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Agglomeration and Specialization Patterns when Firms and Workers are Footloose

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Abstract

In new economic geography models, geographic concentration cant arise because of workers mobility or vertical linkages between firms. We examine a setup that combines those two approaches in conjunction with local congestion costs. We find that, as trade costs are lowered, the geographic concentration of total activity (agglomeration) follows an inverse u-shaped evolution, while the degree of specialization of regions increases. These results shed light on regional development within a country as integration proceeds: when trade costs are hight, firms evenly spread between the regions to supply local demand at low costs, hence diversified regions; at intermediate trade costs, we have coexistence of a diversified core and a specialized periphery and at low trade costs, each industry clusters in one region to fully exploit returns to scale externalities. US city centers and non-metropolitan areas during the period 1850-1990 depict such specialization and agglomeration patterns. These results show that a country's effort to miprove accessibility across its porfolio of places can favor a win-win regional allocation of firms based on each location's competitive advantage.

- **JEL classification:** F12, R12, R15
- **Keywords:** Agglomeration, Specialization, Congestion Cost, Input-Output linkages

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*The findings, interpretations and conclusions expressed in this paper are entirely those of the author. They do not necessarily represent the view of the World Bank, its Executive Directors, or the countries they represent.

I. Introduction

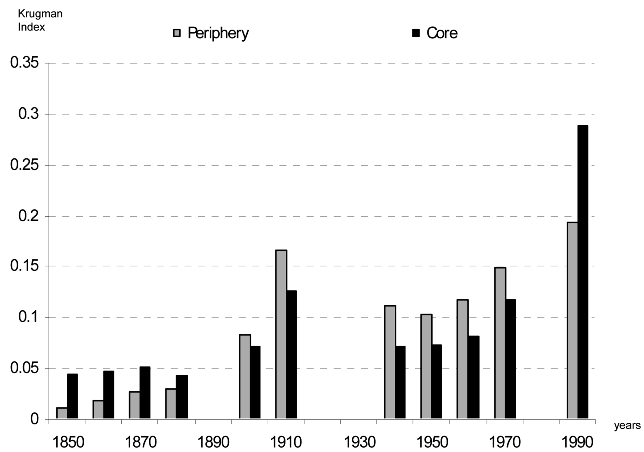
Imagine a city with two districts (a central city and its suburb) and two industries with increasing returns to scale at a stage where transport costs between the two districts are prohibitive. In such a simple model, it is hard to say what will be the concentration pattern of the two industries within the two locations, and its is harder to say how this internal geography might evolve with falling transport costs and what will be its impact on the relative size of the two locations as they become more and more integrated. The intensity of the agglomeration and dispersion forces in presence depends on the degree of substitutability of the two goods, on the intensity of congestion costs, on the budget share of consumers dedicated to each goods, on the intensity of the intra-industry linkages, on the level of transport costs. We can thus imagine extreme cases where all the firms remain evenly spread between the two locations, or the industry with stronger intra-industry linkages remaining totally agglomerated in one region for any transport costs, or an unstable partial location of both industries as trade costs vary.

Abdel-Rahman and Anas (2004) consider this issue of industries mapping in locations to be of primary importance. A glance at the economic geography literature points to two types of outcome. In a two-location two-industry framework, Abdel-Rahman (1996) proposes two configurations: a specialized configuration, where each location receives only one industry, and a diversified configuration, where each location hosts both industries. These configurations are determined by interactions between returns to scale and transport costs: when returns to scale are high, firms have an incentive to concentrate in one location, hence specialization; when transport costs prevail more, firms spread between locations to supply local demand at low cost, hence diversification. The second configuration proposed by Duranton and Puga (2001) points to the coexistence of diversified and specialized locations: the diversified locations are locations where innovating firms locate to develop their ideal production process and then switch to specialized locations for mass production.

In addition to these papers, all the new economic geography models bear some insights on the issue of firms mapping in locations that can be categorized into three types. The first type features dispersion and no specialization at high transports costs, agglomeration and specialization at intermediate transport costs, and finally agglomeration and specialization again at low transports costs (Krugman (1991), Krugman and Venables (1996), and Puga (1999)). Models of the second type feature dispersion and specialization at high transport costs, agglomeration and specialization at intermediate transport costs and finally redispersion and despecialization at low transport costs (Krugman and Venables (1995), Venables (1996), and Puga (1999)). The third type appears in the chapter 16 of Fujita, Krugman and Venables (1999) where there is dispersion at any transport costs associated with no specialization at high transport costs, specialization at intermediate transport costs and de-specialization at low transport costs.

These configurations however do not reflect the evolution of US metropolitan areas. Indeed, a close inspection of the urban evolution in the United States points to a paradoxical outcome of a continuously increasing specialization of city centres and their suburbs combined with an increasing and then decreasing relative agglomeration pattern of these two locations. The census micro data collected and harmonized by the Minnesota Population Centre based on random samples of the American population drawn from fourteen federal censuses between 1850 and 1990 allow us to extract some salient facts on the evolving American cityscapes.[1] We reorganized these data by classifying American workers according to the place where they live (central city or suburb) and according to the sector in which they work (we focused on tradable goods and services to construct four aggregated

Figure 1. Specialization in central and peripheral US metropolitan cities



[1]www.ipums.org
See the reference Section for authors' details.

[2]Financial services include security and commodity brokerage and investment, insurance and real estate. Business services include advertising, accounting, auditing and book-keeping services.

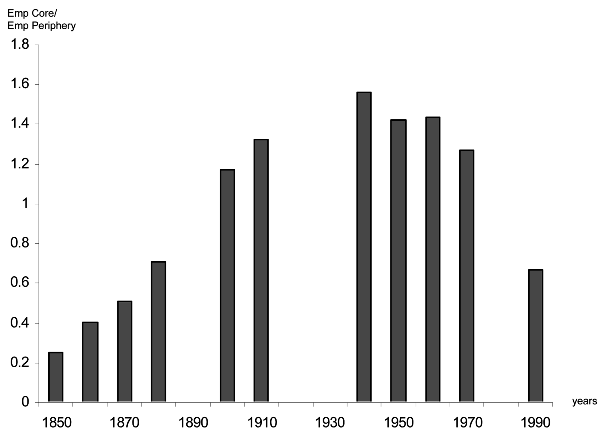
industries: durable manufactured goods, non-durable manufactured goods, financial services and business services).[2]

Figure 1 plots Krugman bilateral specialization index in US city centres (representing the cores) and suburbs (representing the peripheries).[3] This graph indicates an increasing specialization of the cores and the peripheries over the period 1850-1990. A close inspection of the data reveals that city centres have been specializing in financial and business services while suburbs were specializing in manufactures.

Figure 2 shows the evolution of relative employment level between the cores and the peripheries. It seems that central districts first received more workers until a peak around 1940, and then started loosing employment relatively to the suburbs. These two graphs reveal the following spatial evolution: as integration proceeded within American metropolitan areas, specialization has increased monotonically, in parallel with a non-monotonic agglomeration trend where the centre first were gaining and then loosing workers.

The aim of this paper is to build a model that reproduces the outcome described in Figures 1 and 2. We analyze what we believe is a parsimonious model for the

Figure 2. Relative employment in central and peripheral US metropolitan districts



[3]For most of the years, the IPUMS sample includes 1,000 individuals. We dropped years using a different sample size for the sake of coherence. This is why some years are missing in Figures 1 and 2.

[4]Puga (1999) has developed a model that encompasses both the core-periphery and the vertical-linkages models as special cases. Our model differs from that of Puga by two asymmetries we introduce: the two sectors have different intensities of intra-industry linkages, and labour is sector-specific, that is each sector uses a specific type of labour so that workers can move between locations but not between sectors. This, arguably, makes our model more suitable to the analysis of relatively small-scale spatial reallocations such as those occurring in a regional or an urban context.

purpose at hand, building on well known analytical tools of the new economic geography. Since this case is the only missing in the book by Fujita, Krugman and Venables, we opt for a Dixit-Stiglitz monopolistic modelling that is used in this book to complete the map. Our model features both interregionally mobile labour and input-output linkages, thus combining the main locational forces of the core-periphery model initially developed by Krugman (1991) and the vertical linkages model of Krugman and Venables (1995).[4] In addition, our model has two imperfectly competitive sectors with different intensity of vertical linkages, and we add an exogenous congestion cost.[5]

We study our model in terms of its prediction in two dimensions of the spatial economy:

- the spatial distribution of aggregate activity, which we refer to as agglomeration,
- the sectorial composition of locations, which we refer to as specialization.[6]

For some parameter ranges, simulations of the model suggest a simple but striking evolution of the two-location economy as transport costs are gradually reduced. We find that, at early stages of integration, when transport costs are still very high, industries tend to split evenly between the locations, so that sectors co-locate within each location, and there is no specialization. When transport costs fall to some intermediate levels, a core-periphery distinction emerges among the two locations: the strong-linkages industry clusters in one location (the center) that also receives some weak-linkages industry firms. As transport costs keep decreasing, the weak-linkages firms located in the centre start relocating to the periphery. Finally, once trade costs have fallen sufficiently low, locations completely specialize.

The paper proceeds as follows: we present the building blocks of our model in Section 2, while Section 3 reports simulation results that characterize the equilibrium configurations as transport costs vary. We summarize the qualitative behaviour of the model in Section 4, and Section 5 concludes the paper.

