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Monetary and Fiscal Policy in the EMU : Conflict or Coordination?

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Abstract

The common central bank of a monetary union tends to lead a more accommodative monetary policy in order to avoid the default of the moderately indebted member countries whereas the most hardly indebted countries have to default. The optimal inflation rate increases the more numerous are the highly indebted countries in the union, the higher are the interest rates on the risk free capital and the smaller are the interest rates on nominal bonds in the fiscally weak countries. This study considers conflict and coordination between integrated public.

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- **Keywords:** Monetary Policy, Monetary Union, Public Debt, Inflation Rate

I. Introduction

The member countries of a monetary union lose their autonomy to lead independent monetary and exchange rate policies. And their budgetary policies can neither remain widely independent and autonomous. Indeed, the common monetary policy affects the fiscal policies, the budgetary deficits and the public debts, as well as these fiscal policies themselves influence the behavior of the common central bank and contribute to direct and to define the common monetary policy. The Fiscal theory of the price level has particularly contributed to show the impact of the budgetary policy and of the level of indebtedness on the inflation rate

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and on the optimal monetary policy. Moreover, in Europe, the monetary unification has reinforced the narrowness and the complexity of these monetary and fiscal interdependencies; it has modified the public debt management as well as its consequences on the inflation rate.

First, the creation of the Economic and Monetary Union (EMU) has changed public debt management (Favero *et al.*, 2000). Indeed, the speculative demand and the demand for portfolio diversification related to exchange rate variations have disappeared, as all the bonds are now denominated in Euros. Today, the competition between EMU member States in issuing government securities only concerns the liquidity and the risk of default (credit risk) of their assets. Some countries may be tempted to adopt non cooperating behavior in the choice of the issue dates or in the information about the quality of their assets. However, debt structures and maturities (debt duration) have strongly converged in EMU, and an efficient and liquid market for debt instruments has been created, which contributes to ease the conduct of the common monetary policy: the liquidity and the transmission channels of monetary policy are more easily handle-able. This market relies essentially on fixed-rate medium and long term bonds, exchanged on well-integrated markets with large trade volumes, whereas the share of the markets for variable rates securities has much been reduced (Favero *et al.*, 2000; de Haan and Wolswijk, 2005).

The longer term duration of debt implies that the investors support the anti-inflationary policy of the European Central Bank (ECB), as well as it is explained by the greater price-stability provided by an independent central bank. Moreover, it also contributes to isolate the governments' budget from monetary policy and variations in interest rates. Indeed, in the private sector, minimizing interest rates costs or the risks of large fluctuations in these payments are the key considerations for debt management. However, in the public sector and at the level of the European Union (EU), macroeconomic goals also tend to be important. They can include the macroeconomic stabilization (smoothing tax rates, stabilizing public deficits.), the development of national financial markets, or the support of monetary policy (de Haan and Wolswijk, 2005). In this context, Missale (2001) studies the optimal debt management in the framework of the Stability and Growth Pact, introducing deficit stabilization as a new important objective. Then, he finds that a longer maturity structure of conventional debt is optimal if the ECB places a lower weight on output stabilization than national monetary authorities and if the EMU member States are hit by asymmetrical shocks. Besides, the lower the weight assigned by the ECB to output stabilization, the more attractive is inflation indexed debt for deficit stabilization.

In these conditions, how can this more integrated public debt market impact the common monetary policy, and the optimal inflation rate in a monetary union?

In a discretionary monetary policy setting, giving rise to inflation bias, Beetsma and Bovenberg (1999) show that monetary unification relaxes budgetary discipline and increases public debt accumulation. This harms welfare, if the governments are sufficiently myopic and give a higher preference than the society to the current and immediate economic situation. Debt ceilings should then usefully be introduced in a monetary union, in order to allow an independent central bank to preserve price stability. In the same way, Traficante (2009) shows that in a monetary union, the intertemporal solvency constraint for the governments only holds at the aggregate level. Thus, without ceilings on the debt levels, a country could become the permanent debtor of its partners. The coordination between the budgetary policies of the member countries is thus necessary. The coordination between the monetary and budgetary authorities is also necessary, according to Van Aarle *et al.* (1995). Indeed, the authors underline the strategic interaction between the monetary authority which controls monetization and the fiscal authorities which control primary fiscal deficits. A conflict can therefore arise if these authorities have distinct objectives regarding inflation, debt stabilization or public spending. Nevertheless, Van Aarle *et al.* (1997) also show that in a monetary union, the central bank can't really be exploited as common property by undisciplined governments: in case of fiscal difficulties, they can't rely on a more accommodative monetary policy. Debt stabilization is quicker with a common central bank than with individual national central banks, and inflation as well as fiscal deficits are then lower. Indeed, the strategic position of the common central bank strengthens, as governments lose power due to their smaller relative economic size.

Bergin (2000) applies the Fiscal theory of price-level determination to the case of a monetary union. He finds that a rise in the debt level by one member government can raise the common price level, suggesting a role for fiscal solvency rules in a monetary union. In the same way, in a closed economy setting, Leith and Wren-Lewis (2000) assume that when monetary policy seeks to raise interest rate in case of inflationary tensions, a self stabilizing and contractionary budgetary policy is necessary to ensure the long run stability of the model. On the contrary, a fiscal policy that doesn't, by itself, ensure fiscal solvency constrains the monetary policy to be more 'passive' and to accept a possible higher level of inflation by decreasing real interest rates. Woodford (1996) also shows that debt limits are a necessary condition to be able to charge the common central bank of a monetary union with responsibility for maintaining a stable value for the common currency. Indeed, an independent central bank leading a steady non inflationary monetary policy is not sufficient to achieve price stability. A country that shares a common currency with another one exposes itself to price level instability and to the fluctuations in economic activity resulting from the fiscal instability in the other country, even if it is itself a model of fiscal probity.

Besides, Beetsma and Vermeylen (2007) study the implications of monetary unification for real interest rates and relative debt levels. They find that the common currency makes the inflation rates and the risk return characteristics of the participating countries more similar, so that the substitutability of their public debts increases after monetary unification. Then, the share of the debt issued by undisciplined governments increases, as well as the average expected return on the debt. The relative debt levels may thus become a source of tensions for the political sustainability of the monetary union. Nevertheless, Creel and Le Cacheux (2007) contest the hypothesis that the divergences in the inflation dynamics would tend to vanish in a monetary union. They assume that debt levels and real returns on these debts are not fully substitutable in a monetary union, as they contribute to finance more or less productive investments. Therefore, according to them, imposing homogeneous fiscal rules, like those of the Stability and Growth Pact, to the heterogeneous members of a monetary union may be counterproductive.

Furthermore, Jahjah (2000) shows that in a monetary union with an independent central bank and a sufficiently large number of relatively small member countries, the latter tend to accumulate less debt, and thus, that an equilibrium with no inflation and no default on the debt of the governments exists. However, a highly indebted country would be more likely to default if it joins the monetary union than if it remains outside. Therefore, the monetary unification would have substituted a default risk to the inflationary risk, in case of high levels of indebtedness. In the same tradition, the current paper aims at studying the consequences of the various indebtedness levels of the member countries of a monetary union on the optimal inflation rate and on the monetary policy of the common central bank. We show that the optimal inflation rate increases the smaller is the weight given by the central bank to the aim of price stability, the more numerous are the highly indebted countries in the monetary union, and the higher is the interest rate on the risk free capital. Besides, high interest rates on nominal bonds in the fiscally weak countries reduce the optimal inflation rate in the monetary union, but they also increase the probability of default of these countries.

