a simple ex-ante/ex-post model. The domestic output is produced with labor and capital. The economy is composed of \( L \) agents, each supplying inelastically 1 unit of labor. Agents differ only in their endowment of capital. There are three classes, each composed of a third of the population. The ownership of the aggregate stock of capital, \( K \), is distributed among the low, middle and high classes (denoted by \( l, m, h \)) with shares \( s_l, s_m, s_h \), respectively. We assume that \( 0 < s_l < s_m < s_h \). Capital invested at home is subject to a capital tax of \( \phi \). Capital invested abroad via capital flight evades taxes, but is subject to raising transaction costs of tax evasion. We capture this possibility by assuming that the yield on capital invested overseas depends negatively on the magnitude of capital flight. Let \( \delta \) denote the share of capital invested domestically by the representative agent [hence \( 1-\delta \) is invested overseas], and let \( r^* \) be the net yield on capital invested overseas. The gray market investment opportunities are summarized by a reduced form equation,

\[ r^* = r^*(\delta); \frac{dr^*}{d\delta} > 0. \quad (1) \]

Greater effective integration of the domestic capital market with the global market implies lower \( \frac{dr^*}{d\delta} \).
Domestic output, \( y \), is

\[
y = \left[ \bar{L} \right]^\alpha \left[ \delta \bar{K} \right]^{1-\alpha}
\]

(2)

where \( \bar{K} \) is the aggregate capital owned by domestic agents.

The pre-tax real wages and rental rates on capital, \((w, r_k)\), are, respectively

\[
w = MP_L = \alpha \left[ \frac{\delta \bar{K}}{\bar{L}} \right]^{1-\alpha}; r_k = MP_K = (1-\alpha) \left[ \frac{\bar{L}}{\delta \bar{K}} \right]^{\alpha}
\]

(3)

The government budget constraint is

\[
G_0 = \tau w \bar{L} + \phi r_k \delta \bar{K}
\]

(4)

where \( \tau \) is the labor tax rate. The initial equilibrium is characterized by an exogenously given fiscal revenue target, \( G_0 \). This revenue is redistributed among the three classes. Taking the value of the fiscal target and the patterns of redistribution given, the middle class determines the tax rate on capital (\( \phi \)) and on labor (\( \tau \)), at rates that maximize its utility.

A representative capitalist is indifferent to investing at home or overseas (or in the domestic “gray market” activities) if the net returns are equalized–

\[
r^* = (1-\phi)(1-\alpha) \left[ \frac{\bar{L}}{\delta \bar{K}} \right]^{\alpha}
\]

(5)

from which we infer that

\[
\delta = \delta(\phi); \frac{d\delta}{d\phi} = -\frac{\delta}{(1-\phi)[\eta_{r^*,\delta} + \alpha]} < 0
\]

(5')

where \( \eta_{r^*,\delta} = \frac{d \log r^*}{d \log \delta} \)

B. The Political Equilibrium

We turn now to describe the set of taxes supported by the middle class. The net income of class \( i \) (\( i = l, m, h \)) is
\[ R_i = (1 - \tau) \frac{w_L}{3} + (1 - \phi) s_i \bar{K} r_k + (1 - \delta) s_i \bar{K} r^* + q_i G_0 \]  

(6)

where \( q_i \) is the transfer share of class \( i \), and is assumed to be exogenously given.

Taxes are set by the median voter, according to

\[
\text{MAX} [R_m] \quad \tau, \phi
\]

subject to the government budget constraint, (4), and the competitive equilibrium conditions. Applying (3) - (4) to (6) we substitute for the wage tax, obtaining the middle class income as a function of the capital tax

\[
R_m(\phi) = \left[ \frac{\alpha}{3} + (1 - \alpha)[(1 - \phi) s_m + \frac{\phi}{3}] \right] [\bar{L}]^\alpha [\delta \bar{K}]^{1-\alpha} + (1 - \delta) s_m \bar{K} r^* + (q_m - \frac{1}{3}) G_0
\]

(8)

From which we infer that the first order condition defining the optimal tax on capital is

\[
\frac{\partial R_m}{\partial \phi} = \frac{1 - \alpha}{3} y[1 + \frac{\alpha + \phi(1 - \alpha)}{\delta} \frac{\partial \delta}{\partial \phi}] - s_m \Omega = 0
\]

(9)

where \( \Omega = (1 - \alpha) y - \frac{\partial \delta}{\partial \phi} [(1 - \delta) \bar{K} \frac{\partial r^*}{\partial \delta} + (1 - \phi) \frac{(1 - \alpha)^2}{\delta} y - \bar{K} r^*] \).