II. The Model

Our basic setup is as follows. We consider a two-location two-industry model.

[5]Multi-sector models with vertical linkages have been developed by Amiti, 1998, Fujita, Krugman and Venables, 1999, in chapter 16, Tabushi and Thisse, 2006, and Venables, 1999. Our framework differs from theirs by the two asymmetries described in the previous footnote.

[6]The paper by Ricci (1999) also deals with the topic of specialization versus agglomeration but focus on the role of comparative versus absolute advantage.

Trade between the two locations are of iceberg type τ, such that for each unit of a good shipped from location 1, only $1/\tau$ unit arrives in location 2 ($\tau>1$). The economy consists of two monopolistically competitive industries producing differentiated goods x and y under increasing returns to scale.[7] Each variety of each differentiated good is produced by a unique firm. For a differentiated good m ($m= x,y$), the number of varieties produced (and thus the number of firms located) in location r (r=1,2) is denoted $n_{m,r}$. Labour is sector-specific, that is, there are x-type workers and y-type workers. These workers can move between regions but not between sectors.[8] We assume intra-industry input-output linkages, with stronger linkages in industry y than in industry x. There are no inter-industry linkages, so that the interaction between sectors is only through general equilibrium effects. All workers are also consumers, and we write that $\lambda_{m,1}$ workers of industry m are located in region 1 and (1- $\lambda_{m,1}$) are located in region 2, with $0\leq\lambda_{m,1}\leq 1$. Finally, we assume congestion costs within each region: as the number of firms in a region increases, the real wage of that region decreases by a factor δ. This is an easy way to introduce significant congestion costs so as to counterbalance the two agglomeration forces (workers mobility and input-output linkages). These congestion costs can be thought of in a number of ways, such as the opportunity costs of commuting, environmental degradation or costs of immobile factors such as land.

A. Consumers' Side

Let us focus on location 1 (the corresponding results for location 2 are analogously derived). All consumers are identical, and they consume all the varieties produced in the economy. They share the following Cobb-Douglas utility function:

$$U = x^{\mu} y^{1-\mu}, \tag{1}$$

where $0<\mu<1$. Hence consumers spend a share μ of their income on good x and (1-μ) on good y. x and y are Dixit-Stiglitz composites of varieties i:

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[7]Since we do not have a traditional sector, we have to find a numeraire. We cannot use a differentiated good as numeraire because markups vary with the intensity of returns to scale. Since nominal wages are simply set in each labor market, we thus use the wage of industry y in location 2, which will be always defined in the model, as the numeraire.

[8]This assumption is empirically realistic (see for instance Miller, 1984, and Flinn, 1986).

$$x = \left(\int_0^{n_{x,1}} x_1(i)^{(\sigma-1)/\sigma} di + \int_0^{n_{x,2}} x_2(i)^{(\sigma-1)/\sigma} di\right)^{\sigma/(\sigma-1)}, \tag{2}$$

$$y = \left(\int_0^{n_{y,1}} y_1(i)^{(\sigma-1)/\sigma} di + \int_0^{n_{y,2}} y_2(i)^{(\sigma-1)/\sigma} di\right)^{\sigma/(\sigma-1)} \tag{3}$$

The elasticity of substitution σ (with $\sigma>1$) is assumed to be constant and identical for all the varieties of the two goods. Solving the consumer maximization problem yields the following price indices:

$$G_{m,1} = \left[n_{m,1}p_{m,1}^{1-\sigma} + n_{m,2}(\tau p_{m,2})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{4}$$

$$G_{m,2} = \left[n_{m,1}(\tau p_{m,2})^{1-\sigma} + n_{m,2}p_{m,2}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{5}$$

Where $p_{m,r}$ is the equilibrium price of all varieties of good m in location r. We can also derive the demand function for each variety in each location:

$$Q_{m,1} = E_{m,1}p_{m,1}^{-\sigma}G_{m,1}^{\sigma-1} + E_{m,2}p_{m,1}^{-\sigma}\tau^{1-\sigma}G_{m,2}^{\sigma-1}, \tag{6}$$

$$Q_{m,2} = E_{m,1}p_{m,2}^{-\sigma}\tau^{1-\sigma}G_{m,1}^{\sigma-1} + E_{m,2}p_{m,1}^{-\sigma}G_{m,2}^{\sigma-1}, \tag{7}$$

where $Q_{m,r}$ denotes the quantity of a given variety of good m produced in location r, and $E_{m,r}$ is the is the total expenditure on this variety in location r.

B. Producers' Side

We assume that all firms share an identical production technology involving a fixed input F_m specific to each industry m, and a unique constant marginal input γ. Both inputs are expressed in terms of a composite $Z_{m,r}$. Following the chapter 14 of Fujita, Krugman and Venables (1999), we assume that for each location this composite input can be expressed, up to a constant threshold, as $Z_{m,r} = l_{m,r}^{1-\alpha_m}\Psi_{m,r}^{\alpha_m}$, where $l_{m,r}$ denotes the quantity of labour, $\Psi_{m,r}$ is a CES composite of intermediate good for industry m in location r including all the varieties of good m, and α_m represents the share of intermediate inputs in the total production requirement for good m. Following standard simplification practice, we assume that the substitution elasticity among varieties in the composite input Ψ equals the substitution elasticity in consumers' utility function, σ. Importantly, we impose that $\alpha_x < \alpha_y$, so that intermediate inputs have a lower weight in the production technology of industry x than in that of industry y.

A firm's total cost is $F_m + \gamma Q_{m,r}$, where $Q_{m,r}$ is the quantity produced. Profits of a firm in industry m and location r are:

$$\pi_{m,r} = p_{m,r}Q_{m,r} - w_{m,r}^{1-\alpha_m}G_{m,r}^{\alpha_m}(F_m + \gamma Q_{m,r}). \tag{8}$$

Firms with monopoly power set their price such that their marginal revenue equals their marginal cost, where their marginal revenue is $p_{m,r}\left(1 - \frac{1}{\varepsilon}\right)$, ε being the price elasticity of demand. Since, in monopolistic competition ε is approximated by σ, firms' optimization implies that:

$$p_{m,r}\left(1 - \frac{1}{\sigma}\right) = \gamma w_{m,r}^{1-\alpha_m}G_{m,r}^{\alpha_m}. \tag{9}$$

With free entry and exit in all industries, profits are driven to zero in equilibrium. Substituting (9) in (8) at the zero-profit equilibrium yields the optimal level of firm output $Q_m^* = F_m(\sigma - 1)/\gamma$ and the associated optimal input $Z_m^* = F_m + \gamma Q_m^* = F_m\sigma$. Since firms make zero profits in this scenario, their wage bill must be proportional to the total value of production, in accordance with the labour share of inputs, and hence $w_{m,r}\lambda_{m,r} = (1 - \alpha_m)n_{m,r}p_{m,r}Q_m^*$. .

C. Normalizations and Equilibrium

We can make some normalizations that simplify the model without loss of generality. First, following Fujita, Krugman and Venables (1999), we impose that the marginal input requirement equals the constant mark-up, that is $\gamma = (\sigma - 1)/\sigma$, which implies that:

$$p_{m,r} = w_{m,r}^{1-\alpha_m}G_{m,r}^{\alpha_m}. \tag{10}$$

We can also choose the fixed input requirement F_m such that the equilibrium firm production becomes $Q_m^* = (1 - \alpha_m)^{-1}$. The value of a location's wage bill in each of these industries now simplifies to $w_{m,r}\lambda_{m,r} = n_{m,r}p_{m,r}$. Combining this equation with equations (4), (5) and (10) leads to the following expressions for the sectorial price index in the two locations:

$$G_{m,1}^{1-\sigma} = \lambda_{m,1}w_{m,1}^{1-\sigma(1-\alpha_m)}G_{m,1}^{-\alpha_m\sigma} + \lambda_{m,2}w_{m,2}^{1-\alpha(1-\alpha_m)}G_{m,2}^{-\alpha_m\sigma}\tau^{1-\sigma}, \tag{11}$$

$$G_{m,2}^{1-\sigma} = \lambda_{m,1} w_{m,1}^{1-\sigma(1-\alpha_m)} G_{m,1}^{-\alpha_m\sigma} \tau^{1-\sigma} + \lambda_{m,2} w_{m,2}^{1-\sigma(1-\alpha_m)} G_{m,2}^{-\alpha_m\sigma}. \tag{12}$$

It is obvious that the price index of an industry in a given location depends on the industry's wage rate in the location as well as on the price index of that sector in the other location. We can derive the wages associated with the optimal level of production using equations (6) and (7):

$$Q_{m,1} = E_{m,1} p_{m,1}^{-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,1}^{-\sigma} \tau^{1-\sigma} G_{m,2}^{\sigma-1} = Q_{m,1}^{*} = (1-\alpha_m)^{-1}, \tag{13}$$

$$Q_{m,2} = E_{m,1} p_{m,2}^{-\sigma} \tau^{1-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,2}^{-\sigma} G_{m,2}^{\sigma-1} = Q_{m,2}^{*} = (1-\alpha_m)^{-1}, \tag{14}$$

which we can re-write as:

$$\frac{p_{m,1}^{\sigma}}{1-\alpha_m} = E_{m,1} G_{m,1}^{\sigma-1} + E_{m,2} \tau^{1-\sigma} G_{m,2}^{\sigma-1}, \tag{15}$$

$$\frac{p_{m,2}^{\sigma}}{1-\alpha_m} = E_{m,1} \tau^{1-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} G_{m,2}^{\sigma-1}. \tag{16}$$