The structure of the paper is as follows. The second section describes the model and the representative economic agents. The third section defines, according to the levels of indebtedness of three country types in the monetary union, the optimal monetary policy and inflation rate, as well as their determinants. The fourth section analyzes the link between the productive nature of public expenditures and the interest rate on nominal bonds in a given country, and its empirical relevance for the EMU. Finally, the fifth section concludes.

II. The Model

We consider a monetary union of n member countries. In each country, a benevolent government maximizes the utility of a representative agent. This government accumulates a public debt in period 1, and must fully pay back in period 2 this public debt. To repay the debt, according to the monetary policy, the government has the choice between raising taxes, decreasing public expenditures, or defaulting, whose cost is proportional to the amount which is defaulted. The government maximizes a well-being depending on the inflation level, on the economic activity, and on the risk of default. The budgetary policies are decentralized and defined at the national level.

Furthermore, there is a common central bank, which defines the common monetary policy for all the monetary union, and which is concerned about the average welfare in the entire monetary union. Its aim is to achieve price stability, but also to ensure the stability of all the financial system. Without barriers to commodity trade, the inflation rate is supposed to be identical in all the monetary union.

A. The representative agents

There are two periods in our model. In period 1, for example before the advent of the monetary union, the government of the country (i) issues nominal bonds whose interest is (b_i) for an amount ($B_{i,1}$). The share of inflation indexed bonds remains very weak today in the Euro area countries, even for those issuing such bonds (France, Greece, Italy, Germany); so, we neglect here the issuing of inflation indexed bonds. The individuals buy nominal bonds; they also invest in capital ($I_{i,1}$)

whose real interest rate is (r). As capital is highly mobile at the international level, this interest rate (r) is supposed to be identical in all member countries of the monetary union. The individuals also hold cash money (M_i) bearing no interest. We also make the hypothesis that the individuals are perfectly rational, and thus, are indifferent between holding capital or bonds, whose returns must therefore be equalized:

$$(1-d_i)(1+b_i-\pi)=(1+r) \quad (1)$$

Where (π) is the inflation rate in the monetary union, and (d_i) is the probability of default of the country (i) on its debt in period 2.

The real return of the nominal bonds adjusted for the inflation rate and for the probability of default equals the real return of the risk free capital.

Real cash holdings in period (t) are: $M_i=(MM_{i,t}/P_{i,t})$, and this money demand is supposed to be constant. With: ($MM_{i,t}$): nominal money; ($P_{i,t}$): Price level.

So, the seigniorage revenues in period 2, the units of consumption goods to be purchased with newly printed money, are:

$$\frac{(MM_{i,2}-MM_{i,1})}{P_{i,2}}=\frac{MM_{i,2}}{P_{i,2}}-\left(\frac{MM_{i,1}}{P_{i,1}}\right)\left(\frac{P_{i,1}}{P_{i,2}}\right)=M_{i,1}-\frac{M_{i,1}}{1+\pi}=\frac{\pi M_i}{1+\pi}$$

In period 1, the saving of the agents is the share of their resources (labor revenues net of public taxes) which is not consumed. This sparing can be realized in capital or in public debt.

$$S_{i,1}=(1-t_{i,1})Y_{i,1}-C_{i,1}=I_{i,1}+B_{i,1} \quad (2)$$

Where ($S_{i,t}$) is sparing and ($C_{i,t}$) is consumption of the representative agent; ($t_{i,t}$) is the taxation rate; ($Y_{i,t}$) are real labor revenues, in country (i) in period (t).

In period 2, individuals consume all their wealth: labor revenues net of public taxes, the returns on capital and on public bonds, as well as cash holdings net of the seigniorage tax.

$$C_{i,2}=(1-t_{i,2})Y_{i,2}+(1+r)I_{i,1}+(1-d_i)(1+b_i-\pi)B_{i,1}+\frac{M_i}{1+\pi}$$

$$=(1-t_{i,2})Y_{i,2}+(1+r)(1-t_{i,1})Y_{i,1}-(1+r)C_{i,1}+[(1-d_i)(1+b_i-\pi)-(1+r)]B_{i,1}+\frac{M_i}{1+\pi} \quad (3)$$

We can make the hypothesis that for the individuals, the discount rate of the future equals the interest rate. Thus, we suppose that the inter-temporal utility of the representative agent, which must be maximized, has the following expression:

$$U_i = \log C_{i,1} + \frac{1}{1+r}\log C_{i,2} \quad (4)$$

B. The governments

In period 1, for example before the creation of a monetary union, the budgetary constraint of the government (i) implies:

$$G_{i,1} - t_{i,1}Y_{i,1} = B_{i,1} > 0 \quad (5)$$

Where ($G_{i,t}$) are real public expenditures of government (i) in period t.

The government (i) defines its level of indebtedness ($B_{i,1}$), such as condition (1) is always verified, as well as the taxation rate ($t_{i,1}$).[1]

However, the public debt contracted in period 1 can be used to finance various kinds of public projects. Like Creel and Le Cacheux (2007), we suppose that a share (w_i^p) of the public debt intends to finance public expenditures which are productive, whereas a share (w_i^{np}) only finances unproductive expenditures. Financing public expenditures which are productive through issuing public debt can reduce the governments' loss. On the contrary, financing them through raising taxes may be counterproductive, as the welfare of the current generation might be reduced whereas it would not benefit from these expenditures. The real return on the debt due to productive investments is below the risk free interest rate [$b_i^p < r < b_i = \frac{(r+d_i)}{(1-d_i)} + \pi$], as their usefulness can decrease their real *ex post* cost; on the contrary, the real return on the debt due to unproductive investment is above the risk free interest rate [$b_i^{np} > b_i = \frac{(r+d_i)}{(1-d_i)} + \pi > r$]. Nevertheless, neither the

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1 $$\frac{\partial U_i}{\partial t_{i,1}} = \frac{1}{C_{i,1}}\frac{\partial C_{i,1}}{\partial B_{i,1}} + \frac{1}{(1+r)C_{i,2}}\frac{\partial C_{i,2}}{\partial B_{i,1}} = \frac{1}{(1+r)C_{i,2}}[(1-d_i)(1+b_i-\pi)-(1+r)] = 0$$

$$\frac{\partial U_i}{\partial t_{i,1}} = \frac{1}{C_{i,1}}\frac{\partial C_{i,1}}{\partial t_{i,1}} + \frac{1}{(1+r)C_{i,2}}\frac{\partial C_{i,2}}{\partial t_{i,1}} = \frac{1}{(1+r)C_{i,2}}\left[-Y_{i,2}\frac{\partial t_{i,2}}{\partial t_{i,1}} - (1+r)Y_{i,1}\right] = 0$$

Therefore, $\frac{\partial t_{i,2}}{\partial t_{i,1}} = -(1+r)\frac{Y_{i,1}}{Y_{i,2}}$ is the only condition to be verified for the optimal taxation rate ($t_{i,1}$*).

central bank nor the representative agents can distinguish the productive or unproductive pattern of public investment. Therefore, the average interest rate on nominal bonds is:

$$b_i = w_i^{np} b_i^{np} + w_i^p b_i^p \quad (6)$$

In period 2, after the creation of a monetary union, the government (i) sets the tax rate ($t_{i,2}$) and must pay back the totality of the public debt contracted in the previous period. It can default a share (d_i) of this debt; however, the default on one unit of public debt implies a cost ($0<c_i<1$), for example in terms of loss of reputation on the financial markets, which could imply higher future risk premiums. These costs are supposed to be heterogeneous across countries, and to depend on their debt levels. Therefore, in period 2, the budgetary constraint of the government (i) is:

$$t_{1,2}Y_{i,2} + \frac{\pi M_i}{1+\pi} = G_{i,2} + (1 - d_i + c_i d_i)(1 + b_i - \pi)B_{i,1} \quad (7)$$

Indeed, the resources of the government (taxes and seigniorage) are equal to its expenditures: public expenditures, and repayment of the previously contracted public debt, net of the inflation reducing the service of nominal bonds, and net of the probability of default.

