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Quotas and Quality in an International Duopoly

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Abstract

This paper examines possible adjustments to a change in a binding quota in the context of an international duopoly. Consumers directly value embodied quality of goods, which is chosen simultaneously with quantity, and before quantity in a sequential model. Possible responses to a small change in a binding quota are derived. The same three types of equilibria occur in the simultaneous and sequential models. Foreign quality downgrading can occur if domestic quality falls, and is more likely starting with a low quantity of high quality imports. Domestic quality and quantity respond in opposite directions. Welfare effects are discussed.

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I. Introduction

A virtual folk theorem in international economics is that quotas lead to quality upgrading of imports. Theoretical studies of quotas and quality find that import quality rises in response to a quota: the competitive models of Falvey [1979], Santoni and VanCott [1980], Rodriquez [1979], and Mayer [1982]; the monopoly model of Das and Donnenfeld [1987]; and the duopoly model of Das and Donnenfeld [1989]. Patterson [1966], Meier [1973], and MacPhee [1974] have documented increases in quality with quotas in the steel and textile industries. Similar effects have been noted by Feenstra [1984] in automobiles, Aw and Roberts [1986] in footwear, and Anderson [1985] in cheese.

Krishna [1987] argues that a quota imposed on a foreign monopolist can lead to quality downgrading of imports if the value of quality to the average consumer is less than its value to the marginal consumer, a result directly related to the work of Spence [1975]. In a model of vertical product differentiation, Ries [1993] shows that foreign multiproduct Cournot competitors will not upgrade if they produce relatively lower quality goods. Chen [1992] shows that import quality downgrading may be a signal to sustain oligopoly collusion in a supergame when a quota just binds in the free trade collusive equilibrium.

The present paper evaluates the effects of a small change in a binding quota in an international duopoly of one domestic firm and one foreign firm. As in all comparative static analysis, the presumption is that local analysis can be carried over to global issues such as large quota increases. The willingness of consumers to pay for quality is modeled by the demand specification introduced by Mussa and Rosen [1979], and subsequently utilized by Chiang and Spatt [1982], Das and Donnenfeld [1989], and Beard and Ekelund [1991], among others. Two models of noncooperative behavior are employed. Choices of quality and quantity are made simultaneously in the first model, while choice of quality precedes choice of quantity in a sequential model.

If quality is varied through superficial modifications that can be made at the time of manufacture, firms can be said to choose quality and quantity simultaneously. Examples are the choice of upholstery, tires, and options in automobiles. If quality is varied through costly design changes, firms must commit to a choice of quality before production. Examples are a car's interior room, fuel economy, and handling. The surprising conclusion is reached that it is not critical for the range of possible outcomes whether quality is chosen simultaneously or sequentially. This result suggests that the interpretation of product quality may be relatively unimportant when studying quotas.

Three types of equilibria occur for both the simultaneous and sequential models under the assumption that the domestic firm manufactures the higher quality product. In every case, the domestic firm's quality and quantity respond in opposite directions to a quota. A tighter quota leads to either reduced output or reduced quality for the domestic firm.

A tighter quota may also lead to reduced import quality. Import downgrading can occur only when accompanied by domestic downgrading, and is more likely when the market is characterized by a small quantity of high quality imports.

The paper is divided into five sections and a conclusion. Section II outlines assumptions and specifies payoff functions for the international duopoly. Sections III and IV analyze quota constrained equilibria in the case of simultaneous choice of qualities and quantities. Section V evaluates the sequential model in which quality commitments are made prior to Nash quantity competition. Section VI considers the welfare effects of quota changes.

II. Fundamental Assumptions of the Model

The preference model follows Mussa and Rosen [1979]. Assume a large set of potential customers, each with unit demand for the good, buying either one or zero units. A consumer's maximum willingness to pay for the good depends on the amount of embodied quality X. Assume a consumer of type θ is willing to pay up to θX dollars for one unit of the product of quality X in the absence of a better alternative. When two or more versions of the product are available, a consumer chooses the version that yields the greatest difference between value θX and price. If no version yields value at least as large as price, the consumer does not by the product.[1]

1. Formally, it is assumed that a person of type θ who consumes one unit of the good of quality X along with Y dollars of other commodities has utility $Y + \theta X$. This formulation implies X is measured in utility units yielding constant marginal utility. If a physical measure X' of quality were employed, utility would be represented by Y +

Let $f(\theta)$ represent the marginal distribution of willingness to pay among consumers.[2] Suppose there are sufficient consumers that the support of $f(\cdot)$ is the interval $[\underline{\theta}, \bar{\theta}] \subset R^+$. Moreover, suppose $f(\cdot)$ is continuous throughout the interval and differentiable at all interior points.