Figure 1 summarizes a simulation of the association of the middle class capital ownership share \( (s_m) \) with the tax rate on capital \( \phi \). The downward sloping curve traces the political equilibrium values of \( (s_m; \phi) \). In our framework, labor supply is inelastic, whereas the supply of capital for use in the domestic economy is elastic. This in turn implies that the efficient tax on capital is zero. The contours in Figure 1 report the welfare cost of the tax on capital, relative to the efficient benchmark case where the tax on capital is zero. Note that in our example, if the middle class equity share is 10%, it supports a relatively high tax rate on capital (46%), despite its relatively high welfare cost (8%). Increasing the ownership share to 0.2 would reduce the capital tax to about 13%, and reduce the welfare cost to about 1.2%.

We may understand better the above results by noting that the first order
condition determining the capital tax, (9), can be rewritten as a condition equating the marginal benefit of raising the capital tax to the marginal cost (both from the perspective of the middle class). The marginal benefit is defined as the income gain for the middle class were its equity share zero. The marginal cost is defined by the drop the middle class equity income caused by a higher capital tax rate.

**Figure 1**

The Middle Class Capital Ownership Share and Tax Rate on Capital

Notes: 1. The downward sloping curve traces the political equilibrium values of \((s_m, \phi)\).
   
   2. The contours report the welfare cost of capital tax
   
   3. The simulation assumes \(K = 1; L = 1; r' = \delta^3; G_0 = 0.1; \alpha = 2/3\)

\[
MB_\phi = MC_\phi
\]

where

\[
MB_\phi = \frac{1-\alpha}{3} \gamma [1 + \frac{\alpha + \phi(1-\alpha)}{\delta} \frac{\partial \delta}{\partial s}] ; MC_\phi = s_m \Omega
\]

Applying (5') to (9'), collecting terms, it follows that
\[ MB_\phi = \frac{1 - \alpha}{3(1 - \phi)(\alpha + \eta_{r,\delta})} y \left[ \eta_{r,\delta} - \phi(1 + \eta_{r,\delta}) \right]; \]

\[ MC_\phi = s_m \left[ (1 - \alpha) y(1 - \phi)(1 + \eta_{r,\delta}) + \overline{Kr}^* \{ (1 - \delta)\eta_{r,\delta} - \delta \} \right] \frac{1}{(1 - \phi)(\alpha + \eta_{r,\delta})} \]  

(10)

The marginal benefit curve (MB) in Figure 2, panel I, measures the marginal benefits to the middle class resulting from shifting the burden of taxes from labor to capital. The curve corresponds to the example used in Figure 1, where \( \eta_{r,\delta} = 2 \). Starting with zero tax on capital, a higher capital tax rate increases revenue from capitalists, allowing a drop in the wage tax, benefiting the middle class. This effect diminishes as we move upwards on the capital tax Laffer curve - both the tax revenue and the wage gains attributed to the higher tax revenue collected from capital drop due to more intense capital flight. The marginal benefit from the tax will disappear at point \( L \) where \( MB_\phi = 0 \), or \( \phi = \frac{\eta_{r,\delta}}{1 + \eta_{r,\delta}} \). Point \( L \) also determines the tax on capital if the middle class does not own any equity.

The marginal cost stems from the fact that a higher capital tax induces a negative income effect, proportional to the equity position. The income attributed to capital ownership drops both due to the tax on capital, as well as to the induced drop in the gray market return. The sum of both effects defines the marginal cost of the capital tax, and is plotted by the dashed MC curve for \( s_m = 0.066 \) (see Figure 2, panel I). The dotted curve corresponds to the marginal cost if the equity position triples. Note that a higher equity position increases the marginal cost of capital tax for the middle class, shifting MC upwards, inducing a drop in the tax rate.