Using the pricing rule (10) we obtain the following wage equations:

$$\left[w_{m,1}^{1-\alpha_m} G_{m,1}^{\alpha_m}\right]^{\sigma} = (1-\alpha_m)\left[E_{m,1} G_{m,1}^{\sigma-1} + E_{m,2} \tau^{1-\sigma} G_{m,2}^{\sigma-1}\right], \tag{17}$$

$$\left[w_{m,2}^{1-\alpha_m} G_{m,2}^{\alpha_m}\right]^{\sigma} = (1-\alpha_m)\left[E_{m,1} G_{m,1}^{\sigma-1} \tau^{1-\sigma} + E_{m,2} G_{m,2}^{\sigma-1}\right]. \tag{18}$$

Wages in the two sectors are linked through expenditures E, which take into account both final and intermediate consumption. At the zero-profit equilibrium, wages constitute the only source of income. Combining equation (1) with the optimal shares of the composite inputs, we can derive the following expenditure equations:

$$E_{m,r} = \mu_m(\lambda_{x,r} w_{x,r} + \lambda_{y,r} w_{y,r}) + \frac{\alpha_m}{1-\alpha_m} \lambda_{m,r} w_{m,r}, \tag{19}$$

where μ_m is μ for industry x and 1-μ for industry y. The last step is to define the real wage equations. We assume that there are congestion costs, such that the real wage falls with the number of workers in a location. Specifically, we postulate the following real wage equation:

$$\omega_{m,r} = \frac{w_{x,r}(\lambda_{x,r} + \lambda_{y,r})^{-\delta}}{G_{x,r}^{\mu} G_{y,r}^{1-\mu}}, \quad \delta > 0, \tag{20}$$

where the exponent δ represents the real-wage reducing impact of congestion in each location.[9]

The full model consists of the sixteen non-linear equations (11), (12), (17), (18), (19) and (20) for r=1,2 and m=x,y. For a given allocation of labour between industries and locations $\lambda_{m,r}$, these equations define the short-run equilibrium, that is, the market clearing price indices and wages. In the long run, sectorial labour moves between locations in response to real wage differences.[10]

We can summarize this model by describing the locational forces at work. There are two agglomeration forces: forward and backward linkages. These forces are due to the fact that firms tend to locate close to both the final and intermediate goods big markets. There are two dispersion forces: the market crowding effect (within each sector) and the congestion cost (within as well as across sectors). We now explore how these forces combine to shape the internal geography of our two-location economy as transport costs decrease between the two locations.

III. Numerical Analysis

We are interested in the model's predictions regarding agglomeration and specialization at different levels of transport costs. Our definition of these location features is in terms of numbers of workers (rather than in, say, output values). Since the equilibrium equations derived in the previous section are highly non-linear, the model is not analytically tractable, and we have to resort to numerical analysis to explore equilibria.[11]

In the following, we will describe the equilibrium regime for a baseline set of parameters: $\alpha_x = 0.29$, $\alpha_y = 0.31$, $\delta = 0.45$, $\mu = 0.50$ and $\sigma = 4$. The last two parameters values ensure the results not to depend on asymmetries (namely higher returns to scale or higher consumption share for one of the two goods) other than the two asymmetries assumed in this paper: different intensity of intra-industry

[9]We are following here Fujita, Krugman and Venables (1999) in their chapter 18. Krugman and Livas (1996) explicitly deal with this congestion issue by including land rent. Since our model is already quite complex (two monopolistic industries with intra-industry linkages) we also opt for this simplest version as Fujita, Krugman and Venables.

[10]We imply the usual *ad hoc* migration dynamics whereby the flow of migrants is a linear function of the real wage difference between the two locations (see Baldwin *et al.*, 2003, chapter 2, for a thorough discussion).

[11]All of the numerical computations were done using the software GAMS. See the website www.gams.com for a description of the software.

linkages, and inter-industry labour immobility. We may notice that these values are similar to those generally used in the new economic geography literature (see for instance the book by Fujita, Krugman and Venables, 1999). For these parameters values, the model accommodates three types of equilibria that correspond to the stylized facts observed in the US metropolitan data over the period 1850-1990:

1) both industries are completely concentrated in a different location (completely concentrated equilibrium),

2) industry y is completely concentrated in one location and industry x is unevenly spread between the two locations (partially concentrated equilibrium),

3) both industries are evenly spread between the two locations (completely dispersed equilibrium).

In terms of *agglomeration*, the distribution of aggregate labour (and hence activity) across locations, equilibrium types 1 and 3 represent perfect dispersion, and type 2 represents partial agglomeration. In terms of *specialization*, i.e. locations' relative industry shares, equilibrium type 3 is completely diversified, type 1 is completely specialized and type 2 is incompletely specialized.

A. The Completely Concentrated Equilibrium

As a first step, we explore the conditions under which a completely specialized equilibrium (where workers of each industry are completely concentrated in one location) is sustainable. Henceforth we assume that, if complete specialization applies, industry x clusters in location 1 and industry y in location 2, so that $\lambda_{x,1} = \lambda_{x,2} = 1$ and $\lambda_{x,2} = \lambda_{y,1} = 0$. For these values of $\lambda_{m,r}$, the congestion cost parameter δ does not matter in the real wage equations (20) and combining equations (11)-(12) and (20) yields the following relations between nominal and real wages:

$$\frac{\omega_{x,2}}{\omega_{x,1}} = \tau^{1-2\mu}\frac{w_{x,2}}{w_{x,1}}, \tag{21}$$

$$\frac{\omega_{y,1}}{\omega_{y,2}} = \tau^{2\mu-1}\frac{w_{y,1}}{w_{y,2}}. \tag{22}$$

Using the previous conditions on $\lambda_{m,r}$, equations (11)-(12) and (19) simplify, and we can substitute them into the wage equations to obtain the expressions relevant to track $\omega_{m,r}/\omega_{m,s}$ evolves with falling transport costs:

$$\frac{w_{x,1}^{(1-\alpha_x)}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)} + \mu(1 + w_{y,2}) w_{x,1}^{\sigma(1-\alpha_x)-1} \tag{23}$$

$$\frac{w_{x,2}^{(1-\alpha_x)\sigma}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)} \tau^{1-\sigma-\alpha_x\sigma} + \mu w_{x,1}^{\sigma-\alpha_x\sigma-1} \left(\frac{\tau^{1-\sigma-\alpha_x\sigma} + \tau^{\sigma-\alpha_x\sigma-1}}{2} + w_{y,2}\tau^{\sigma-\alpha_x\sigma-1} \right) \tag{24}$$

$$\frac{w_{y,1}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu-\delta) w_{y,2}^{\sigma(1-\alpha_y)-1} \left(w_{x,1}\tau^{\sigma-\alpha_y\sigma-1} + \frac{\tau^{1-\sigma-\alpha_y\sigma} + \tau^{\sigma-\alpha_y\sigma-1}}{2} \right) + \left(1-\mu-\delta + \frac{\alpha_y}{1-\alpha_y} \right) w_{y,2}^{\sigma(1-\alpha_y)} \tau^{1-\sigma-\alpha_y\sigma} \tag{25}$$

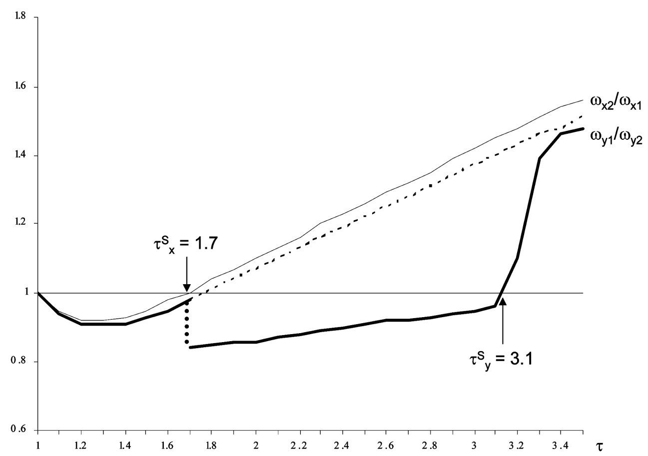
$$\frac{w_{y,2}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu-\delta)(1 + w_{x,1}) w_{y,2}^{\sigma(1-\alpha_y)-1} + \left(1-\mu-\delta + \frac{\alpha_y}{1-\alpha_y}\right) w_{y,2}^{\sigma(1-\alpha_y)} \tag{26}$$

These expressions combined with equations (21) and (22) help to get τ_x^S (sustain point of industry *x*) and τ_y^S (sustain point of industry *y*) and as shown in Appendix 1, we will have $\tau_x^S < \tau_y^S$. For the simulations, we focus on the baseline set of parameters given above ($\alpha_x = 0.29$, $\alpha_y = 0.31$, $\delta = 0.45$, $\mu = 0.50$ and $\sigma = 4$), compute the relevant relative real wages and plot them for $\tau \leq \tau_x^S$.