Assume a firm's selection of quality involves fixed costs which are sufficiently large to make each firm sell units of only one quality in the neighborhood of the equilibrium.

A marginal value of quality function $H(Q)$ is specified by

$$\int_{H(Q)}^{\bar{\theta}} f(\theta)d\theta = Q. \tag{1}$$

If customers were lined up starting with those who value quality most, $H(Q)$ would describe the marginal value of quality to customer Q.

If two firms offer products of the same quality X and produce a total output Q, market clearing would require both firms to charge the price P where total quantity demanded by all consumers who value the product at least P would equal Q. This equilibrium would occur if and only if $P = XH(Q)$.

Suppose, however, the two firms offer products of low and high qualities $X_1 < X_h$, at prices P_1 and P_h respectively. Then a consumer of type θ would either:

(i) buy the good with quality X_h if $P_h \le \theta X_h$ and $(X_h - X_1)\theta \ge P_h - P_1$;
(ii) buy the good with quality X_1 if $P_1 \le \theta X_1$ and $(X_h - X_1)\theta \le P_h - P_1$; or
(iii) buy nothing if $P_h > \theta X_h$ and $P_1 > \theta X_1$.

Market clearing requires that (P_1, X_1, Q_1) and (P_h, X_h, Q_h) satisfy

$$P_1 = X_1 H(Q_1 + Q_h) \tag{2i}$$

$$P_h = X_1 H(Q_1 + Q_h) + (X_h - X_1) H(Q_h), \tag{2ii}$$

where Q_1 and Q_h are quantities of the low and high quality products.

$\theta U(\hat{X})$, $U' \ge 0$ and $U'' \le 0$. The two measures of quality are monotonic transformations of each other, $X = U(\hat{X})$.

2. If the distribution function $f(\theta)$ is reinterpreted, the assumption that each consumer buys either one or zero units can be relaxed. Assume that consumer taste can be characterized by a vector $(\theta_1, \theta_2, \theta_3, \ldots)$ where θ_i represents the consumer's willingness to pay for the quality of unit i.

The demand system specified by (2) is utilized. Note that a product of higher quality commands a higher price, $P_h > P_1$. The assumption that $f(\theta)$ is differentiable implies $H(Q)$ is twice continuously differentiable. Therefore, (1) implies

$$dH(Q)/dQ = -\{f[H(Q)]\}^{-1} \leq 0 \quad (3a)$$

$$d^2H(Q)/dQ^2 = -\{f'[H(Q)]\}\{f[H(Q)]\}^{-2}. \quad (3b)$$

Differentiation of (2) implies that the inverse demand functions $P_h(Q_h, X_h, Q_1, X_1)$ and $P_1(Q_h, X_h, Q_1, X_1)$ are continuous in both quantities and qualities, differentiable everywhere in quantities, and piecewise linear in qualities.[3] Further, $\partial P_h/\partial Q_h < 0$, $\partial P_h/\partial X_h > 0$, $\partial P_h/\partial Q_1 < 0$, and $\partial P_h/\partial X_1 < 0$. Similarly, $\partial P_1/\partial Q_h < 0$, $\partial P_1/\partial X_h < 0$, $\partial P_1/\partial Q_1 < 0$, and $\partial P_1/\partial X_1 > 0$. The demand system in (2) exhibits intuitive responses. Some second and higher order derivatives of $P_1(\cdot)$ and $P_h(\cdot)$ depend on derivatives of the marginal density function $f(\theta)$.

