International Trade and the Risk Premium in the Currency Forward Market

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Abstract

In this paper we present an intertemporal model of the spot and forward markets for foreign exchange. We analyze the implications of central bank interventions on the spot market for the risk premium in the currency forward market and discuss the consequences for the allocation of exchange rate risk and for the volume of international trade. As a main result we find that exchange rate volatility does not generate systematic risk and hence does not adversely affect international trade as long as the monetary authorities do not exogenously intervene in the foreign exchange spot market. (JEL Classification: F11, F31, F33)

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I. Introduction

With the breakdown of the Bretton Woods System and the beginning of floating between the major currencies, exchange rates of many industrial countries have become highly volatile. A popular conjecture claims that exchange rate instability imposes an adjustment burden on the real sector of an economy. This adjustment burden is caused by the exchange rate related uncertainty in international transactions and manifests itself in the form of a reduced volume of international trade (Cushman [1986], Broll, Wahl and Zilcha [1995], Broll and Eckwert [1996]). Most work in this vein uses a partial equilibrium approach focusing on exporters or importers concerned with revenues in home currency. In the absence of well developed forward markets there is some unequivocal evidence of a negative impact of exchange rate risk on export and import volumes.

The situation is different if forward markets for foreign exchange exist on which the exchange rate risk can be hedged. In this case the decision problem of an international firm exhibits a classical separation property according to which the export/import level of the firm only depends on the forward rate (Ethier [1973], Kawai [1981]). Thus, under proper optimization, the probability distribution of the exchange rate will affect the real economy only to the extent that it affects the forward rate. If investors were risk-neutral, then the forward market would be unbiased, i.e. the market should set the forward rate equal to the expected future spot rate. In this case, greater volatility (mean preserving spread) of the foreign exchange rate will have no effects on the allocation of resources, and therefore on foreign trade. Yet, if the participants in the foreign exchange market are risk-averse, then the forward market might be biased through the presence of a risk premium. Moreover, the risk premium could vary over time and thereby affect international trade.

In this paper we argue that the question whether or not exchange rate volatility creates an adjustment burden for the real economy critically depends on the behavior of the central bank. As long as the central bank refrains from intervening in the foreign exchange spot market, the exchange rate risk is non-systematic and therefore cannot spill over into the real sector of the economy. However, if the central bank acts as a buyer or a
seller on the spot market then the exchange rate fluctuations are turned into a systematic risk which affects the real economy. Thus, our analysis suggests that (costly) real sector adjustments must be attributed to active intervention policies rather than to exchange rate variability.

The plan of the paper is as follows. Section II lays out an intertemporal model of an open economy and derives the optimal behavior of the market participants. In Section III we study the implications of the central bank policy on the foreign exchange spot market for the determination of the forward rate and for the flows of international trade. A summary and some conclusions are provided in Section IV.

II. Exports, Imports and the Markets for Foreign Exchange

Consider a two-period model with three types of economic agents: a central bank, importers, and exporters. At date 0, the forward market for foreign exchange opens and the contracts on the import and export markets are concluded. We assume that invoicing occurs in the foreign currency and that all balances are settled at date 1 (one-period trade credit). In order to close open positions for foreign currency the agents can trade on a spot foreign exchange market at time 1. The random one period ahead spot foreign exchange rate is \( S \).

**Central Bank:** The monetary authorities act on the spot and forward markets for foreign exchange. On the forward market the central bank stands ready to satisfy the private agents' demand for foreign currency by selling forward exactly the amount of foreign exchange which is needed to clear the market. Thus the forward position of the central bank, \( B_f \) (amount of foreign currency sold forward at date 0), will be determined endogenously in equilibrium. On the spot market central bank interventions take the form of exogenous foreign currency swaps. We denote by \( B_s \) the central bank's supply of foreign currency on the spot market at date 1. \( B_s \) is a deterministic policy variable which is known to all agents when the markets open.¹

¹. This grossly simplified specification of the central bank intervention strategy on the forward market and on the spot market is a theoretical abstraction not designed to
Exporting Firms: The export sector produces a quantity $X$ of an export good at deterministic domestic costs $C(X)$, and sells on competitive world markets at price $P$. Thus, if calculated in terms of domestic currency, the export revenues, $SPX$, are uncertain. The cost function is strictly increasing and convex, i.e. $C'(X) > 0$, $C''(X) > 0$. Let $K$ be the amount of foreign currency sold forward against the forward rate $F$ at date 0. Then the profit function of the export sector takes the form

$$\Pi_{EX} = SPX - C(X) + (F - S)K.$$  \hfill (1)

The exporter evaluates his random domestic profits according to a risk-averse von Neumann-Morgenstern utility function $U_{EX}: IR_+ \rightarrow IR$, $U'_{EX} > 0$, $U''_{EX} < 0$.