So far, we have consciously neglected a relevant but complicating fact: for $\tau > \tau_x^S$, $\lambda_{x,1}$, is no longer equal to 1. In fact, for $\tau > \tau_x^S$, we neglect the impact of congestion costs. In order to take into account these congestion costs, we have to use the full expressions for the price index, expenditures and nominal and real wages only with the condition $\lambda_{y,2} = 1$. This condition simplifies the industry *y* price index, but we now have an additional variable, $\lambda_{x,1}$. To close the model, we have to use the fact that workers in industry *x* migrate between locations 1 and 2 until the equalization of the real wage within the two locations. This yields a system of nine equations with nine unknowns described in Appendix 2. The next step is to solve these nine non-linear equations numerically for different values of transport costs. Figure 3 combines these two simulation exercises.

The sustain points are τ_x^S =1.7 and τ_y^S =3.1 for the baseline parameters set. Figure 3 shows that the two industries are completely concentrated in different locations at

Figure 3. Sustain point of the two industries



low trade costs ($\tau < \tau_x^S$). For intermediate trade costs ($\tau_x^S < \tau < \tau_y^S$), complete concentration of the *x* industry in location 1 is no longer sustainable while industry *y* remains clustered in location 2. In this intermediate range, because of the existence of the congestion costs, industry *x* will not necessarily spread evenly across the two locations (an issue explored in the following subsection). For high trade costs ($\tau > \tau_y^S$), the agglomeration of the strong-linkages industry is not sustainable either, and neither of the two industries is completely concentrated in one location.

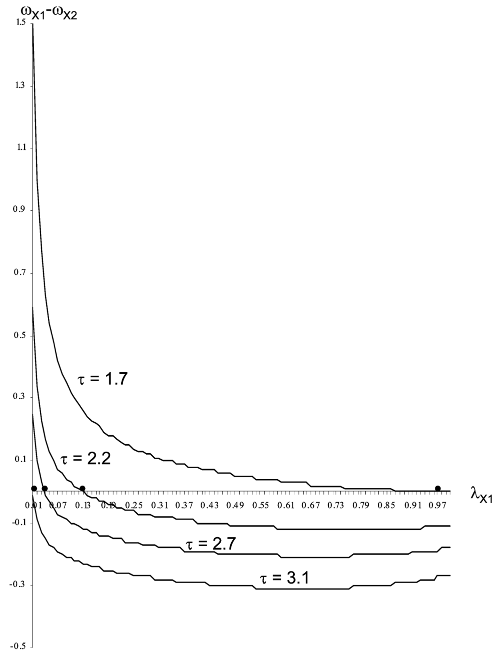
B. The Partially Concentrated Equilibrium

Figure 3 has shown an incomplete-specialization range for trade costs for which the concentration of the weak-linkages industry in location 1 was not sustainable while the strong-linkages industry remained clustered in location 2. Now, we examine what happens to the non-concentrated industry *x* in this transport costs range.

In the incomplete-specialization range, we have $\lambda_{y,1} = 0$ and $\lambda_{y,2} = 1$, i.e. industry *y* remains concentrated in location 2. We choose the wage of this industry in location 2 as the numeraire, setting $w_{y,2} = 1$ (see footnote 7). The complete concentration of industry *y* in location 2 is sustainable as long as the real wage in this location is higher than that in location 1 ($\omega_{y,2} > \omega_{y,1}$). These conditions simplify the equilibrium equations (11), (12), (17), (18), (19) and (20), as presented in Appendix 3.

The analysis consists in simultaneously solving these nine non-linear equations

Figure 4. Real wage differential in industry *x*



for different values of transport costs. We then compute the real wage differential in industry x, $\omega_{x,1} - \omega_{x,2}$, and plot it against $\lambda_{x,1}$. Figure 4 depicts the results for our baseline set of parameters ($\alpha_x = 0.29$, $\alpha_y = 0.31$, $\delta = 0.45$, $\mu = 0.50$ and $\sigma = 4$).

The simulations lead to a consistent set of qualitative results:

· when trade costs are low ($\tau \leq \tau_x^S$), the real wage gap is in favour of location 1 ($\omega_{x,1} - \omega_{x,2} > 0$), inciting industry x's workers to locate in location 1, hence a full concentration of industry x in this location;

· when trade costs increase, the real wage gap is negative for high values of $\lambda_{x,1}$ and positive for low values, so that we have a stable partial equilibrium $\lambda^*_{x,1}$;

· when trade costs are high ($\tau \geq \tau_x^S$), the real wage gap is in favour of location 2($\omega_{x,1} - \omega_{x,2} < 0$), inciting industry x's workers to locate in location 2, hence a full concentration of industry x in this location.

These results reflect the sustain point analysis of the basic core-periphery model: when trade costs are low, firms can supply both markets at low cost, and because of vertical linkages they have an incentive to agglomerate in one location (location 1 for *x*-firms and location 2 for *y*-firms). When transport costs increase, it becomes costly to supply remote consumers, and some *x*-firms will relocate to location 2, and the *x*-firms share in location 1 will be $\lambda^*_{x,1}$ rather than 1. It is obvious that the share of relocating *x*-firms in location 2 ($1 - \lambda^*_{x,1}$) will be higher the higher the transport costs. On the other hand, since the size of location 2 increases because of the relocation of some *x*-firms, induced congestion costs increase. These higher congestion costs will attenuate the incentive of *x*-firms to relocate in location 2, yielding a stable partial concentration of *x*-firms in location 2. For higher trade costs, both industries collapse in one location (here, location 2 that primarily received *y*-firms). This smooth variation of industry *x* share for this intermediate transport cost is key result of our study since it departs from the catastrophic variation of the basic core periphery model. Outside this transport costs range, our model yields the usual core-periphery model result (dispersion at high trade costs and agglomeration at low trade costs) for the baseline parameter values.

C. The Completely Dispersed Equilibrium

Now we turn to the stability analysis of the perfectly dispersed equilibrium, where both industries are spread evenly across the two locations ($\lambda_{x,1} = \lambda_{x,2} = \lambda_{y,1} = \lambda_{y,2} = 0.5$). The existence of such equilibrium is well-established in the new economic geography literature. However, our model differs from existing economic geography models in two key ways: we assume an asymmetry in the intensity of intra-industry linkages (industry *x* is assumed to have lower input-output linkages) and two types of labour, specific to each industry (workers can move between locations but only within the same industry). These two asymmetries substantially affect the usual perfectly dispersed equilibrium when trade costs are high.

We are interested in the following question: "starting from a perfectly dispersed equilibrium, how does a reallocation of labour between locations affect relative real wages?" If relative wages change in favour of the location that receives the labour inflow, then the initial configuration was not a stable equilibrium. Conversely, if relative wages change in favour of the location from which labour has migrated, then the initial configuration was a stable equilibrium.

A specificity of our model is that we have two state variables: the weak-linkage industry labour allocation (λ_x) and the strong-linkages industry labour allocation (λ_y). This increases the complexity of the perfectly dispersed equilibrium. To make the model tractable, we make the following assumption: $d\lambda_x/d\lambda_y = d\omega_x/d\lambda_y$ and $d\lambda_y/d\lambda_x = d\omega_y/d\lambda_x$. This means that we assume that a reallocation of labour in

[12]This assumption is an ad-hoc way to link migration to the real wage of the two industries. An alternative approach would be to set different levels of λ_x and λ_y exogenously and to combine the final effects, but this would increase the complexity of the simulations beyond the scope of this paper.

a given industry affects labour in the other industry through the variation induced in the real wage of this latter industry.[12]

We are interested in the variation in real wages due to labour reallocation $d\omega_x/d\lambda_x(d\omega_y/d\lambda_y)$. A positive value in this variation suggests that labour reallocation implies a real wage gain; hence the perfectly dispersed equilibrium breaks. To solve the model, we focus on price indices, expenditures, nominal and real wages equations. The perfectly dispersed equilibrium implies that $G_{x,1}=G_{x,2}=G_x$, $G_{y,1}=G_{y,2}=G_y$, $w_{x,1}=w_{x,2}=w_x$, $w_{y,1}=w_{y,2}=w_y$. First, we have to evaluate the symmetric equilibrium values of the variables and then totally differentiate the system formed by price indices, expenditures and nominal and real wages equations.[13] These steps are described in detail in Appendices 4 and 5. Our model is symmetric in the sense that $d\lambda_{x,1}=-d\lambda_{x,2}=d\lambda_x$, $dw_{y,1}=-dw_{y,2}=dw_y$, $dw_{x,1}=-dw_{x,2}=dw_x$, $dw_{y,1}=-dw_{y,2}=dw_y$, $dG_{x,1}=-dGG_{x,2}=dG_x$, $dG_{y,1}=-dG_{y,2}=dG_y$, $d\omega_{x,1}=-d\omega_{x,2}=d\omega_x$, $d\omega_{y,1}=-d\omega_{y,2}=d\omega_y$. At the perfectly dispersed equilibrium, we find that (see Appendix 5 for the derivation of these results):

$$w_{x,1} = w_{x,2} = (1-\alpha_x)/(1-\alpha_y)$$
$$w_{y,1} = w_{y,2} = 1$$

$$G_{x,1} = G_{x,2} = (1-\alpha_x)/(1-\alpha_y)[(1+\tau^{1-\sigma})/2]^{1/[1-\sigma(1-\alpha_x)]}$$
$$G_{y,1} = G_{y,2} = [(1+\tau^{1-\sigma})/2]^{1/[1-\sigma(1-\alpha_y)]}$$

Total differentiation around the perfectly dispersed equilibrium yields ten equations of interest: the derivative of G_m with respect to λ_m, the derivative of w_m with respect to λ_x and λ_y, and the derivative of ω_m with respect to λ_x and λ_y.[14] Using the same baseline set of parameters (α_x=0.29, α_y=0.31, δ=0.45, μ=0.50 and σ=4), we can simultaneously solve these equations for different levels of transport costs. This allows us to plot $d\omega_x/d\lambda_x(d\omega_y/d\lambda_y)$ against τ. As long as these derivatives are negative, indicating that migration of workers to the other location reduces their real wage, perfect dispersion is a stable equilibrium.