Therefore, in period 2, the share of debt defaulted by the government (i) is:

$$d_i = \frac{(1+\pi)G_{i,2} + (1+\pi)(1+b_i-\pi)B_{i,1} - (1+\pi)t_{i,2}Y_{i,2} - M_i\pi}{(1-c_i)(1+\pi)(1+b_i-\pi)B_{i,1}} \quad (8)$$

In period 2, the benevolent government sets the tax rate ($t_{i,2}$) in order to maximize the inter-temporal utility of the representative agent (U_i), taking the inflation rate (π) as given.

Thus, (3) and (4) imply: $\frac{\partial U_i}{\partial t_{i,2}} = \frac{1}{(1+r)C_{i,2}} \frac{\partial C_{i,2}}{\partial t_{i,2}} = 0$, and so:

Therefore, the optimal tax rate ($t_{i,2}$*) is such as: $\frac{\partial G_{i,2}}{\partial t_{i,2}} = c_i Y_{i,2}$ (9)

For the country (i), according to equation (8), the share of the debt which is defaulted is therefore (see Appendix A):

$d_i^* = 1$ if: $t_{i,2}Y_{i,2} \le G_{i,2} + c_i(1+b_i-\pi)B_{i,1} - \frac{\pi M_i}{1+\pi}$, that is to say if: $\pi \le \pi_{Li}$ (10)

Indeed, if the fiscal resources aren't enough to cover at least the cost of default on the debt, the country (i) is sure to default on all its debt.

$$0 \le d_i^* = \frac{(1+\pi)G_{i,2} + (1+\pi)(1+b_i-\pi)B_{i,1} - (1+\pi)t_{i,2}^* Y_{i,2} - M_i\pi}{(1-c_i)(1+\pi)(1+b_i-\pi)B_{i,1}} \le 1 \qquad (11)$$

if: $G_{i,2} + c_i(1+b_i-\pi)B_{i,1} - \frac{\pi M_i}{1+\pi} \le t_{i,2}^* Y_{i,2} \le G_{i,2} + (1+b_i-\pi)B_{i,1} - \frac{\pi M_i}{1+\pi}$, i.e. if: $\pi_{Li} \le \pi \le \pi_{Hi}$

If the fiscal resources are moderate, enough to cover the cost of default on the debt but insufficient to repay all the debt, the country (i) default on part of its debt.

$$d_i^* = 0 \text{ if: } t_{i,2}^* Y_{i,2} \ge G_{i,2} + (1+b_i-\pi)B_{i,1} - \frac{\pi M_i}{1+\pi} \text{ that is to say if: } \pi \ge \pi_{Hi} \qquad (12)$$

If the fiscal resources are sufficient to pay the public expenditures, the country (i) doesn't default; there is even a fiscal surplus.

So, if the inflation rate is too low, if monetary policy is too contractionary, servicing the public debt is so costly that the optimal choice for the government (i) is to default on all its debt; no equilibrium with a positive public debt can then exist. As the inflation rate increases, the share of the debt defaulted decreases, and finally, for a sufficiently high level of inflation, the government always chooses to repay all its debt without defaulting. Furthermore, to allow the government (i) to meet its commitments, the necessary level of inflation is an increasing function of the stock of its nominal debt ($B_{i,1}$), of the interest rate on this public debt (b_i), of its public expenditures to be financed ($G_{i,2}$) and of the cost of default (c_i) (see Appendix A). On the contrary, it is a decreasing function of the economic growth and of the real labor revenues ($Y_{i,2}$) as well as of the real cash holdings in this country (M_i).

C. The common central bank

The common central bank aims at minimizing the deviations of average inflation from a target ($\bar{\pi}$); in particular, price stability has been defined as the main aim of the European Central Bank. However, it also aims at preserving the stability of the financial system and at avoiding the risks of default from one government. Indeed, in the event of huge fiscal difficulties, the central bank can be forced to accommodate its monetary policy in order to avoid a fiscal crisis. Even if this risk can be null in good times and with fiscally strong governments, it cannot be avoided in trouble times and if some governments encounter large fiscal problems. Furthermore, we have seen that the recent fiscal difficulties of the Greek government to finance its public debt on the financial markets necessitate empirically, for the monetary authority, to lead a more accommodative monetary policy. The loss function of the common central bank is therefore as follows:

$$L^M = a^M (p-\bar{\pi})^2 + \Sigma_i (\gamma_i d_i) \tag{13}$$

Where (γ_i) is the relative weight, for example in terms of relative GNP, of the country (i) in the monetary union; (a^M) is the relative weight related to price stability in comparison with the risk of default for the central bank.

Let's suppose a Stackelberg equilibrium where the monetary authority sets the inflation rate (π) according to the anticipated reaction function of the governments. Indeed, we make the hypothesis that the monetary authority is unable to commit, and takes into account the budgetary situation of the governments in order to fix the optimal inflation rate. Thus, $\partial L^M/\partial\pi = 0$ implies the following inflation rate:

$$\pi = \bar{\pi} - \frac{1}{2\alpha^M}\sum_{i=1}^{n}\gamma_i\frac{\partial d_i}{\partial\pi} \geq \bar{\pi} \tag{14}$$

Therefore, in order to prevent the default from some governments, the optimal inflation rate fixed by the monetary authority must always be above its target.

III. The Optimal Monetary Policy

A. Three types of countries

Let's suppose that there are 3 groups of countries in the monetary union: (A), (B) and (C), whose relative weights are such as: $\gamma_A+\gamma_B+\gamma_C=1$.

- a group (C) of fiscally weak and very indebted countries, whose relative weight is (γ_C), and whose level of indebtedness is such as:

$$(1+b_i-\bar{\pi})B_{i,1} \geq \frac{(t^*_{i,2}Y_{i,2}-G_{i,2})}{c_i} + \frac{\bar{\pi}M_i}{c_i(1+\bar{\pi})} \qquad (\forall i \in C) \tag{15}$$

- a group (B) of intermediary countries, whose relative weight is (γ_B), and whose level of indebtedness is such as:

$$(t^*_{i,2}Y_{i,2}-G_{i,2}) + \frac{\bar{\pi}M_i}{(1+\bar{\pi})} < (1+b_i-\bar{\pi})B_{i,1} < \frac{(t^*_{i,2}Y_{i,2}-G_{i,2})}{c_i} + \frac{\bar{\pi}M_i}{c_i(1+\bar{\pi})}(\forall i \in B) \tag{16}$$

Thus, these type (B) countries are liable to default, but only on a part of their debt.

- a group (A) of fiscally strong and very weakly indebted countries, whose relative weight is (γ_A), and whose level of indebtedness is such as:

$$(1+b_i-\bar{\pi})B_{i,1} \leq (t^*_{i,2}Y_{i,2}-G_{i,2})+\frac{\bar{\pi}M_i}{(1+\bar{\pi})} \qquad (\forall i \in A) \tag{17}$$

Thus, as ($-\pi \leq -\bar{\pi}$), these fiscally strong countries never default, and (d_A=0).