After some rearrangements the first order conditions for the exporter’s decision problem

$$\max_{x, k} E U_{EX}(\Pi_{EX}),$$

can be stated as

$$C'(X) = PF,$$  \hfill (2)

$$EU'_{EX}(\Pi_{EX})(F - S) = 0.$$  \hfill (3)

Observe that the equation system (2)-(3) exhibits a recursive structure which implies that the export decision can be separated from the hedging decision. In particular, according to equation (2), the export decision $X$ depends only on the forward rate $F$, while the optimal hedge $K$ is also a function of the exporter’s preferences and of the distribution of the spot exchange rate $S$. This is a consequence of the well-known separation theorem (Ethier [1973], Danthine [1978]).

replicate the real behavior of central banks in any given period of time. Such an abstraction seems acceptable because the results derived in this paper depend neither on the specific way in which equilibrium is achieved on the forward market nor on the decision rule on the part of the central bank which may have led to the currency swap on the spot market. Therefore, in an effort to keep the complexity of the theoretical framework low, we have modeled the central bank activities in this stylized manner, thereby avoiding the technical complications which arise from more sophisticated central bank intervention strategies.
The optimal export volume can directly be read off equation (2) and is an increasing function of the forward rate. Equation (3) determines the hedging position of the export sector. The term $EU_{EX}^F$ represents the utility loss of paying the forward price for one unit of foreign currency. At the optimum this marginal loss matches the expected marginal utility gains, $EU_{EX}^S$, provided by the future value of the foreign currency unit on the spot exchange market.

**Importing firms:** The import sector imports an intermediate good, $Y$, from abroad and incurs random costs of $SQY$ domestic currency units, where $Q$ is the fixed foreign currency unit price. The imported good is transformed into a final good by means of a technology described by the neoclassical production function $G(Y), G'(Y) > 0, G''(Y) \leq 0$. Final goods are sold on a domestic market, where the importers face the inverse demand function $P_d(G(Y)), \epsilon[P_d, G] = -\frac{\partial P_d}{\partial G} P_d > 0$. $P_d(G(Y))$ is the unit price of the final good at which the quantity $G(Y)$ can be sold. Since the importers sell the final good at a deterministic price and buy the intermediate good at a random price, their net revenues are uncertain. The importers may hedge against the revenue risk by contracting on the currency forward market. Denoting by $L$ the amount of foreign currency sold forward the profit function of the import sector reads:

$$\Pi_{IM} = P_d(G(Y))G(Y) - SQY + (F - S)L. \quad (4)$$

The necessary and sufficient conditions for the importer’s decision problem

$$\max_{Y,L} EU_{IM}(\Pi_{IM}),$$

are

$$P_d G'(Y) (1 - \epsilon[P_d, G]) = FQ, \quad (5)$$

$$EU_{IM}'(\Pi_{IM})(F - S) = 0. \quad (6)$$

Equations (5) and (6) lend themselves to an analogous interpretation as

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2. The transformation of the intermediate good into a final good in the home country may be necessary because the transformation technology is not available abroad. Another interpretation involves transportation costs which can be modeled as shrinkage of the good: If a quantity $Y$ is bought abroad by the import sector, then quantity $G(Y)$ (assumed to be less than $Y$ in this case) arrives in the home country.
equations (2) and (3). Again the agent’s decision problem exhibits a recursive structure and the import decision can be separated from the hedging decision.

Note from equations (3) and (6) that

$$\text{Cov}[U'_{EX}(\Pi_{EX}), S] = 0,$$

$$\text{Cov}[U'_{IM}(\Pi_{IM}), S] = 0. \quad (7)$$

holds, if and only if the forward market is unbiased, i.e. $F = ES$. By the properties of the utility and profit functions (7) and (8) are equivalent to $K = PX$ and $L = -QY$. Thus, the speculative positions of importers and exporters vanish if and only if the risk premium on the forward market is zero.