Figure 5 shows that when trade costs are very high, the two industries split

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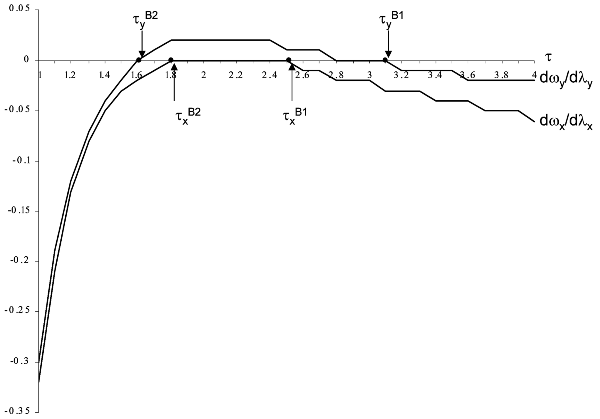
[13]The linear approximation of the function $y=f(x)$ around x^* and $y=f(x^*)$ involves computing $dy = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j}(x^*)dx_j$. This is derived in Appendix 5.

[14]Note that the derivative of $G_x(G_y)$ with respect to $\lambda_y(\lambda_x)$ is zero because of the chosen functional forms.

Figure 5. Stability of the perfectly dispersed equilibrium



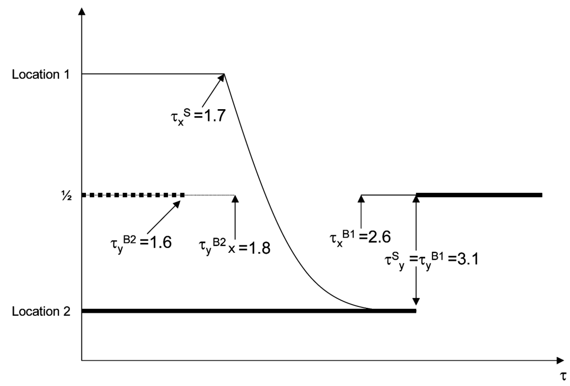
evenly between the two locations to supply local consumers at low cost. Dispersion forces are stronger than agglomeration forces. As transport costs decrease, we reach a first break point τ_y^{B1} =3.1 at which the strong-linkages industry deviates from the symmetric equilibrium to concentrate in the centre. Note that for our baseline set of parameters (α_x=0.29, α_y=0.31, δ=0.45, μ=0.50 and σ=4), $\tau_y^{B1} = \tau_y^{S}$. The symmetric equilibrium in industry *x* breaks at a lower transport cost τ_x^{B1} =2.6.

As transport costs keep decreasing, we reach a reverse break point first in industry *x*, τ_x^{B2} =1.8 and then in industry *y*, τ_y^{B2} =1.6. These symmetric equilibria are obviously unstable because for instance for τ just higher than τ_y^{B2} =1.6, a worker switching its location will face lower wage forcing him to relocate back.

D. Summing Up the Equilibria: The Bifurcation Diagram

In this section, we sum up the previous findings on firms location as transport and congestion costs vary. Our baseline set of parameters (α_x=0.29, α_y=0.31, δ=0.45, μ=0.50 and σ=4) suggests a coherent pattern. At high transport costs, the two industries spread evenly between the two locations. As transport costs fall, the strong-linkages industry deviates first from the symmetric equilibrium to completely concentrate in one location. Meanwhile, the weak-linkages industry remains evenly spread between the two locations until a critical level of transport costs, τ_x^{B1}, below which this industry partially concentrates in location 2. As transport costs keep decreasing, agglomeration forces also matter more in industry *x*, and we end up with a full concentration of industry *x* in location 1. For very low transport costs, we can have a stable full concentration of each industry in one

Figure 6. Bifurcation diagram



location, or an unstable even spread of the two industries between the two locations.

Figure 6 illustrates the typical spatial evolution generated by the baseline set of parameters. The bold lines represent industry *y* (with strong linkages), the fine lines represents industry *x* (with weak linkages), and the dashed line (both bold and fine) represent unstable equilibria. τ_m^{B1} and τ_m^{S} represent the break point and the sustain point of industry *m* respectively.

Figure 6 reveals a rich pattern of firms' location for trade costs in the range $[\tau_x^S;\tau_y^{B1}]$, and this differentiates our model from existing economic geography models cited in the introduction: the weak-linkages industry is unevenly spread between the two locations, while the strong-linkages industry is totally concentrated in location 2.

Until now, we have focused on a set of baseline parameters. In the next section, we test the robustness of our findings when departing from these baseline parameter values.

E. Robustness

One of the challenges of this paper was to retrieve relevant information from sixteen strongly non-linear equations representing price indices, wages, real wages and expenditures for two locations and two industries. We used different parameter combinations to analyze specialized equilibria and symmetric equilibria. It appeared that for intermediate values of α_x and α_y with α_x closer to α_y, and intermediate values of congestion costs, the results were close to that obtained with the baseline parameter combinations. However, for higher values of the parameters (α_x, α_y, δ and μ), the equations system was not numerically solvable.

(1) The completely specialized equilibrium

The sustain point analysis can be reproduced for a wide range of parameters. For an overview of the impact of these parameters on $\omega_{m,r} / \omega_{m,s}$, we organized the simulations in two ways. First, we set $\alpha_x = 0.25$, $\alpha_y = 0.5$, and let μ vary from 0.1 to 0.9. Secondly, we set μ=0.5 and let α_x and α_y vary from 0.1 to 0.9, with $\alpha_x < \alpha_y$.

These simulations lead to the following result: when consumers prefer more *y*-good, the real wage curve of this industry moves to the right, and this industry remains clustered in location 2 for higher value of transport costs. Conversely, the complete concentration of the weak input-output linkages industry breaks for lower transport costs. The reverse pattern holds when consumers shift expenditure towards the good (i.e. $\mu > 0.5$). When the intensities of the input-output linkages increases (with $\alpha_x < \alpha_y$), the simulations indicate that the real wage curve of the two industries moves to the right and therefore the industries remain clustered for higher value of transport costs.

To sum up, these simulations show that final expenditure shares and intensity of intermediate inputs are substitutable concentration forces in this model: a higher expenditure share or a higher intensity of intra-industry linkages reinforce concentration.

(2) The partially concentrated equilibrium

The analysis of incompletely specialized equilibria, where industry is totally concentrated in location 2 while industry is unevenly spread between locations, yields similar results for a wide range of parameter values different from the baseline set of parameters. At high transport costs, location 2 receives all the *x*-firms and more than half of y-firms. As transport costs fall, the weak-linkages industry moves away from the strong-linkages industry, and location 1's share in *x*-firms increases. Once transport costs are low enough, the weak-linkages industry completely concentrates in location 1 and the strong-linkages industry completely concentrates in location 2.

(3) The completely dispersed equilibrium

The simulations yield various configurations depending on the intensity of intra-industry linkages and congestion costs. We can summarize these results with the following five scenarios:

· For any intensity of intra-industry linkages and congestion cost and for very low transport costs, the symmetric equilibrium is always stable for the two industries.

· $d\omega_m / d\lambda_m > 0$ for higher transport costs, hence the symmetric equilibrium is never stable.

· $d\omega_m / d\lambda_m < 0$ for any level of transport costs, hence the symmetric equilibrium is always stable.

· $d\omega_y / d\lambda_y < 0$ for higher transport costs while $d\omega_x / d\lambda_x < 0$ for any level of trade costs: the symmetric equilibrium is never stable in the strong-linkages industry while it is always stable in the weak-linkages industry.

· For intermediate values of α_x and α_y with α_x closer to α_y, and intermediate values of congestion costs, the results were close to that obtained with the baseline parameter combinations.

(4) Summing up the equilibria

One particularity of the baseline set of parameters was that $\tau_y^{B1} = \tau_y^S$. This nice results does not hold for other parameters values: for some combinations of α_x and α_y, $\tau_y^{B1} < \tau_y^S$, and for some other combinations, $\tau_y^{B1} > \tau_y^S$. The first case is usual in the new economic geography literature, but the second is not. In this latter case, when industry *y* firms deviate from the perfectly dispersed equilibrium, they do not directly totally concentrate in the core. This situation leaves room for some partial non-stable concentration that could not be obtained numerically.