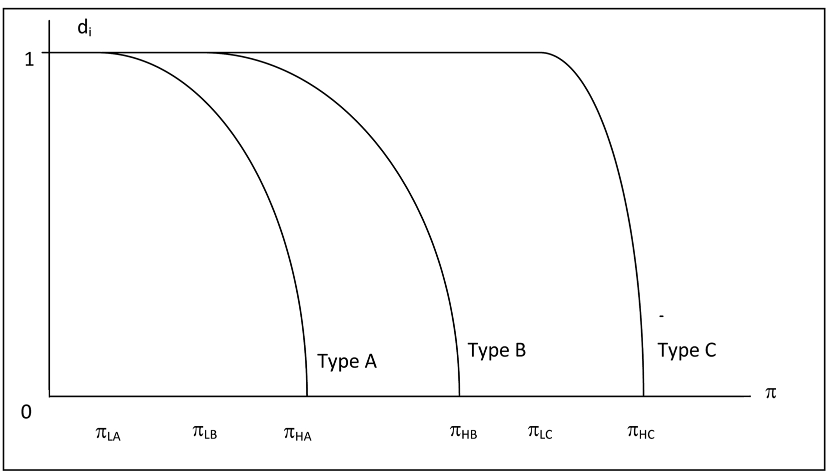
As type (C) countries are the most hardly indebted in real terms, we also make the following hypothesis on the debt levels:

$$\frac{c_C B_{C,1}}{M_C} > \frac{B_{B,1}}{M_B} > \frac{B_{A,1}}{M_A} > \frac{c_B B_{B,1}}{M_B} \qquad b_C > b_B > b_A = r+\pi$$

$$\frac{c_C B_{C,1}}{(t^*_{C,2}Y_{C,2}-G_{C,2})} > \frac{B_{B,1}}{(t^*_{B,2}Y_{B,2}-G_{B,2})} > \frac{B_{A,1}}{(t^*_{A,2}Y_{A,2}-G_{A,2})} > \frac{c_B B_{B,1}}{(t^*_{B,2}Y_{B,2}-G_{B,2})} \tag{18}$$

According to equations (10), (11) and (12), for each type of country, there is then a lower limit for the inflation rate (π_{Li}) under which the country is sure to default on all its debt, and an upper limit (π_{Hi}) beyond which the country is sure to repay all its debt (see Appendix B for their respective positions). Figure 1 then represents

Figure 1. Probability of default and inflation rate



the probability of default of type (A), (B) and (C) countries on their debt, according to the inflation rate in the monetary union.[2]

B. The optimal monetary policy

With a positive risk free real interest rate (r>0), no equilibrium exists where the investors would accept to lend to an excessively indebted country (i) which would be sure to default (d_i=1). Indeed, the interest rate on the nominal debt of this country would tend *ad infinitum…*

With an intermediary level of indebtedness, given the risk free interest rate (r), the return on the nominal debt of the government (i) must be fixed at the level: $b_i = \frac{r+d_i}{1-d_i} + \pi \geq r + \bar{\pi}$, in order to make the public debt as attractive as the risk free asset for the investors. Finally, the real return on the bonds can be fixed at the level ($b_A = r + \pi$) without risk of default, for type (A) countries. So, the maximal debt level that a type (A) country can accumulate without being rationed is:

$$B_{A,1}^{max} = \frac{(t_{A,2}^{*} Y_{A,2} - G_{A,2})}{(1+r)} + \frac{\pi M_A}{(1+r)(1+\pi)}.$$

In this framework, combining the previous equations, the optimal monetary policy verifies (see Appendix B):

$$\pi^* = \pi_{HA} = 2\sqrt{\frac{-p}{3}}\cos\left[\frac{1}{3}acos\left(\frac{-Q}{2}\sqrt{\frac{27}{-P^3}}\right) + \frac{2pi}{3}\right] + \frac{(2+2b_B+\bar{\pi})}{3} \quad (19)$$

$$P = \frac{-(1+b_B-\bar{\pi})^2}{3} \qquad Q = \frac{4\alpha^M \bar{\pi}^2(3+3b_B-\bar{\pi}) + 4\alpha^M(1+b_B)^2(1+b_B-3\pi) - 27\gamma_B(1+r)}{54\alpha^M}$$

For this optimal inflation level, type (A) countries never default, type (B) countries default on part of their debt, whereas type (C) countries default on all their debt.

Proposition 1: The less indebted type (A) countries never default. They would not default on their debt, even if the monetary authority would decide to stick to its target (π;¯) and to lead a strict inflation targeting policy.

[2] π_{LA}, π_{LB} and π_{LC} are respectively the inflation rates (π_{Li}) defined in (10) for types A, B and C countries, whereas π_{HA}, π_{HB} and π_{HC} are respectively the inflation rates (π_{Hi}) defined in (12) for types A, B and C countries (see Appendix A).

Proposition 2: If the public debt is moderate in type (B) countries, the monetary authority can choose to accommodate its monetary policy ($\pi > \bar{\pi}$), which is more inflationary than expected, in order to avoid the default of these countries. The central bank pursues this monetary policy as long as avoiding the default from some governments is worth the cost in terms of deviating from its inflation target.

Proposition 3: The monetary authority doesn't accommodate to avoid the default of the highly indebted type (C) countries. Our model shows that at the optimal inflation rate, these countries have to default on all their debt. Therefore, upon perfect information, financial funds would not accept to buy bonds of these countries, which could thus be forced to leave the monetary union.

Therefore, creating a monetary union between very heterogeneous countries regarding their budgetary and fiscal situation can have harmful consequences. Indeed, in a monetary union, the fiscally strong and virtuous type (A) countries have to finance their public debt at a higher cost than if they had remained independent, because the inflation rate is then higher. On the opposite, fiscally weak type (C) countries can be forced to default on all their debt in a monetary union, without being rescued by the common central bank.

Nevertheless, the measure in which the central bank accepts to deviate from its inflation target depends on the weight given to price stability (a^M). Indeed, if this weight is negligible ($a^M \rightarrow 0$), the central bank tends to accommodate its monetary policy to the most fiscally weak countries and the inflation rate is then very high. On the contrary, if the weight given to price stability is very high ($a^M \rightarrow \propto$), the central bank fixes the inflation rate at the level corresponding to its target ($\pi = \bar{\pi}$), even if some countries have then to default.[3]

C. The factors of a high inflation rate

Beyond a weak preference of the central bank for price stability (a^M) or the presence of a high inflation target ($\bar{\pi}$), according to equation (19), there are other parameters of a high inflation rate in a monetary union. First, the optimal monetary policy highly depends on the relative size of the fiscally weak type (B) countries in the monetary union (γ_B) (see Figure 2). Indeed, the more these fiscally weak countries are numerous in the monetary union, the more the central bank tends to deviate from the targeted level of inflation and to accommodate its monetary policy. On the contrary, the monetary authority is less likely to have an inflationary

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[3]The estimations of equation (19) are made with Excel, using the following basic calibration: b_B=0.04; π;⁻ =0.02; γ_B=0.05; a^M=2; r=0.015. This calibration is also used for Figures (2) and (3).

policy to improve the situation of a single small country in a bad fiscal position.