**III. The Risk Premium in the Forward Market**

In this section we analyze how central bank interventions on the spot market affect the risk premium on the forward market and international trade. The forward market at date 0 clears if $L + K + B_f = 0$ holds. At the final date 1, no private actor wishes to hold foreign currency. Therefore, at this date the excess supply of foreign exchange on the spot market equals $(PX - K) - (L + QY) + B_s$, where $PX - K$ and $-(L + QY)$ are, respectively, the amounts of foreign currency supplied by the export sector and by the import sector. Thus, using the equilibrium condition for the forward market and defining the trade balance as the difference between exports and imports, $T := PX - QY$, the clearing condition for the spot market can be stated as $T + B_f + B_s = 0$. Recall that the central bank intervention on the spot market, $B_s$, is a policy parameter while the forward rate, $F$, and the forward position of the central bank, $B_f$, are endogenously determined in equilibrium. This leads us to the following

**Definition: (Equilibrium)** An equilibrium consists of a forward rate $F$ and a forward exchange market intervention by the central bank, $B_f$, such that the forward foreign exchange market and the spot foreign exchange market clear:

$$L + K + B_f = 0, \quad \text{(forward market)} \quad (9)$$

$$T + B_f + B_s = 0. \quad \text{(spot market)} \quad (10)$$
Let $\tilde{S} = ES$, and denote by $R = \tilde{S} - F$ the premium on the currency forward market. The following theorem claims that in equilibrium the premium on the forward market is zero, if and only if the central bank does not intervene on the spot market.

**Theorem 1:** In equilibrium the currency forward market is unbiased, i.e. $F = ES$ holds, if and only if the central bank does not intervene on the currency spot market. The forward market is upward biased (positive risk premium), if the central bank intervention on the spot market, $B_s$, is negative. The forward market is downward biased (negative risk premium), if the central bank intervention on the spot market is positive.

**Proof:** To prove the theorem it is sufficient to show that

(i) $B_s = 0$ implies $R = 0$,

(ii) $B_s (\prec) 0$ implies $R (\prec) 0$,

Combining equations (9) and (10) and using the definition of the trade balance $T$, we obtain the equilibrium relationship

$$(PX - K) - (QY + L) + B_s = 0$$

(11)

With $B_s = 0$ equation (11) is satisfied if and only if there is no risk premium in the forward market. To see this, note that $R = 0$ implies the equalities (7) and (8). However, by the definition of $\Pi_{EX}$ and $\Pi_{IM}$, (7) and (8) hold if and only if $K = PX$ and $L = -QY$, from which (11) follows immediately. Similarly, in view of (3) and (6), $R (\prec) 0$ implies a negative (positive) sign for the covariances in (7) and (8). From this we conclude $PX (\prec) K$ and $QY (\prec) -L$, and hence $B (\prec) 0$ by (11). Thus we have shown that $R (\neq) 0$ implies $B_s (\neq) 0$ which proves (i), and $R (\prec) 0$ implies $B_s (\prec) 0$ which proves (ii).

The central economic mechanism which underlies the result in Theorem 1 can best be understood by concentrating on the behavior of the speculative parts of the hedges of the export sector, $PX - K$, and of the import sector, $L + QY$. Since the agents are symmetrically informed and risk-averse they all speculate in the same direction, i.e. they all take positive speculative positions on the forward market if the risk premium is negative and negative speculative positions if the risk premium is positive. Speculation is absent only if the risk premium vanishes. However, no private agent wants
to hold foreign currency beyond date 1. Thus, at date 1, all traders supply
the total of their foreign currency positions on the spot market. In case the
central bank does not intervene on the spot exchange market, the specula-
tive positions of the agents must cancel out in the aggregate. This in turn
implies a zero risk premium, because importers and exporters never take
opposite speculative positions on the forward market.

Now consider the case where the central bank buys foreign currency on
the spot market, \( i.e. B_s < 0 \). Market clearing then requires that the private
agents supply foreign currency on the spot market. The private agents will
do so only if they have taken negative speculative positions on the forward
market. However, at date 0 an agent has no incentive to take a negative
speculative forward position unless there is a positive risk premium on the
forward market.

In the terminology of modern finance theory, Theorem 1 says that
exchange rate fluctuations do not constitute a systematic risk factor as long
as the central bank does not intervene on the foreign exchange spot mar-
ket. In the absence of exogenous interventions on the spot market, risks
resulting from movements of the exchange rate can be fully diversified,
because any appreciation of the home country’s currency is linked to a de-
preciation of the foreign country’s currency. Thus, if the spot market clears,
the exchange rate losses of the agents on one side of the market are just off-
set by exchange rate gains of the agents on the other side of the market, so
that the exchange rate risks cancel out in the aggregate.