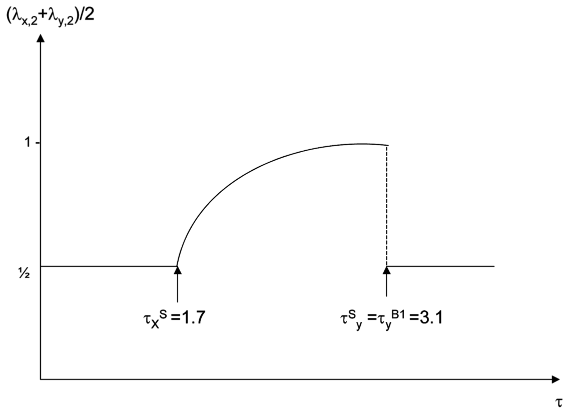
IV. Agglomeration and Specialization Pattern

Simulations of our model yield a rich set of locational predictions that are summarized in the bifurcation diagram of Figure 6. The behaviour of our model becomes even clearer when we illustrate the equilibria (only the stable ones) of our model separately in terms of agglomeration, specialization and co-location. The following graphs focus on location 2 assumed to be the core.

A. Agglomeration

We define *agglomeration* in terms of the locational allocation of total labour. The typical configuration of equilibrium agglomeration levels at different levels of transport costs within location 2 is represented in Figure 7 that is derived from the bifurcation diagram plotted in Figure 6.

Figure 7. Comparative static of agglomeration



We find that agglomeration follows a bell-shape trajectory as trade costs are lowered. Total labour (and hence aggregate activity) is evenly spread between the two locations when transport costs are high and low. At intermediate transport costs, location 2's size increases while location 1's size decreases. As transport costs keep decreasing, the size of location 2 starts decreasing until perfect dispersion corresponding to two equalized locations' sizes.

B. Specialization and Co-location

Specialization can be defined in our model using the Herfindahl index, $H=(\lambda_{x,2}/(\lambda_{x,2}+\lambda_{y,2}))^2+(\lambda_{y,2}/(\lambda_{x,2}+\lambda_{y,2}))^2$, with $0.51 \leq H \leq 1$. This index is traced for different levels of transport costs in Figure 8.

Figure 8. Specialization and co-location

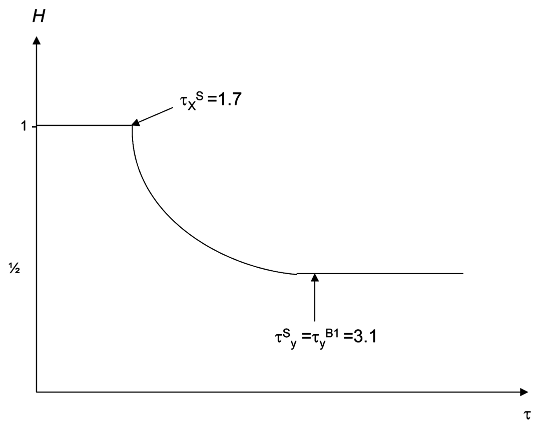


Figure 8 shows an increasing specialization of location 2 as transport costs are lowered. We have no specialization at high transport costs, since the two industries are evenly distributed between the two locations. As transport costs are lowered, some *y*-firms in location 1 relocate to location 2, hence increasing specialization in this location. As transport costs keep decreasing, the *x*-firms located in location 2 start relocating in location 1 and the specialization of location 2 in *y*-firms is reinforced while that of location 1 in *x*-firms is also reinforced. At low transport costs, the strong-linkages industry (*y*) is totally agglomerated in location 2 while the weak-linkages industry is totally concentrated in location 1; hence a perfect industrial specialization of each location.

Our results thus appear to depart from the other Dixit-Stiglitz-Krugman models: at high transport costs, we have dispersion associated with no specialization, at intermediate transport costs we have partial agglomeration and partial specialization, and finally at low transport costs, we have re-dispersion associated with perfect specialization. This locational evolution is consistent with the stylized facts on US city centres and suburbs over the period 1850-1990 described in the introduction.

V. Conclusion

We have tracked locational equilibria in an integrating economy consisting of two locations, using a Dixit-Stiglitz framework with two industries, two industry-specific interregionally mobile production factors and exogenous locational congestion costs. We assumed that the two industries had different intensities of intra-industry linkages, and workers were allowed to move between regions but not between sectors. These assumptions make our model more suitable to the analysis of relatively small-scale spatial reallocations such as those occurring in a regional or an urban context. We found that, at early stages of integration, industries tend to evenly split between locations so that sectors co-locate within each location with no specialization. When transport costs fall to an intermediate level, a core-periphery distinction emerges among the two locations: the strong-linkages industry totally clusters in one location (the core) which also receives some weak-linkages industry firms. As transport costs further decrease, the weak-linkages firms located in location 2 relocate to location 1 until full agglomeration. Finally, once transport costs have fallen sufficiently low, locations completely specialize, and industries no longer co-locate. However, at those advanced levels of integration, the peripheral location recaptures activity from the core, so that the overall degree of agglomeration is reduced. The threshold values of transport cost, as well as the uniqueness or multiplicity of equilibria in certain parameter ranges, depend on the calibration of the model, in particular with respect to the expenditure shares of the two industries and to the importance of locational congestion costs.

Although our model accommodates the locational patterns of Abdel-Rahman (1996) and Duranton and Puga (2001), it is not analytically tractable, leading us to heavily rely on simulations. The footloose economy framework proposed by Baldwin et al (2003) would have been more tractable. However, by sticking to the Dixit-Stiglitz framework, this paper provides case missing in the Fujita, Krugman and Venables (1999) collection of model. The results yielded are not surprising but point to fact in a small geographical scale such as an urban system, it is most of time hard to disentangle agglomeration, specialization and diversification features. At least, our results seem to reproduce the stylized facts on US metropolitan employment patterns over the period 1850-1990 as described in the introduction.

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Appendix 1: Comparing the Sustain Points

The condition $\omega_{x,1} > \omega_{x,2}$ translates to $w_{x,1} > \tau^{1-2\mu} w_{x,2}$ using equation (21), and this latter condition combined with equations (23) and (24) imply that:

$$w_{x,1} > \frac{\mu(1-\alpha_x)}{\mu + (1-\mu)\alpha_x} \frac{\tau^{2(1-\mu)\sigma(1-\alpha_x)-1} - 1}{1 - \tau^{1-2\mu\sigma(1-\alpha_x)}} w_{y,2}\,. \qquad (27)$$

Equation (26) implies that $w_{x,1} = w_{y,2}$ and the condition found above becomes:

$$\mu(1-\alpha_x)\tau^{2(1-\mu)\sigma(1-\alpha_x)-1} + [\mu + (1-\mu)\alpha_x]\tau^{1-2\mu\sigma(1-\alpha_x)} < \alpha_x + 2\mu(1-\alpha_x) \quad (28)$$

which is equivalent to

$$\mu(1-\alpha_x)\tau^{2(1-\mu)\sigma(1-\alpha_x)-1} + [\mu + (1-\mu)\alpha_x]\tau^{1-2\mu\sigma(1-\alpha_x)} \le 1 \quad \text{for } \mu \le 1/2 \quad (29)$$

After some algebra, we can show that for $w_{x,1} = w_{y,2}$, $\mu \le 1/2$ and $\alpha_x < \alpha_y$, equation (29) imply that $\omega_{y,2} > \omega_{y,1}$, that is the complete agglomeration of the weak input-output industry in region 1 implies the complete agglomeration of the strong input-output linkages industry in region 2 and thus $\tau_x^S < \tau_y^S$.

Appendix 2: Equations Solving for the Agglomerated Equilibrium of the Strong-Linkages Industry

Firstly, let recall that $\lambda_{x,2} = 1 - \lambda_{x,1}$ in our model. If only industry is completely concentrated in one location, we have $\lambda_{y,2} = 1$ and $\lambda_{y,1} = 0$ and the price index for industry are simplified. The other equations become:

Price indices equations

$$G_{x,1}^{1-\sigma} = \lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma} \tau^{1-\sigma}$$

$$G_{x,2}^{1-\sigma} = \lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} \tau^{1-\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}$$

Wage equations

$$w_{x,1}^{(1-\alpha_x)\sigma} = (1-\alpha_x)\left(\begin{array}{c}\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}G_{x,1}^{\sigma-\alpha_x\sigma-1} + \\ \left(\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2} + \mu w_{y,2}\right)\tau^{1-\sigma}G_{x,1}^{-\alpha_x\sigma}G_{x,2}^{\sigma-1}\end{array}\right)$$

$$w_{x,2}^{(1-\alpha_x)\sigma} = (1-\alpha_x)\left(\begin{array}{c}\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}G_{x,1}^{\sigma-1}G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma} + \\ \left(\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2} + \mu w_{y,2}\right)G_{x,2}^{\sigma-\alpha_x\sigma-1}\end{array}\right)$$

$$w_{y,1}^{(1-\alpha_y)\sigma} = (1-\alpha_y)\left(\begin{array}{c}(1-\mu)\lambda_{x,1}w_{x,1}\tau^{\sigma-\alpha_y\sigma-1}w_{y,2}^{\sigma(1-\alpha_y)-1} + \\ \left(\mu\lambda_{x,2}w_{x,2} + \left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)w_{y,2}\right)w_{y,2}^{\sigma(1-\alpha_y)-1}\tau^{1-\sigma-\alpha_y\sigma}\end{array}\right)$$

$$w_{y,2}^{(1-\alpha_y)\sigma} = (1-\alpha_y)\left(\begin{array}{c}(1-\mu)\lambda_{x,1}w_{x,1}w_{y,2}^{\sigma(1-\alpha_y)-1} + \\ \left(\mu\lambda_{x,2}w_{x,2} + \left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)w_{y,2}\right)w_{y,2}^{\sigma(1-\alpha_y)-1}\end{array}\right)$$