A greater integration with the global capital market implies that a given tax rate induces greater capital flight, exacerbating the welfare cost of the capital tax, and leading to a lower capital tax rate. This effect is captured by comparing the two panels of Figure 2. Greater integration implies a smaller \( \eta_{r,\delta} \). Indeed, reducing it from 2 to 1 implies that the capital tax rate supported by the middle class drops from 2/3 to 1/2 if the middle class equity share is zero [and from 10% to 0% if the middle class equity share is 20%].
Figure 2
The Marginal Cost and Marginal Benefit of Capital Taxes
[the Middle Class Perspectives]

I - drawn for $\eta_r, \delta = 2$

I - drawn for $\eta_r, \delta = 1$
**C. Social Security Privatization**

We turn now to evaluate the impact of social security privatization. We assume that the privatization scheme is revenue neutral at the initial tax rates. A transfer of equity ownership from the high to the middle class leads to a rise in the net income of the middle class. It is assumed that there is a matching drop in the direct transfer from the government to the middle class, so that the net income effect is zero. This “compensation” is summarized by (11) –

\[
\frac{dq_m}{ds_m} = -\frac{\partial R_m}{\partial q_m} = -\frac{G_0}{(1-\alpha)(1-\phi) + (1-\delta)Kr^*} < 0
\]  

(11)

A similar adjustment (but in the opposite direction) takes place for the high class.

The net effect of the equity infusion to the middle class can be traced with the help of our analysis from the previous section, summarized by (10). Note that the results in the previous section were independent from the transfer share of the middle class, \(q_m\). Hence (independently of the income effects), social security privatization tends to reduce the capital tax supported by the middle class.

**D. Discussion**

- The model described in Section II is a static version, designed to deliver the argument in the simplest manner. The logic of our discussion can be applied to a dynamic, overlapping generation model.

- Privatization is not a panacea. Partial privatization, with excessive regulatory intervention by the government, may induce significant welfare costs. Examples –
  - Attempts to force funds freed by social security privatization to be invested mostly in domestic government bonds [as has been the case in Mexico and Bolivia] amounts to an implicit tax on the new pool of privatized savings. It is equivalent to “forced saving,” used to fund public debt at below the market interest rate. Our analysis can be extended to address the above issues in a model that allows for
restrictions on capital mobility, along the line of Aizenman and Guidotti [1994]—considering an economy where there are no lump sum taxes, and all tax collection is costly. In these circumstances the treasury would rely on restrictions on capital mobility as a second best policy aiming at increasing the domestic tax base.

- Attempts to restrict the privatized social security funds to be invested mostly in domestic assets would limit the private sector's gains from global diversification. These gains may be especially large in emerging markets characterized by limited GDP diversity, and relatively high reliance on commodity trade.
- Non competitive management of the privatized social security funds may encourage waste and rent seeking, as apparently has been the case in Chile (see Diamond [1993]).
- With uncertainty, social security privatization has other effects that are ignored by the present paper (e.g., it increases the exposure of the middle class to equity risk).

III. Import Competing Goods, Protection, and Privatization

Various developing countries are characterized by inflated public sector that operates under the shield of a high level of protection. In this section we illustrate that the logic of our discussion is applicable to these circumstances. We consider the simplest example—a two goods, small economy, where the government owns a fraction \( s_g \) of the import competing sector. Exportables, \( X \), and importables, \( Y \), are produced using the services of labor \( (L) \), and a sector specific immobile capital. The international price of both goods is normalized to 1. Labor is fully mobile between sectors, and all markets are competitive. Government utility, \( U_g \), is a weighted average of consumer's welfare (denoted by \( W \), evaluated in terms of good \( X \)) and government's income (denoted by \( R \))^6 –

\[
U_g = \alpha W + (1 - \alpha) R 
\]  
(12)

6. For a derivation of such a reduced form in a political economy equilibrium see Grossman and Helpman (1996).
Government’s revenue is the sum of its revenue from a tariff (the rate of which is \( \tau \)), plus any surplus stemming from the ownership of productive assets (\( \Pi \))

\[
R = \tau IM_y + \Pi = \tau IM_y + s_y[(1 + \tau)Y^s - \omega L_y]
\]  

(13)

where \( IM_y \) denotes the imports of good \( y \), \( Y^s \) is the domestic production of importables, \( \omega \) the wage, and \( L_y \) the labor employed in sector \( y \).

A. ‘Optimal’ Tariff and Public Sector’s Ownership

The government determines the tariff at a rate that maximizes (12), leading to a first order condition given by

\[
0 = \alpha \frac{\partial W}{\partial \tau} + (1 - \alpha) \frac{\partial R}{\partial \tau}
\]  

(14)

Alternatively, as shown in the Appendix

\[
0 = \alpha \left[ -IM_y - s_y (Y^s - L_y \frac{d\omega}{d\tau}) \right] + (1 - \alpha) [IM_y (1 - \eta_{IM,\tau}) + s_y (Y^s - L_y \frac{d\omega}{d\tau})] \]