Central bank interventions are normally justified on the grounds that they
contribute to more stable exchange rates and thereby limit the risks of
international trade. Our model suggests that just the opposite might be
true: if the central bank does not intervene on the spot exchange market
and hence the forward market is unbiased, then the risk allocation in the
economy is optimal in the sense that all risks will be perfectly hedged at no
cost. The situation is different if the central bank acts as a buyer or a seller
on the spot market. According to Theorem 1 interventions on the spot mar-
et leave the forward market biased, which means that the exchange rate
fluctuations are turned into a systematic risk which can no longer be
hedged at no cost. However, if hedging becomes costly then the exchange
rate risk will no longer be fully hedged. Hence, in equilibrium, the profits of
the import sector and of the export sector will both be random. Since the distribution of the economy-wide risk factor, the stochastic exchange rate, has not changed, we may conclude that central bank interventions on the spot exchange market prevent full risk diversification and therefore result in a suboptimal allocation of the exchange rate risk.

**Corollary:** Assume that the elasticity of the inverse demand function is constant and less than 1, i.e. $\epsilon[P_d,G] = \alpha$, $\alpha \in ]0,1[$. Let $\tilde{T}$ be the equilibrium trade balance associated with the central bank policy $B_s = 0$ (no spot market intervention). Then $T < \tilde{T}$ if and only if $B_s > 0$.

The corollary is an immediate consequence of Theorem 1: If the central bank intervention, $B_s$, is positive, then the forward market will be downward biased, i.e. $F > S$. According to equations (2) and (5) the export value depends positively and the import value depends negatively on the forward rate. Hence the trade balance is an increasing function of the forward rate and therefore is larger in a situation in which the forward market is downward biased as compared to a situation in which the forward market is unbiased.

By the above result the central bank policy on the foreign exchange spot market not only affects the financial sector of an economy but also has an impact on international trade. Monetary disturbances are transmitted into the real economy via the forward foreign exchange market and may cause significant fluctuations in the volume and direction of international trade.

**IV. Conclusions**

This paper studies the implications of equilibrium on the spot and forward markets for foreign exchange. In the absence of central bank interventions on the foreign exchange spot market our study provides a theoretical basis for the empirical evidence which suggests that forward foreign exchange markets are approximately unbiased (Hodrick [1987], Goodhard [1988], Macklem [1991], Beetsma [1995], Malliaropulos [1995]). Our paper also demonstrates that risk premia on these markets need not necessarily be understood as a disequilibrium phenomenon. Positive or negative risk premia on the forward market may be caused by central bank interventions in the foreign exchange spot market.
Since the breakdown of the Bretton Woods System there have been countless occasions on which monetary authorities have officially intervened in the spot markets for foreign exchange. Even if the central banks agree that the time path of the exchange rate should be determined by market forces, it might nevertheless be in their interest to smooth out the daily erratic movements of the exchange rate without affecting its time trend. In a generalized version of our model such interventions would lead to a risk premium on the forward market, which fluctuates randomly around zero.

The central bank policy on the foreign exchange spot market not only affects the financial sector of the economy but also has an impact on international trade. Exogenous interventions on the spot market are transmitted into the real economy via the forward market for foreign exchange. Such interventions affect both the volume and the direction of international trade flows.

Due to the simplicity of the modeling approach the analysis in this work is subject to a number of limitations. Most importantly, in our model the random spot exchange rate constitutes the only source of uncertainty which affects the agents' decision problems. In economies with multiple risk factors (endowment risks, dividend risks, policy risks etc.) agents might be willing to hold long positions or short positions on the forward exchange market even if the risk premium is zero. For example, agents who face some endowment risk in the future could construct a partial hedge at no costs by selling forward foreign currency, if the spot exchange rate is positively correlated to the endowment risk. In such a generalized framework the interactions of the spot exchange rate with other risk factors could generate a positive or negative risk premium on the forward market even if no central bank intervention occurs on the spot market.

In the real world central bank interventions on the spot exchange market are often compensatory with respect to stochastic shocks rather than autonomous as we assumed in our analysis. This feature could be dealt with in a slightly generalized version of our model in which the interventions by the central bank are modeled as a stochastic activity. In such a more general context a negative correlation between \( B_s \) and some stochastic shock would indicate that the intervention is compensatory with respect to that shock.

It is also worth mentioning that the analysis in this paper assumes away all informational asymmetries across agents. Obviously, expectations about
the distribution of the future spot exchange rate may affect the forward rate. If these expectations depend on private information which is asymmetrically distributed in the economy, then part of this information will be revealed by the equilibrium prices. The process of information transmittal may affect the forward rate through its impact on the agents' expectations.

References