Real wage equations

$$\frac{\omega_{x,2}}{\omega_{x,1}} = \frac{w_{x,2}(\lambda_{x,2}+1)}{w_{x,1}\lambda_{x,1}^{-\delta}}\frac{G_{x,1}^{\mu}\tau^{1-\mu}}{G_{x,2}^{\mu}}$$

$$\frac{\omega_{y,1}}{\omega_{y,2}} = \frac{w_{y,1}\lambda_{x,1}^{-\delta}}{w_{y,2}(\lambda_{x,2}+1)^{-\delta}}\frac{G_{x,2}^{\mu}}{G_{x,1}^{\mu}\tau^{1-\mu}}$$

$$w_{x,1}\lambda_{x,1}^{-\delta}G_{x,2}^{\mu} = w_{x,2}(1+\lambda_{x,2})^{-\delta}G_{x,1}^{\mu}\tau^{1-\mu}$$

Appendix 3: Equations Solving the Real Wage Differential in the Weak-Linkages Industry

In the incomplete-specialization range, we have $\lambda_{y,1} = 0$ and $\lambda_{y,2} = 1$ i.e. industry remains concentrated in location 2. We choose the wage of this industry in location 2 as the numeraire, setting $w_{y,2} = 1$ (see footnote 7). The complete concentration of industry in location 2 is sustainable as long as the real wage in this location is higher than that in location 1 ($\omega_{y,2} > \omega_{y,1}$). Here again, the price indexes for industry are simplified and the other equations become:

Price indices equations

$$G_{x,1}^{1-\sigma} = \lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma} \tau^{1-\sigma}$$

$$G_{x,2}^{1-\sigma} = \lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} \tau^{1-\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}$$

Wage equations

$$w_{x,1}^{(1-\alpha_x)\sigma} G_{x,1}^{\alpha_x\sigma} = \begin{bmatrix} (\mu(1-\alpha_x) + \alpha_x)\lambda_{x,1} w_{x,1} G_{x,1}^{\sigma-1} + ((\mu(1-\alpha_x) + \alpha_x)\lambda_{x,2} w_{x,2} \\ +\mu(1-\alpha_x))G_{x,2}^{\sigma-1} \tau^{1-\sigma} \end{bmatrix}$$

$$w_{x,2}^{(1-\alpha_x)\sigma} G_{x,2}^{\alpha_x\sigma} = \begin{bmatrix} (\mu(1-\alpha_x) + \alpha_x)\lambda_{x,1} w_{x,1} G_{x,1}^{\sigma-1} \tau^{1-\sigma} + ((\mu(1-\alpha_x) + \alpha_x)\lambda_{x,2} w_{x,2} \\ +\mu(1-\alpha_x))G_{x,2}^{\sigma-1} \end{bmatrix}$$

$$w_{y,1}^{(1-\alpha_y)\sigma} \tau^{\alpha_y\sigma} = \begin{bmatrix} \mu(1-\alpha_y)\lambda_{x,1} w_{x,1} \tau^{1-\sigma} + (\lambda_{x,2}(1-\mu)(1-\alpha_y) w_{x,2} \\ +(1-\mu)(1-\alpha_y) + \alpha_y)\tau^{1-\sigma} \end{bmatrix}$$

Real wage equations

$$\omega_{x,1} G_{x,1}^{\mu} \tau^{1-\mu} = w_{x,1}^{-\delta} \lambda_{x,1}^{-\delta}$$

$$\omega_{x,2} G_{x,2}^{\mu} = w_{x,2}(1 + \lambda_{x,2})^{-\delta}$$

$$\omega_{y,1} G_{x,1}^{\mu} \tau^{1-\mu} = w_{y,1}^{-\delta} \lambda_{x,1}^{-\delta}$$

$$\omega_{y,2} G_{x,2}^{\mu} = (1 + \lambda_{x,2})^{-\delta}$$

Appendix 4: Stability Analysis of the Dispersed Equilibria

If we substitute the expenditures equations in the wage equations, the expressions we have to totally differentiate are the following:

Price indices equations

$$G_{x,1} = \left(\lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma} \tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$G_{x,2} = \left(\lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} \tau^{1-\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$G_{y,1} = \left(\lambda_{y,1} w_{y,1}^{1-\sigma(1-\alpha_y)} G_{y,1}^{-\alpha_y\sigma} + \lambda_{y,2} w_{y,2}^{1-\sigma(1-\alpha_y)} G_{y,2}^{-\alpha_y\sigma} \tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$G_{y,2} = \left(\lambda_{y,1} w_{y,1}^{1-\sigma(1-\alpha_y)} G_{y,1}^{-\alpha_y\sigma} \tau^{1-\sigma} + \lambda_{y,2} w_{y,2}^{1-\sigma(1-\alpha_y)} G_{y,2}^{-\alpha_y\sigma}\right)^{\frac{1}{1-\sigma}}$$

Wage equations

$$w_{x,1} = (1-\alpha_x)^{\frac{1}{(1-\alpha_x)\sigma}} \left(\begin{array}{l} \left[\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1} w_{x,1} + \mu\lambda_{y,1} w_{y,1}\right] G_{x,1}^{\sigma-\alpha_x\sigma-1} + \\ \left[\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2} w_{x,2} + \mu\lambda_{y,2} w_{y,2}\right] G_{x,1}^{-\alpha_x\sigma} G_{x,2}^{\sigma-1} \tau^{1-\sigma} \end{array} \right)^{\frac{1}{(1-\alpha_x)\sigma}}$$

$$w_{x,2} = (1-\alpha_x)^{\frac{1}{(1-\alpha_x)\sigma}} \left(\begin{array}{l} \left[\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1} w_{x,1} + \mu\lambda_{y,1} w_{y,1}\right] G_{x,1}^{\sigma-1} G_{x,2}^{-\alpha_x\sigma} \tau^{1-\sigma} + \\ \left[\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2} w_{x,2} + \mu\lambda_{y,2} w_{y,2}\right] G_{x,2}^{\sigma-\alpha_x\sigma-1} \end{array} \right)^{\frac{1}{(1-\alpha_x)\sigma}}$$

$$w_{y,1} = (1-\alpha_y)^{\frac{1}{(1-\alpha_y)\sigma}} \left(\begin{array}{l} \left[(1-\mu)\lambda_{x,1} w_{x,1} + \left(1-\mu + \frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,1} w_{y,1}\right] G_{y,1}^{\sigma-\alpha_y\sigma-1} + \\ \left[(1-\mu)\lambda_{x,2} w_{x,2} + \left(1-\mu + \frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,2} w_{y,2}\right] G_{y,1}^{-\alpha_y\sigma} G_{y,2}^{\sigma-1} \tau^{1-\sigma} \end{array} \right)^{\frac{1}{(1-\alpha_y)\sigma}}$$

$$w_{y,2} = (1-\alpha_y)^{\frac{1}{(1-\alpha_y)\sigma}} \left(\begin{array}{l} \left[(1-\mu)\lambda_{x,1} w_{x,1} + \left(1-\mu + \frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,1} w_{y,1}\right] G_{y,1}^{-\alpha_y\sigma} G_{y,2}^{\sigma-1} \tau^{1-\sigma} + \\ \left[(1-\mu)\lambda_{x,2} w_{x,2} + \left(1-\mu + \frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,2} w_{y,2}\right] G_{y,2}^{\sigma-\alpha_y\sigma-1} \end{array} \right)^{\frac{1}{(1-\alpha_y)\sigma}}$$

Real wage equations

$$\omega_{x,1} = \frac{w_{x,1}(\lambda_{x,1} + \lambda_{y,1})^{-\delta}}{G_{x,1}^{\mu} G_{y,1}^{1-\mu}}$$

$$\omega_{x,2} = \frac{w_{x,2}(\lambda_{x,2} + \lambda_{y,2})^{-\delta}}{G_{x,2}^{\mu} G_{y,2}^{1-\mu}}$$

$$\omega_{y,1} = \frac{w_{y,1}(\lambda_{y,1} + \lambda_{x,1})^{-\delta}}{G_{x,1}^{\mu} G_{y,1}^{1-\mu}}$$

$$\omega_{y,2} = \frac{w_{y,2}(\lambda_{x,2} + \lambda_{y,2})^{-\delta}}{G_{x,2}^{\mu} G_{y,2}^{1-\mu}}$$