(15)

where \( \eta_{IM,\tau} \) is the import’s elasticity with respect to the tariff rate. The first term of the RHS of (15) weighs the impact of a marginal tariff on consumer’s welfare. Welfare drops by the adverse income effect of the higher cost of imports [proportional to the level of imports] and by a drop in the private’s sector income [which in the Appendix is shown to equal the increase in public enterprise surplus]. The second terms weigh the impact of the tariff on government income. It is the sum of the net change in tariff revenue plus the resultant increase in government surplus. Alternatively,

\[
(1 - 2\alpha) [IM_y + s_y (Y^s - L_y \frac{d\omega}{d\tau})] = (1 - \alpha) IM_y \eta_{IM,\tau}
\]

(15’)

The term \( 1-2\alpha \) is the government’s utility gain from transferring a unit of purchasing power from public to government use \([-\alpha + (1-\alpha)]\). This is a measure of the degree to which the government’s agenda diverges from that of the private sector’s agenda. It must be positive if the government
prefers not to rebate the tariff income to the public, as we assume it to be the case. The LHS of (15') measures the marginal benefit of a higher tariff [from the government perspective], equal to the product of $1 - 2\alpha$ times the extra revenue from tariff (at the given level of imports) plus the increase in rents associated with importables. The RHS is the marginal cost, induced by a drop in tariff income due to the drop in imports triggered by the tariff hike.

Equation (15') reveals that a higher ownership share of the public sector $s_g$ increases the marginal benefit of the tariff, without impacting on the marginal cost. Hence, it will lead to a higher optimal tariff. Formally, by applying the implicit function theorem to (15) we infer that increasing the government’s ownership share of importables would increase the optimal tariff [see the Appendix for derivation].

$$\frac{d\tau}{ds_g} = \frac{(1 - 2\alpha)(Y^s - L_y\frac{d\omega}{d\tau})}{-\frac{\partial^2 U^g}{\partial\tau^2}} > 0$$

In signing the RHS of (16) we note that the second order condition for maximizing government utility implies $-\frac{\partial^2 U^g}{\partial\tau^2} > 0$. Note also that $Y^s - L_y\frac{d\omega}{d\tau}$ measures the impact of a marginal tariff change on rents in sector $Y$. In the appendix we show that $Y^s - L_y\frac{d\omega}{d\tau} > 0$. Hence, a higher share of importables produced by the government would increase the ‘optimal’ tariff chosen by the government, as it increases the marginal benefit of a tariff hike. Alternatively, a lower public ownership of the import competing sector would reduce the benefits from the tariff [from the government’s perspective], resulting in lower protection. This suggests that successful privatization would lead overtime to a drop in protection.

IV. Discussion

- Our analysis of protection and public sector ownership considered the case where the public enterprises are generating rents. Frequently, howev-
er, public enterprises in developing countries are not profitable. This is consistent with the logic of our paper once we recognize that the rent of public enterprises may be used [and abused] towards other political goals [e.g., a politically motivated expansion of employment, financing political activities, etc.]. Under these circumstances the rent relevant for our discussion is the “gross rent,” before subtracting all the costs of politically motivated activities.\footnote{Note that privatization in such an economy may curb the political rent seeking, increasing the resultant efficiency gains.}

- Lobbying for protection [as in Grossman and Helpman (1994)] may mitigate the impact of privatization, as the new owners will use part of their rent to lobby. Yet, lobbying would not reverse the above results – in the initial equilibrium the government got directly all the surplus in the activity under its ownership, whereas in the lobbying game it will get in most cases only a fraction of that surplus. Note also that with uncertainty, privatization may lead to other large indirect effects, like reducing government’s unfounded contingent liabilities.

- Our analysis points out that privatization in developing countries may entail positive externalities by changing the menu of taxes. The two examples described above illustrated that in economies where taxes are determined by the median voter, privatization may lead to efficiency gains by reducing the tax on capital [with social security privatization] and tariffs [with public enterprise liberalization]. Earlier literature dealing with privatization of public enterprise pointed out that, unless privatization is accompanied by liberalization measures, it is unlikely to result in significant gains in economic efficiency [see van de Walle (1989)]. Hence, a public enterprise privatization effort is more likely to succeed if it is part of a comprehensive rationalization and trade liberalization efforts. Similar conclusions apply for social security privatization – its gains are greater and more transparent if it is accomplished with liberalization of financial markets, allowing greater international diversification and freer management of private funds.