Suppose sales of the foreign firm are subject to a binding quota, while sales of the domestic firm are unrestricted. Assume production technologies of the two firms differ, but both exhibit constant returns to scale with respect to output and decreasing returns with respect to quality. Total production cost of the domestic firm is $[c_d(X_d)]Q_d$ and total cost of the foreign firm is $[c_f(X_f)]Q_f$, where d and f refer to the domestic and foreign firm. Each unit cost function $c_i(\cdot)$ is strictly convex and twice continuously differentiable.[4]

Profit functions for the domestic and foreign firm are given by

$$\pi_d(Q_d, X_d, Q_f, X_f) = [p_d(\cdot) - c_d(X_d)]Q_d \quad (4i)$$

$$\pi_f(Q_d, X_d, Q_f, X_f) = [p_f(\cdot) - c_f(X_f)]Q_f. \quad (4ii)$$

As the demand system (2) indicates, the manner in which quality enters a firm's inverse demand and profit functions depends on whether the firm

3. Inverse demands are piecewise linear in qualities because the derivative of one firm's inverse demand with respect to either its own quality or its rival's quality changes abruptly at the point where both qualities are equal.
4. Measuring quality in physical units rather than utility units would a require a transformation of the unit cost functions $c_i(X_i)$ and their derivatives. See note 1.

produces the high or low quality product. Properties of the quota constrained Nash equilibrium must, therefore, depend on the relative quality of imports. This paper examines the case in which the foreign firm initially produces the low quality product. This choice is motivated by a desire to apply this model to industries in which the domestic firm competes with a low quality or less opulent foreign substitute, as in the automobile industry of the early 1980s.

III. Simultaneous Choice of Quality and Quantity

In the simultaneous model, firms select outputs and qualities contemporaneously. In the absence of a strictly binding quota, any interior Nash equilibrium $(Q_d^*, X_d^*, Q_f^*, X_f^*)$ must satisfy the first order condition

$$0 = \partial\pi^d/\partial Q_d = \partial\pi^d/\partial X_d = \partial\pi^f/\partial Q_f = \partial\pi^f/\partial X_f. \tag{5}$$

Suppose a quota of the form $Q_f \le \bar{Q}_f < Q_f^*$ is imposed. If the quota $\bar{Q}_f$ is sufficiently close to Q_f^*, a new Nash equilibrium emerges in which the quota is strictly binding in the sense that $\partial\pi^f/\partial Q_f > 0$. Using (2) and (4), such a quota constrained Nash equilibrium must satisfy the first order conditions

$$\begin{aligned}\partial\pi^d/\partial Q_d = {} & X_f H(Q_d + Q_f) + (X_d - X_f)H(Q_d) - c_d \\ & + Q_d[X_f H'(Q_d + Q_f) + (X_d - X_f)H'(Q_d)] = 0 \end{aligned} \tag{6i}$$

$$\partial\pi^d/\partial X_d = [H(Q_d) - c_d']Q_d = 0 \tag{6ii}$$

$$\partial\pi^f/\partial X_f = [H(Q_d + Q_f) - c_f']Q_f = 0, \tag{6iii}$$

where the derivative of each firm's cost function with respect to quality is written $c_i' \equiv dc_i/dX_i$. Sufficient second order conditions for the Nash equilibrium are

$$\begin{vmatrix} \partial^2\pi^d/\partial Q_d^2 & \partial^2\pi^d/\partial Q_d\,\partial X_d \\ \partial^2\pi^d/\partial X_d\,\partial Q_d & \partial^2\pi^d/\partial X_d^2 \end{vmatrix} > 0 \tag{7i}$$

$$\partial^2\pi^d/\partial Q_d^2 < 0, \quad \partial^2\pi^d/\partial X_d^2 < 0, \text{ and } \partial^2\pi^f/\partial X_f^2 < 0. \tag{7ii}$$

Interest focuses on the derivatives of X_d^* and X_f^* with respect to $\bar{Q}_f$. Differentiating the system (6) and accounting for all derivatives that are identically equal to zero implies the desired derivatives must satisfy the linear system

$$\begin{bmatrix} \partial^2\pi^d/\partial Q_d^2 & \partial^2\pi^d/\partial Q_d\partial X_d & \partial^2\pi^d/\partial Q_d\partial X_f \\ \partial^2\pi^d/\partial X_d\partial Q_d & \partial^2\pi^d/\partial X_d^2 & 0 \\ \partial^2\pi^f/\partial X_f\partial Q_d & 0 & \partial^2\pi^f/\partial X_f^2 \end{bmatrix} \begin{bmatrix} dQ_d^*/\overline{Q}_f \\ dX_d^*/\overline{Q}_f \\ dX_f^*/\overline{Q}_f \end{bmatrix} = -\begin{bmatrix} \partial^2\pi^d/\partial Q_d\partial Q_f \\ 0 \\ \partial^2\pi^f/\partial X_f\partial Q_f \end{bmatrix} \tag{8}$$