The calibration of our parameters also shows that the situation of a monetary union could very quickly deteriorate with the number of member countries which encounter fiscal difficulties. Indeed, with a share of only 5% of fiscally weak type (B) countries in the monetary union, the inflation rate would remain around 2.5%. Nevertheless, this inflation rate could rise afterwards very quickly, and attain around 30% in the event of a share of 60% of type (B) countries! Nevertheless, these type (B) countries are only those which are likely to default at least on part of their debt, whereas in the EMU in Europe, for example, the default of one member country is not a conceivable situation, today.

Furthermore, the optimal inflation rate is also a slightly increasing function of the real interest rate on the risk free capital (r) (see Figure 3). Indeed, if this interest rate is very high, the investors ask for a high return to service the debt. Therefore, the risks of default are sizeable, and the inflation rate must be very high in order to avoid the default of some moderately indebted countries. On the contrary, if this interest rate is weak, the service of the debt is less costly. Thus, the inflation rate necessary to avoid the default of some countries can be smaller.

Finally, the optimal inflation rate is a slightly decreasing function of the interest rate on nominal bonds in the fiscally weak type (B) countries (b_B) (see Figure 3). Indeed, the higher this interest rate, the higher are the risks of default, because of

Figure 2. Optimal inflation rate and share of fiscally weak countries (γ_B)

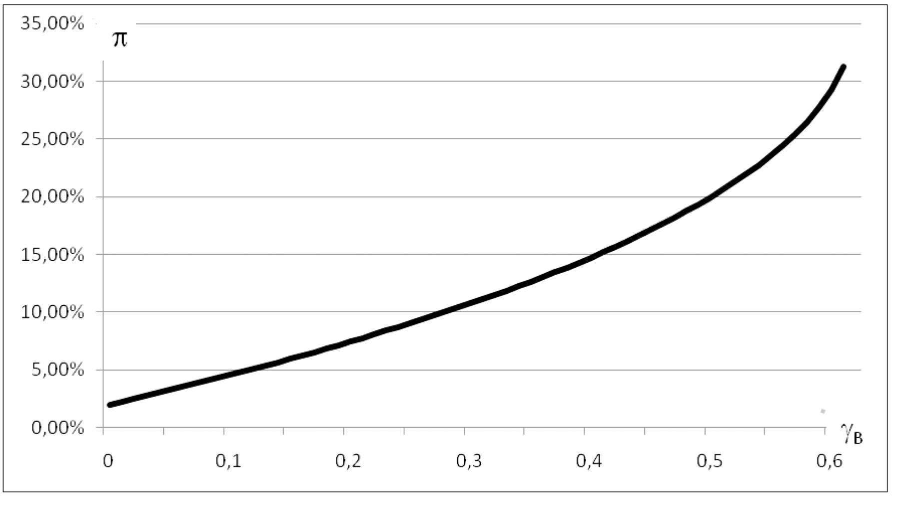
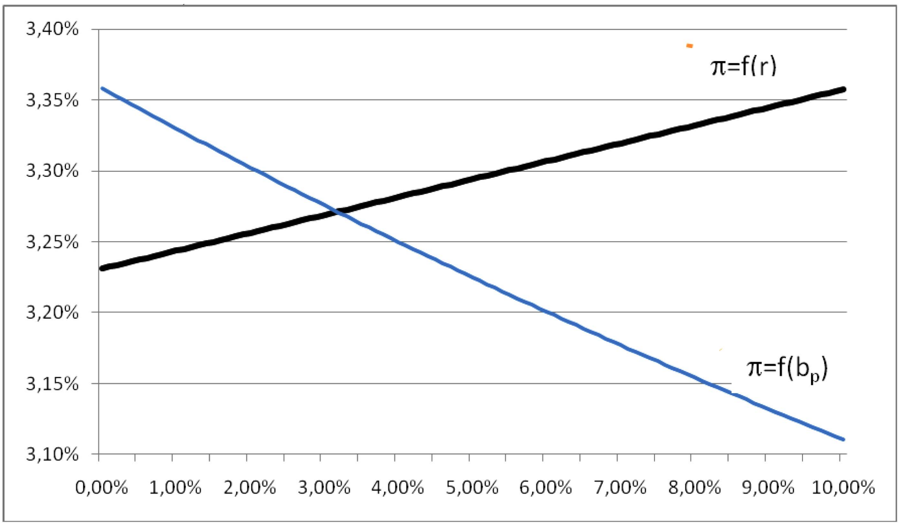


Figure 3. Optimal inflation rate, nominal risk free interest rate (r) and interest rate on nominal bonds (b_B)



the cost of the debt repayment for the type (B) countries. If the central bank gives a non negligible weight to price stability, it has then interest in letting type (B) countries defaulting on a share of their debt growing with the interest rate on nominal bonds (b_B), as an accommodating monetary policy would not be worse the deviation from its inflation target.

IV. The Nature of Public Expenditures

According to our model, in a monetary union, a moderate interest rate on nominal bonds in the already heavily indebted type (B) countries (b_B) has thus a fundamental advantage: it reduces the risk of default of these countries (d_B).

Indeed, Equation (8) implies: $$\frac{\partial d_i}{\partial b_i} = \frac{(1+\pi)(t_{i,2}Y_{i,2}-G_{i,2})+M_i\pi}{(1-c)(1+\pi)(1+b_i-\pi)^2 B_{i,1}} > 0 \qquad (20)$$

The stability of the financial system is therefore more preserved if (b_B) is small. However, the central bank has then interest in avoiding the default of these heavily indebted type (B) countries, as the cost of an accommodating and more expansionary monetary policy is then smaller; the inflation rate is thus higher (see the previous section). In this framework, the productive or unproductive nature of public expenditures can have a fundamental influence on the interest rate on nominal bonds.

A. Productivity of public expenditures and interest rate on nominal bonds

Some empirical studies have underlined the ambiguous nature of public expenditures on the economic growth. Indeed, productive public expenditures can be beneficial and sustain the economic growth, whereas to the contrary, unproductive public expenditures are mainly harmful. For example, Creel and Poilon (2008) apply a VAR methodology to France, Italy, Germany, the United-Kingdom and the United-States between 1960 and 2004; they also analyze a panel of 6 European countries between 1969 and 2002 and a panel of French regions. Then, they show that public investments, and even public capital but to a lesser extent, are significant determinants of output. A 'golden rule of public finance', where government borrowing should not exceed public capital formation, could thus be beneficial. Indeed, it would avoid the current negative incentives of the Stability and Growth Pact, resulting in cuts in capital formation in many European countries, whereas such expenditures are mainly productive. On the contrary, Checherita and Rother (2010) study the impact of debt on per capita GDP growth in 12 EU countries between 1970 and 2010. They find a non linear impact of debt on growth with a turning point at about 90-100% of GDP, beyond which debt would begin to have a deleterious impact on long term growth. Nevertheless, even before this level, annual changes of the public debt ratio and of the deficit-to-GDP ratio would be negatively associated with per capita GDP growth. This effect would be due to higher long term interest rates crowding out private investment. Therefore, whereas productive public expenditures would be beneficial, unproductive expenditures would mainly be harmful. In this framework, it seems important to distinguish the nature of public expenditures in our model.

For a credible inflation targeting policy, when the central bank can credibly fix the inflation level in conformity with its target ($\pi = \overline{\pi}$), all countries in the monetary union must have sustainable and moderate indebtedness levels. The monetary union should only be composed of weakly indebted type (A) countries (see Appendix C). As soon as there are types (B) or (C) countries in the monetary union liable to default on a share of their debt, the inflation level has to be beyond the level targeted by the central bank. Furthermore, the interest rate on nominal bonds in these most indebted countries is then beyond ($r + \overline{\pi}$).