- Our analysis suggests that privatization and liberalization are complementary steps, reinforcing each other through the political process. This result should be qualified as it ignores other obstacles to privatization. Fre-
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...quent, the parties that would lose from privatization and liberalization are easier to identify, and are better organized than the parties that would gain from it. This implies that the adjustment of taxes to the new economic environment would be protracted and time consuming. In these circumstances a modified version of our model continues to hold, once proper discounting is applied to the costs and the benefits of privatization.

* The indirect effects of privatization described in the paper are external to the privatized activity. Hence, these benefits are not accounted for in a conventional cost benefit assessment of the privatized projects. Our examples illustrate that ignoring these effects may lead one to underestimate the potential gains of privatization.

References


Appendix

This Appendix summarizes the steps leading to (16). First, note that the optimal consumption of the private sector is obtained by

$$\text{MAX}_{X^c,Y^c}[u(X^c,Y^c) - \lambda(X^c + (1 + \tau)Y^c - I)]$$ (A1)

where $u$ is a neoclassical utility function, $X^c, Y^c$ are the consumption of exportables and importables, respectively, and $I$ is the private sector income -

$$I = X^s + (1 + \tau)Y^s - s_s[(1 + \tau)Y^s - \omega L_y]$$ (A2)

From the first order conditions corresponding to (A1) we infer that

$$dW = \frac{du}{u_X} = dX^c + (1 + \tau)dY^c$$ (A3)

where $u_X$ is the marginal utility of $X$. From the budget constraint $[I = X^c + (1 + \tau)Y^c]$ and (A2) it follows

$$dX^c + (1 + \tau)dY^c = dI - Y^c d\tau =$$

$$\{dX^s + (1 + \tau)dY^s\} + Y^s d\tau - s_s[(1 + \tau)dY^s - \omega dL_y] - s_s[Y^s d\tau - L_y d\omega] - Y^c d\tau.$$

Recall that labor is mobile between the two sectors, hence

$$\frac{dX^s}{dL_x} = \omega; \ (1 + \tau) \frac{dY^s}{dL_y} = \omega; \ \text{and} \ dL_x + dL_y = 0,$$ (A5)

from which we infer that

$$dX^s + (1 + \tau)dY^s = \omega[dL_x + dL_y] = 0$$

$$(1 + \tau)dY^s - \omega dL_y = 0$$ (A6)

Applying (A6) to (A3) and (A4) we infer that

$$dW = Y^s d\tau - s_s[Y^s d\tau - L_y d\omega] - Y^c d\tau = -IM_y d\tau - s_s[Y^s d\tau - L_y d\omega].$$ (A7)
Equation (15) is obtained by substituting (A7) for $dW$ in (14), after collecting the various terms.

We conclude the Appendix by showing that $Y^s - L_y \frac{d\omega}{d\tau} > 0$, a condition that is applied in signing (16). Note that equilibrium in the labor market implies that

$$L_x + L_y = \bar{L} \quad \text{(A8)}$$

where $\bar{L}$ is the supply of labor. Thus,

$$\theta_L \hat{L}_x + \theta_L \hat{L}_y = 0, \quad \theta_L + \theta_L = 1, \quad \text{(A9)}$$

where $\theta_L; \theta_L$ correspond to the labor share in the relevant sectors. Applying (A5) we infer that

$$\hat{L}_x \eta_{X,L_x} = -\omega; \quad \hat{L}_y \eta_{Y,L_y} + \hat{t} = -\omega \quad \text{(A10)}$$

where $\eta_{X,L_x} = -\frac{d \log^2 X^s}{d \log L_x^2}; \quad \eta_{Y,L_y} = -\frac{d \log^2 Y^s}{d \log L_y^2}$.

Applying (A9) to (A10) we infer that

$$\frac{d\omega}{d\tau} = \frac{\omega}{1 + \tau} \Omega \quad \text{where} \quad \Omega = \frac{\theta_L / \eta_{Y,L_y}}{\theta_L + \theta_L / \eta_{Y,L_y}} < 1. \quad \text{(A11)}$$

Thus,

$$Y^s - L_y \frac{d\omega}{d\tau} = Y^s - L_y \frac{\omega}{1 + \tau} \Omega = \frac{1}{1 + \tau}[(1 + \tau)Y^s - L_y \omega \Omega] > \frac{1}{1 + \tau}[(1 + \tau)Y^s - L_y \omega], \quad \text{(A12)}$$

Note that $(1 + \tau)Y^s - L_y \omega$ are the rents in sector $Y$, hence $(1 + \tau)Y^s - L_y \omega > 0$, and $Y^s - L_y \frac{d\omega}{d\tau} > 0$. 