Cramer's rule yields

$$dQ_d^*/d\overline{Q}_f = -(\partial^2\pi^d/\partial X_d^2)I_d/\Delta \tag{9i}$$

$$dX_d^*/d\overline{Q}_f = -(\partial^2\pi^d/\partial Q_d\partial X_d)I_d/\Delta \tag{9ii}$$

$$dX_f^*/d\overline{Q}_f = -(\partial^2\pi^f/\partial Q_d\partial X_f)I_f/\Delta, \tag{9iii}$$

where:

$$I_d \equiv \frac{\partial^2\pi^d}{\partial Q_d\partial Q_f}\frac{\partial^2\pi^f}{\partial X_f^2} - \frac{\partial^2\pi^f}{\partial Q_f\partial X_f}\frac{\partial^2\pi^d}{\partial Q_d\partial X_f} \tag{10i}$$

$$I_f \equiv \frac{\partial^2\pi^d}{\partial X_d^2}\frac{\partial^2\pi^d}{\partial Q_d\partial Q_f} + \frac{\partial^2\pi^d}{\partial Q_d^2}\frac{\partial^2\pi^d}{\partial X_d^2} - \left(\frac{\partial^2\pi^d}{\partial Q_d\partial X_d}\right)^2, \tag{10ii}$$

and Δ is the determinant of the 3×3 matrix in (8).

Comparison of (9i) and (9ii), combined with (6) yields

Proposition 1: A change in a quota restriction induces changes in the equilibrium domestic quantity and quality in opposite directions:

$$sgn\,[dX_d/dQ_f] = -sgn\,[dQ_d/dQ_f].$$

Consider the last row of (8). Whenever (6iii) is satisfied, $\partial^2\pi^f/\partial Q_f\partial X_f = Q_fH' + H - c_f' = Q_fH'(Q_d+Q_f) = \partial^2\pi^f/\partial Q_d\partial X_f$, which implies

$$\frac{dQ_d^*}{d\overline{Q}_f} + 1 = -\frac{\partial^2\pi^f/\partial X_f^2}{\partial^2\pi^f/\partial Q_d\partial X_f}\frac{dX_f^*}{d\overline{Q}_f} \tag{11}$$

By (7ii), $\partial^2\pi^f/\partial X_f^2 < 0$. Hence, the term in front of dX_f^*/dQ_f in (11) must be negative. If $dQ_d^*/dQ_f > 0$, then $dX_f^*/dQ_f < 0$. Proposition 1 and (11) together imply

Proposition 2: Regardless of the distribution of tastes, the comparative statics of any interior Nash equilibrium with $X_f^* < X_d^*$ must follow one of three patterns:

	dQ_d^*/dQ_f	dX_d^*/dQ_f	dX_f^*/dQ_f
I	–	+	+
II	–	+	–
III	+	–	–

This array presents the possible comparative statics when the domestic firm produces the higher quality product. Mathematically, there are eight potential qualitative effects that a quota change could have on domestic quantity, domestic quality, and foreign quality, but only three are possible. If domestic output falls with a quota, both domestic quality and foreign quality must rise. Domestic upgrading cannot occur unless foreign quality also improves. Examples in the following section show that equilibria of each of the three types can be obtained using a simple distribution of tastes.

These results are more general than those of Ries [1993], where foreign multiproduct oligopolistic firms in Cournot competition would not upgrade quality, while domestic firms necessarily increase output and might downgrade. The model of Ries leads to sign pattern I of Proposition 2.

A little reflection on the structure of the model makes the results in Proposition 2 transparent. As Spence [1975] and others have argued, profit maximization requires that a firm choose the quality at which the marginal cost of quality equals its marginal benefit to the marginal customer. This principle holds in any static model with the firm free to choose quality. Determining how a quota change affects a firm's choice of quality is equivalent to determining the change in the value of quality to the marginal consumers. If the marginal cost of quality increases with quality ($c_i'' > 0$), a firm would increase (decrease) quality if its new marginal consumers value quality more (less) than its previous marginal consumers.