Appendix 5: Price Indices and Nominal Wages at the Symmetric Equilibria

Let consider the optimal input allocation rules:

$$\frac{X_r^1}{\lambda_{x,r}} = \frac{\alpha_x}{1-\alpha_x} \frac{w_{x,r}}{G_{x,r}} \tag{30}$$

$$\frac{Y_r^1}{\lambda_{y,r}} = \frac{\alpha_y}{1-\alpha_y} \frac{w_{y,r}}{G_{y,r}} \tag{31}$$

where X_r^I and Y_r^I indicate the quantities of goods x and y used as firms intermediate consumption in region r. Dividing these two relations yields:

$$\frac{w_{x,r}}{w_{y,r}} = \frac{Y_r^1 G_{x,r}}{Y_r^1 G_{y,r}} \frac{\alpha_y(1-\alpha_y)}{\alpha_x(1-\alpha_y)} \tag{32}$$

Then, let consider the optimal final consumption demand:

$$X_r^F = \mu \frac{Income}{G_{x,r}} \tag{33}$$

$$Y_r^F = (1-\mu) \frac{Income}{G_{y,r}} \tag{34}$$

where X_r^F and Y_r^F indicate the quantities of goods x and y used as final consumption goods in region r. Dividing these two relations yields:

$$\frac{G_{x,r}}{G_{y,r}} = \frac{\mu}{1-\mu}\frac{Y_r^F}{Y_r^F} \tag{35}$$

Combining (32) and (35) yields:

$$\frac{w_{x,r}}{w_{y,r}} = \frac{\mu}{1-\mu}\frac{\alpha_y(1-\alpha_x)}{\alpha_x(1-\alpha_y)}\left(\frac{X_r^I}{Y_r^I}\frac{Y_r^F}{X_r^F}\right) \tag{36}$$

Since we consider the symmetric equilibrium, we can assume the following conditions:

$X_r^I = \alpha_x K_1, Y_r^I = \alpha_y K_1, X_r^F = \mu K_2, Y_r^F = (1-\mu)K_2$ where K_1 and K_2 are constants. With these conditions, (36) becomes:

$$w_{x,r} = \frac{1-\alpha_x}{1-\alpha_y}w_{y,r}. \tag{37}$$

If we take as numeraire the wage in industry in location 2, we have following values for the symmetric equilibrium:

$$w_{x,1} = w_{x,2} = \frac{1-\alpha_x}{1-\alpha_y},\ w_{y,1} = w_{y,2} = 1,$$

$$G_{x,1} = G_{x,2} = \frac{1-\alpha_x}{1-\alpha_y}\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{1}{1-\sigma(1-\alpha_x)}},$$

$$G_{y,1} = G_{y,2} = \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{1}{1-\sigma(1-\alpha_y)}},$$

Using the total differentiation expression, we get the following expressions:

Differentiated price indices equations

$$\frac{dG_x}{d\lambda_x} = Z\left(W^{\frac{1}{1-\sigma(1-\alpha_x)}}\left(\frac{2(1-\alpha_x)}{(1-\sigma)(1-\alpha_y)} + \frac{1+\alpha_x\sigma-\sigma}{1-\sigma}\frac{dw_x}{d\lambda_x}\right) - \frac{\alpha_x\sigma}{1-\sigma}\frac{dG_x}{d\lambda_x}\right)$$

$$\frac{dG_y}{d\lambda_y} = Z\left(W^{\frac{1}{1-\sigma(1-\alpha_y)}}\left(\frac{2}{1-\sigma} + \frac{1+\alpha_y\sigma-\sigma}{1-\sigma}\frac{dw_y}{d\lambda_y}\right) - \frac{\alpha_y\sigma}{1-\sigma}\frac{dG_y}{d\lambda_y}\right)$$

Differentiated wage equations

$$\frac{dw_x}{d\lambda_x} = \Theta^{\frac{1}{(1-\alpha_x)\sigma}} \left(\begin{array}{l} \Theta^{1-\sigma(1-\alpha_x)} Z \left(\begin{array}{l} \frac{2}{\sigma}\frac{(1-\alpha_x)\mu+\alpha_x}{1-\alpha_y} + \frac{2(1-\mu)}{\sigma}\frac{dw_y}{d\lambda_x} \\ + \frac{(1-\alpha_x)\mu+\alpha_x}{(1-\alpha_x)\sigma}\frac{dw_x}{d\lambda_x} + \frac{\mu}{\sigma}\frac{dw_y}{d\lambda_x} \end{array} \right) + \\ \frac{\sigma-\alpha_x\sigma-1-(\sigma+\alpha_x\sigma-1)\tau^{1-\sigma}}{2(1-\alpha_x)\sigma} W^{-\frac{2-(1-\alpha_x)\sigma}{1-(1-\alpha_x)\sigma}} \frac{dG_x}{d\lambda_x} \end{array} \right)$$

$$\frac{dw_x}{d\lambda_y} = \Theta^{\frac{1}{(1-\alpha_x)\sigma}-\sigma(1-\alpha_x)+1} Z \left(\begin{array}{l} \frac{2}{\sigma}\frac{(1-\alpha_x)\mu+\alpha_x}{1-\alpha_y}\frac{dw_x}{d\lambda_y} + \frac{2(1-\mu)}{\sigma} + \\ \frac{(1-\alpha_x)\mu+\alpha_x}{(1-\alpha_x)\sigma}\frac{dw_x}{d\lambda_y} + \frac{\mu}{\sigma}\frac{dw_y}{d\lambda_y} \end{array} \right)$$

$$\frac{dw_y}{d\lambda_x} = \Phi^{\frac{1}{(1-\alpha_y)\sigma}-\sigma(1-\alpha_y)+1} Z \left(\begin{array}{l} \frac{2}{\sigma}\frac{1-\alpha_x}{1-\alpha_y} + \frac{2((1-\alpha_y)(1-\mu)+\alpha_y)}{(1-\alpha_y)\sigma}\frac{dw_y}{d\lambda_x} + \\ \frac{(1-\mu)}{\sigma}\frac{dw_x}{d\lambda_x} + \frac{(1-\alpha_y)(1-\mu)+\alpha_y}{\sigma}\frac{dw_y}{d\lambda_x} \end{array} \right)$$

$$\frac{dw_y}{d\lambda_y} = \Phi^{\frac{1}{(1-\alpha_y)\sigma}} \left(\begin{array}{l} \Phi^{1-\sigma(1-\alpha_y)} Z \left(\begin{array}{l} \frac{2}{\sigma}\frac{(1-\alpha_x)}{1-\alpha_y}\frac{dw_x}{d\lambda_y} + \frac{2((1-\mu)(1-\alpha_y)+\alpha_y)}{(1-\alpha_y)\sigma} \\ + \frac{(1-\mu)}{\sigma}\frac{dw_x}{d\lambda_y} + \frac{(1-\mu)(1-\alpha_y)+\alpha_y}{\sigma}\frac{dw_y}{d\lambda_y} \end{array} \right) + \\ \frac{\sigma-\alpha_y\sigma-1-(\sigma+\alpha_y\sigma-1)\tau^{1-\sigma}}{2(1-\alpha_y)\sigma} W^{-\frac{2-(1-\alpha_y)\sigma}{1-(1-\alpha_y)\sigma}} \frac{dG_y}{d\lambda_y} \end{array} \right)$$

Differentiated real wage equations

$$\frac{d\omega_x}{d\lambda_y} = \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{1-\mu} W^{\Gamma}\left(-\delta\left(1+\frac{d\omega_y}{d\lambda_x}\right) + \frac{1-\alpha_y}{1-\alpha_x}\frac{dw_x}{d\lambda_x} - \mu\frac{1-\alpha_y}{1-\alpha_x}(W)^{-\frac{1}{1-\sigma(1-\alpha_x)}}\frac{dG_x}{d\lambda_x}\right)$$

$$\frac{d\omega_x}{d\lambda_y} = \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{1-\mu} W^{\Gamma}\left(-\delta\left(\frac{d\omega_y}{d\lambda_y}+1\right) + \frac{1-\alpha_y}{1-\alpha_x}\frac{dw_x}{d\lambda_x} - (1-\mu)(W)^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_y}\right)$$

$$\frac{d\omega_y}{d\lambda_x} = \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{-\mu} W^{\Gamma}\left(-\delta\left(1+\frac{d\omega_y}{d\lambda_x}\right) + \frac{dw_y}{d\lambda_x} - \mu\frac{1-\alpha_y}{1-\alpha_x}(W)^{-\frac{1}{1-\sigma(1-\alpha_x)}}\frac{dG_x}{d\lambda_x}\right)$$

$$\frac{d\omega_y}{d\lambda_y} = \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{-\mu} W^{\Gamma}\left(-\delta\left(\frac{d\omega_x}{d\lambda_y}+1\right)+\frac{dw_y}{d\lambda_y}-(1-\mu)(W)^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_y}\right).$$

In these equations, we have set:

$$\Gamma = \frac{\mu\sigma(\alpha_x-\alpha_y)+\sigma-\alpha_x\sigma-1}{(1-\sigma+\alpha_x\sigma)(1-\sigma+\alpha_y\sigma)}$$

$$\Theta = (1-\alpha_x)\mu+\alpha_x+\mu(1-\alpha_y)$$

$$\Phi = (1-\alpha_y)(1-\mu)+\alpha_y+(1-\mu)(1-\alpha_x)$$

$$Z = \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}$$

$$W = \frac{1+\tau^{1-\sigma}}{2}$$