More precisely, according to Equations (1) and (6), the real return on the debt related to unproductive public expenditures is: $b_i^{np} = x_1\left[\frac{(r+d_i)}{(1-d_i)}+\pi\right] \quad x_1 > 1$ (21).

On the contrary, the real return on the debt related to productive public expenditures is:

$$b_i^p = x_2\left[\frac{(r+d_i)}{(1-d_i)}+\pi\right] \qquad x_2 < 1 \tag{22}$$

So, we obtain:

$$b_i = w_i^p\left[\frac{w_i^{np}}{w_i^p}x_1 + x_2\right]\left[\frac{(\gamma+d_i)}{(1-d_i)}+\pi\right] \tag{23}$$

Therefore, according to Equation (23), (bi) is small and the risks of default in a given country (i) member of a monetary union are more limited if the productive public expenditures are more numerous in comparison with the unproductive expenditures in this country (i). In these conditions, it is interesting to look at the empirical results regarding the link between the interest rates on nominal bonds and the productive nature of public expenditures, in the European Economic and Monetary Union.

B. Empirical relevance of our model for Europe

Except in the case of Denmark and Sweden, outside the EMU, Table 1 shows that the government bond yields are generally smaller for the member countries of

Table 1. Government bond yields, 10 years' maturity, 2007

EMU countries							
Austria	Belgium	Finland	France	Germany	Greece	Ireland	Italy
4.05	4.06	4.05	4.07	4.02	4.29	4.04	4.26
Netherlands	Portugal	Slovenia	Spain				
4.07	4.18	4.54	4.07				
Non EMU countries							
Bulgaria	Cyprus	Denmark	Hungary	Latvia	Lithuania	Malta	Poland
4.42	4.51	4.02	6.74	5.63	4.58	4.73	5.50
Slovakia	Sweden	UK					
4.49	3.93	4.94					

Source: Eurostat
All data were not available on Eurostat for the year 2008; and it was the beginning of the financial crisis; thus, the data are those of 2007.

the EMU than for countries which are not member of the monetary union. Indeed, the possibility to benefit from moderate interest rates has always been one of the main lure of the membership in EMU for the European countries. Thus, for the EMU countries, the government bond yields were nearly the same and quite moderate in 2007. However, these yields were still higher in Slovenia, which has just joined the EMU in 2007. They were also higher in Greece, in Italy and in Portugal. Indeed, the financial markets tended to consider that the risks of default on their debts were higher for these fiscally weak European countries.

Therefore, according to our model, these high interest rates on the debt of three heavily indebted EMU countries can contribute to increase their probability of default on a share of their debt (see Equation (20)). Indeed, during the spring 2010, the fast growth in the interest rates on the Greek public debt causes its financing difficulties and the anxiety of the financial markets about the solvability of the Greek government, even if they could also have contributed to maintain the low level of inflation in the EMU (see Figure 3).

In this framework, the classification of public expenditures between those which can be considered as productive and those which are unproductive is a very difficult task. Nevertheless, the empirical economic studies[4] generally conclude that public expenditures in the following fields are particularly productive: Health, Transport and Communication, Education, Housing and community amenities. So, in this framework, Table 2 shows that these productive expenditures represent a particularly weak share of the global public expenditures in Greece (18.80%). Indeed, a proportional higher percentage of public expenditures in Greece than in the other European countries are intended to defense, general public services or social protection. This can contribute to explain the relatively high yields on Greek bonds reported in Table 1. In the same way, in Italy (29.95%) or in Portugal (30.14%), the relative weak share of productive public expenditures can also be explained by the relative importance of expenditures related to general public services, for example.

[4]See for example: Aschauer (1989), Devarajan *et al*. (1996), Kneller *et al*. (1999), or Wang and Davis (2005), for the empirical studies about the public expenditures which are productive.

Table 2. General government expenditures, in millions of Euros, in 2008

	Health	Transport and communic	Education	Housing & community amenities	Total	Share of productive expenditures
Belgium	25 541.0	:	20 436.1	1 197.1	172 426.4	:
Bulgaria	1 597.0	1 025.0	1 434.2	530.9	12 729.1	36.04%
Czech Rep.	10 665.8	7 843.9	6 900.0	1 625.2	63 472.3	42.59%
Denmark	18 098.3	:	16 308.6	1 091.3	120 770.9	:
Germany	165 480.0	38 280.0	96 310.0	16 940.0	1 090 780.0	29.06%
Estonia	832.6	474.1	1 074.0	100.3	6 408.4	38.71%
Ireland	14 227.7	6 104.4	9 715.5	4 337.5	76 407.4	45.00%
Greece	12 266.0	1 225.0	7 471.0	762.0	115 581.0	18.80%
Spain	65 943.0	25 571.0	50 585.0	11 038.0	446 910.0	34.27%
France	152 753.0	:	114 012.0	36 687.0	1 027 044.0	:
Italy	111 661.0	33 584.0	72 528.0	11 671.0	766 134.0	29.95%
Cyprus	516.8	162.5	1 342.2	435.8	7 345.6	33.45%
Latvia	1 119.2	799.2	1 494.0	293.3	8 933.4	41.48%
Lithuania	1 604.2	759.0	1 871.2	121.9	12 045.3	36.17%
Luxembourg	1 753.5	1 215.9	1 719.2	238.4	14 628.2	33.68%
Hungary	5 169.5	3 915.1	5 486.9	1 034.4	51 962.7	30.03%
Malta	318.2	187.4	311.3	45.0	2 556.8	33.71%
Netherlands	35 570.0	:	31 258.0	6 290.0	273 553.0	:
Austria	21 826.4	6 426.4	15 042.3	1 618.3	137 927.6	32.56%
Poland	18 339.5	11 857.3	20 844.2	3 997.0	156 787.1	35.10%
Portugal	10 695.1	2 926.3	10 023.2	-567.6	76 557.2	30.14%
Romania	5 858.6	:	6 659.0	1 897.2	52 508.6	:
Slovenia	2 265.2	946.6	2 288.9	316.7	16 423.5	35.42%
Slovakia	4 328.7	:	2 139.0	409.2	22 542.7	:
Finland	13 005.0	4 157.0	10 935.0	778.0	91 121.0	31.69%
Sweden	22 984.2	8 959.5	22 781.0	2 413.3	173 886.1	32.86%
U-Kingdom	134 873.4	32 371.8	114 965.8	23 886.1	860 245.1	35.58%

: missing data
Source: Eurostat, General government expenditures by function

V. Conclusion

In our model, if some countries have very high levels of indebtedness, the monetary authority faces a trade-off between inflation and the default risk. In this case, in a closed economy, a national central bank could be tempted to ease its monetary policy in response to the budgetary difficulties of the national government. But in a monetary union with many countries where only one of them has budgetary difficulties, the marginal benefit of inflation is reduced. The monetary policy of the common central bank is therefore less accommodative (high interest rates and low inflation) in case of a fiscal crisis, and the costs of servicing the debt are higher. In fact, the budgetary constraint is thus tighter for the members of a monetary union.

More precisely, our model shows that the common central bank of a monetary union tends to lead a more accommodative monetary policy in order to avoid the default of the member countries whose levels of indebtedness remain moderate. On the contrary, the most heavily indebted member countries have to default at least on a part of their debt, as for the central bank, the cost of the deviation from its inflation target is then higher than the benefits to avoid the default of one country. More precisely, our model shows that in a monetary union, the optimal inflation rate increases the smaller is the weight given by the common central bank to the aim of price stability and the more numerous are the highly indebted countries in the union. Besides, the optimal inflation rate increases when is the interest rate on the risk free capital is high the interest rate on nominal bonds in the fiscally weak countries is low. Indeed, the higher and growing interest rates on the Greek public debt have largely contributed to increase the probability of default of this country during the spring 2010, even if they could also have contributed to maintain the low level of inflation in the monetary union. Finally, we can mention that high government bond yields in a country like Greece can be explained, among other things, by the high share of weakly productive public expenditures in this country.

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Appendix

A. Determination of the share of debt which is defaulted

- $d_i = 1$ if: $t_{i,2}Y_{i,2} \leq G_{i,2} + c_i(1+b_i-\pi)B_{i,1} - \frac{\pi M_t}{1+\pi}$ that is to say if: $\pi \leq \pi_{Li}$ such as:

$$\left\{\pi-\left[\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-c_ib_iB_{i,1})+\sqrt{\Delta}}{2c_iB_{i,1}}\right]\right\}\left\{\pi-\left[\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-c_ib_iB_{i,1})+\sqrt{\Delta}}{2c_iB_{i,1}}\right]\right\}\leq 0$$

With $\Delta = [t_{i,2}{*}Y_{i,2} - G_{i,2} + M_i - ci(2+b_i)B_{i,1}]^2 + 4c_iM_iB_{i,1}$.

$$\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-c_ib_iB_{i,1})-\sqrt{\Delta}}{2c_iB_{i,1}}\leq 0\leq\pi\leq\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-c_ib_iB_{i,1})-\sqrt{\Delta}}{2c_iB_{i,1}}$$

$$\pi_{L,i}=\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-c_ib_iB_{i,1})-\sqrt{[t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_t-c_t(2+b_i)B_{i,1}]^2+4c_iM_iB_{i,1}}}{2c_iB_{i,1}}$$

- $d_i{*}=0$ if: $t_{i,2}Y_{i,2} \geq G_{i,2} + (1+b_i-\pi)B_{i,1} - \frac{\pi M_t}{1+\pi}$ that is to say if: $\pi \leq \pi_{Hi}$ such as:

$$\left\{\pi-\left[\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-b_iB_{i,1})+\sqrt{\Delta}}{2B_{i,1}}\right]\right\}\left\{\pi-\left[\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-b_iB_{i,1})-\sqrt{\Delta}}{2B_{i,1}}\right]\right\}\leq 0$$

With $\Delta = [t_{i,2}{*}Y_{i,2} - G_{i,2} + M_i - (2+b_i)B_{i,1}]^2 + 4M_iB_{i,1}$.

$$\pi\leq\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-b_iB_{i,1})-\sqrt{\Delta}}{2B_{i,1}}\leq 0 \;\; or \;\; \pi\geq\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-b_iB_{i,1})+\sqrt{\Delta}}{2B_{i,1}}$$

$$\pi_{H,i}=\frac{-(t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_i-b_iB_{i,1})+\sqrt{[t_{i,2}^{*}Y_{i,2}-G_{i,2}+M_t-(2+b_i)B_{i,1}]^2+4M_iB_{i,1}}}{2B_{i,1}}$$

B. Optimal monetary policy

With $b_C > b_B > b_A$, we have:

$$\pi_{LB}-\pi_{LA}=-\sqrt{\left[\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}+M_A}{2c_AB_{A,1}}-1-\frac{b_A}{2}\right]^2+\frac{M_A}{c_AB_{A,1}}}+\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}+M_A}{2c_AB_{A,1}}-\frac{b_A}{2}$$
$$+\sqrt{\left[\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}+M_B}{2c_BB_{B,1}}-1-\frac{b_B}{2}\right]^2+\frac{M_B}{c_BB_{B,1}}}-\left(\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}+M_B}{2c_BB_{B,1}}\right)+\frac{b_B}{2}>0$$

if: $\frac{M_{A,i}}{B_{A,1}}>\frac{M_B}{B_{B,1}}$; $\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}}{B_{A,1}}>\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}}{B_{B,1}}$

$$\pi_{HA}-\pi_{LB}=-\sqrt{\left[\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}+M_A}{2B_{A,1}}-1-\frac{b_A}{2}\right]^2+\frac{M_A}{B_{A,1}}}-\left(\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}+M_A}{2B_{A,1}}\right)+\frac{b_A}{2}$$
$$-\sqrt{\left[\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}+M_B}{2c_BB_{B,1}}-1-\frac{b_B}{2}\right]^2+\frac{M_B}{c_BB_{B,1}}}+\left(\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}+M_B}{2c_BB_{B,1}}\right)-\frac{b_B}{2}>0$$

if: $\frac{M_B}{c_BB_{B,1}}>\frac{M_A}{B_{A,1}}$; $\frac{t_{B,2}^{*}Y_{B,2}-G_{B,2}}{c_BB_{B,1}}>\frac{t_{A,2}^{*}Y_{A,2}-G_{A,2}}{B_{A,1}}$; $(b_B - b_A)$ small

$$\pi_{HB}-\pi_{HA}=-\sqrt{\left[\frac{t^*_{A,2}Y_{A,2}-G_{A,2}+M_A}{2B_{A,1}}-1-\frac{b_A}{2}\right]^2+\frac{M_A}{B_{A,1}}}+\left(\frac{t^*_{A,2}Y_{A,2}-G_{A,2}+M_A}{2B_{A,1}}\right)-\frac{b_A}{2}$$
$$+\sqrt{\left[\frac{t^*_{B,2}Y_{B,2}-G_{B,2}+M_B}{2B_{B,1}}-1-\frac{b_B}{2}\right]^2+\frac{M_B}{B_{B,1}}}-\left(\frac{t^*_{B,2}Y_{B,2}-G_{B,2}+M_B}{2B_{B,1}}\right)+\frac{b_B}{2}>0$$
$$\text{if: } \frac{M_A}{B_{A,1}}>\frac{M_B}{B_{B,1}} \;;\; \frac{t^*_{A,2}Y_{A,2}-G_{A,2}}{B_{A,1}}>\frac{t^*_{B,2}Y_{B,2}-G_{B,2}}{B_{B,1}}$$

$$\pi_{LC}-\pi_{HB}=-\sqrt{\left[\frac{t^*_{C,2}Y_{C,2}-G_{c,2}+M_C}{2_{c_C}B_{c,1}}-1-\frac{b_c}{2}\right]^2+\frac{M_c}{_{c_C}B_{c,1}}}-\left(\frac{t^*_{c,2}Y_{c,2}-G_{c,2}+M_c}{2_{c_C}B_{c,1}}\right)+\frac{b_c}{2}$$
$$-\sqrt{\left[\frac{t^*_{B,2}Y_{B,2}-G_{B,2}+M_B}{2B_{B,1}}-1-\frac{b_B}{2}\right]^2+\frac{M_B}{B_{B,1}}}+\left(\frac{t^*_{B,2}Y_{B,2}-G_{B,2}+M_B}{2B_{B,1}}\right)-\frac{b_B}{2}>0$$
$$\text{if: } \frac{M_B}{B_{B,1}}>\frac{M_C}{c_C B_{C,1}} \;;\; \frac{t^*_{B,2}Y_{B,2}-G_{B,2}}{B_{B,1}}>\frac{t^*_{C,2}Y_{C,2}-G_{C,2}}{c_C B_{C,1}}$$

(1) and (8) imply: $t_{i,2}{*}Y_{i,2}-G_{i,2}=[(1-c_i)(1+r)+c_i(1+b_i-\pi)]B_{i,1}-\frac{\pi M_t}{1+\pi}$ (B1)

• If $\pi<\pi_{HA}$: type (A) countries would default on a part of their debt, which is incompatible with the definition of these countries.

• If $\pi\in[\pi_{HA};\pi_{HB}]$: type (A) countries never default; type (B) countries default on part of their debt; type (C) countries default on all their debt. $b_B>b_A=r+\pi$.

Using the definition of (π_{HB}), of the type (B) countries, and ($\bar{\pi}<\pi<\pi_{HB}$), we have:

$$t_{B,2}Y_{B,2}-G_{B,2}=(1+b_B-\pi_{HB})B_{B,1}-\frac{\pi_{H,B}M_B}{1+\pi_{HB}}<(1+bB-\bar{\pi})B_{B,1}-\frac{\pi M_B}{1+\pi}$$

(B1) and the definition of (π_{HA}) imply:

$$t_{A,2}{*}Y_{A,2}-G_{A,2}=(1+b_A-\pi)B_{A,1}-\frac{\pi M_B}{1+\pi}=(1+b_A-\pi_{HA})B_{A,1}-\frac{\pi_{HA}M_A}{1+\pi_{HA}}$$

So: $\pi^*=\pi_{HA}<\pi_{HB}$.

• If $\pi\in]\pi_{HB};\pi_{LC}]$: types (A) and (B) countries never default; type (C) countries default on all their debt. Therefore, we have $\partial d_i^*/\partial\pi=0$ $(\forall i)$, and $\pi=\bar{\pi}$.

Using the definition of (π_{HB}), of the group (B) countries, and of ($\pi=\bar{\pi}>\pi_{HB}$), we have:

$$t_{B,2}{*}Y_{B,2}-G_{B,2}=(1+b_B-\pi_{HB})B_{B,1}-\frac{\pi_{HB}M_B}{1+\pi_{HB}}>(1+b_B-\pi)B_{B,1}-\frac{\pi M_B}{1+\pi}>t_{B,2}{*}Y_{B,2}-G_{B,2}.$$

Therefore, the equations are incompatible.

• If $\pi \in]\pi_{LC};\pi_{HC}[$: type (A) and (B) countries never default; type (C) countries default on a part of their debt. Therefore, we have $b_A=b_B=r+\pi$.

Thus, using ($\bar{\pi}<\pi<\pi_{HC}$), the definition of group (C) countries and of (π_{HC}), we have:

$$t_{C,2}*Y_{C,2}-G_{C,2} = (1+b_C-\pi_{HC})B_{C,1} - \frac{\pi_{HC}M_C}{1+\pi_{HC}} < (1+b_C-\pi)B_{C,1} - \frac{\pi M_C}{1+\pi}$$

$$(1+b_C-\bar{\pi})B_{C,1} - \frac{\pi M_C}{1+\pi} > (1+bC-\bar{\pi})B_{C,1} - \frac{\pi M_C}{c_C(1+\pi)} \geq \frac{(t^*_{C,2}Y_{C,2}-G_{C,2})}{c_c} > (t_{C,2}*Y_{C,2}-G_{C,2})$$

Therefore, the equations are incompatible.

• If $\pi \geq \pi_{HC}$: Types (A), (B) and (C) countries never default on their debt. Therefore, we have $\partial d_i*/\partial\pi = 0$ $(\forall i)$, and $\pi = \bar{\pi}$.

$$(1+b_C-\bar{\pi})BC,1 - \frac{\pi M_C}{c_C(1+\pi)} < (1+b_C-\bar{\pi})B_{C,1} - \frac{\pi M_C}{(1+\pi)} = (1+bc-\pi)B_{C,1} - \frac{\pi M_C}{(1+\pi)}$$

$$\leq (1+b_C-\pi_{HC})B_{C,1} - \frac{\pi_{HC}M_C}{1+\pi_{HC}} = t_{C,2}*Y_{C,2} - G_{C,2} < \frac{(t^*_{C,2}Y_{C,2}-G_{C,2})}{c_C}$$

However, this is incompatible with the definition of the countries from the group (C).

The optimal monetary policy is therefore: $\pi^*=\pi_{HA}$, where type (C) countries default on all their debt and type (B) countries default on part of their debt.

Moreover, (B1) and (11) imply: $d^*_B = \frac{(b_B-\pi-r)}{(1+b_B-\pi)}$. So, with $d_A=0$ and $d_C=1$:

$$\pi^* = \bar{\pi} - \frac{1}{2\alpha^M}\gamma_B\frac{\partial d^*_B}{\partial\pi} = \bar{\pi} + \frac{\gamma_B(1+r)}{2\alpha^M(1+b_B-\pi)^2}$$

So, we have: $\pi^3-(2+2b_B+\bar{\pi})\pi^2 + (1+2b_B+b_B^2+2\bar{\pi}+2\bar{\pi}\,b_B)$

$$\pi = (1+bB)^2\bar{\pi} + \frac{\gamma_B(1+r)}{2a^M}.$$

We can solve this cubic equation of the third degree:

$$p = \frac{-(1+b_B-\bar{\pi})^2}{3} \qquad Q = \frac{4a^M\bar{\pi}^2(3+3b_B-\bar{\pi})+4a^M(1+b_B)^2(1+b_B-3\bar{\pi})-27\gamma_B(1+r)}{54a^M}$$

$$\Delta = \left(\frac{Q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = -\frac{\gamma_B(1+r)}{216a^M}\left[4(1-\bar{\pi})^3 - \frac{27\gamma_B(1+r)}{2a^M} + 12b_B(1-\bar{\pi})(1-\bar{\pi}+b_B) + 4b_B^3\right] < 0$$

provided (a^M) is not excessively small and (γ_B) is not too high.

Therefore, our equation has three real solutions:

$$\pi = 2\sqrt{\frac{-P}{3}}\cos\left[\frac{1}{3}\operatorname{acos}\left(\frac{-Q}{2}\sqrt{\frac{27}{-P^3}}\right) + \frac{2m(Pi)}{3}\right] + \frac{(2+2b_B+\bar{\pi})}{3} \quad \textit{with } m = 0, 1, 2.$$

The only solution avoiding an excessive (100%) inflation rate is then the one for m=1.

C. Credible inflation targeting policy

A credible inflation targeting policy implies: $\pi = \bar{\pi}$.

Type (C) countries could not enter the monetary union, as they would be sure to default (d_C=1), and thus, as nobody would want to buy their debt.

$\pi_{LA} < \pi_{LB} < \pi_{HA} < \pi_{HB}$ (see Appendix B)

As mentioned in Appendix B, the optimal monetary policy is: $\pi^* = \bar{\pi} = \pi_{HA}$, where the type (B) countries default on a part of their debt. Moreover, we have:

$$\pi^* = \bar{\pi} = \pi_{HA} = \bar{\pi} - \frac{1}{2a^M}\gamma_B\frac{\partial d_B^*}{\partial \pi} = \bar{\pi} + \frac{\gamma_B(1+r)}{2a^M(1+b_B-\pi)^2}$$

Thus, γ_B=0, and the type (B) countries can't exist.