The value of quality to the domestic firm's marginal consumers is $H(Q_d)$. Since $H'(Q_d) < 0$, domestic quantity and the value of quality to its marginal customers move in opposite directions. The domestic firm's profit maximizing quantity and quality thus move in opposite directions, as reflected by Proposition 1. The value of quality to the foreign firm's marginal consumers is $H(Q_d+Q_f)$. When a tighter quota leads to a reduction in Q_d, the sum Q_d+Q_f falls and $H(Q_d+Q_f)$ rises. When the foreign firm's new marginal consumers value quality more, foreign quality rises as in case III.

When a tighter quota leads to an increase in Q_d, there are two possibilities. If the increase in Q_d is smaller than the reduction in the quota ($dQ_d^*/d\bar{Q}_f > -1$), $Q_d + Q_f$ falls and the new marginal consumers of the foreign firm value quality more than its previous marginal consumers. Foreign quality then rises as in case II. Alternatively, if the increase in Q_d exceeds the reduction in the quota ($dQ_d^*/d\bar{Q}_f < -1$), $Q_d + Q_f$ rises and the foreign firm's new marginal consumers value quality less. Foreign quality then falls as in case I.

IV. Simultaneous Choice with Uniform Distribution of Consumer Types

The example of this section is based on the assumption of a uniform distribution of consumer marginal valuation of quality, $f(\theta)$. Analysis of this special case is motivated by two considerations. First, the uniform distribution can be used to construct examples that yield the three equilibria in Proposition 2, which suggests that each is equally plausible. Second, this analysis will facilitate study of the model of sequential choice.

When $f(\theta)$ is uniform, $H(Q)$ is linear,

$$H(Q) = \bar{\theta} - hQ,$$

where h is a positive constant. Calculating the first order conditions and performing the static exercise yields a special case of system (8),

$$\begin{bmatrix} -2hX_d & -hQ_d & -h\bar{Q}_f \\ -hQ_d & -Q_d c_d'' & 0 \\ -h\bar{Q}_f & 0 & -\bar{Q}_f c_f'' \end{bmatrix} \begin{bmatrix} dQ_d^*/d\bar{Q}_f \\ dX_d^*/d\bar{Q}_f \\ dX_f^*/d\bar{Q}_f \end{bmatrix} = \begin{bmatrix} hX_f \\ 0 \\ h\bar{Q}_f \end{bmatrix} \qquad (8')$$

Solving this system, or equivalently, substituting into equations (9) and (10) implies

$$dQ_d^*/d\bar{Q}_f = c_d'' Q_d I_d/\Delta \qquad (13i)$$

$$dX_d^*/d\bar{Q}_f = -hQ_d I_d/\Delta \qquad (13ii)$$

$$dX_f^*/d\bar{Q}_f = h\bar{Q}_f I_f/\Delta, \qquad (13iii)$$

where:

$$I_d = (hQ_f)\ [X_f c_f'' - hQ_f] \quad (14i)$$

$$I_f = (hQ_d)\ [(2X_d - X_f)c_d'' - hQ_d] \quad (14ii)$$

$$\Delta = hQ_f Q_d[hQ_f c_d'' - c_f''\ (2X_d c_d'' - hQ_d)]. \quad (14iii)$$

The relative locations of the three types of equilibria that arise in (Q_f, X_f) space are illustrated in Figure 1. The boundary Δ=0 separates regions of negative and positive, and has slope equal to the expression in brackets in (14iii). When Δ = 0, $h\ Q_f c_d'' = c_f''(2X_d c_d'' - hQ_d)$, which implies $2X_d c_d'' > hQ_d$. The slope of the locus Δ = 0 thus depends on the sign of $c_f'''(X_f)$. Figure 1 is drawn under the assumption of a positive slope. Regardless of its slope, $\Delta < 0$ to the left and $\Delta > 0$ to the right.

The boundary defined by I_d=0 passes through the origin and has slope $h_f/(X_f c_f + c_f'')$. This slope is positive, provided that c_f'' does not diminish too rapidly, or specifically that $X_f c_f''$ is an increasing function of X_f. If this slope is never positive, there cannot be an equilibrium with positive Q_f and X_f. I_d is positive (negative) at any point to the left (right) of the I_d=0 boundary.

The boundary defined by I_f=0 is horizontal. I_f is negative at any point above and positive at any point below. Note that all three boundaries intersect at the equilibrium point.[5]

An assumption commonly employed to reduce the set of potential Nash equilibria is stability to a tatonment adjustment process in which firms in each round make optimal choices given their rival's choice in the previous round. This stability requires a negative Δ. A proof is in Appendix B.

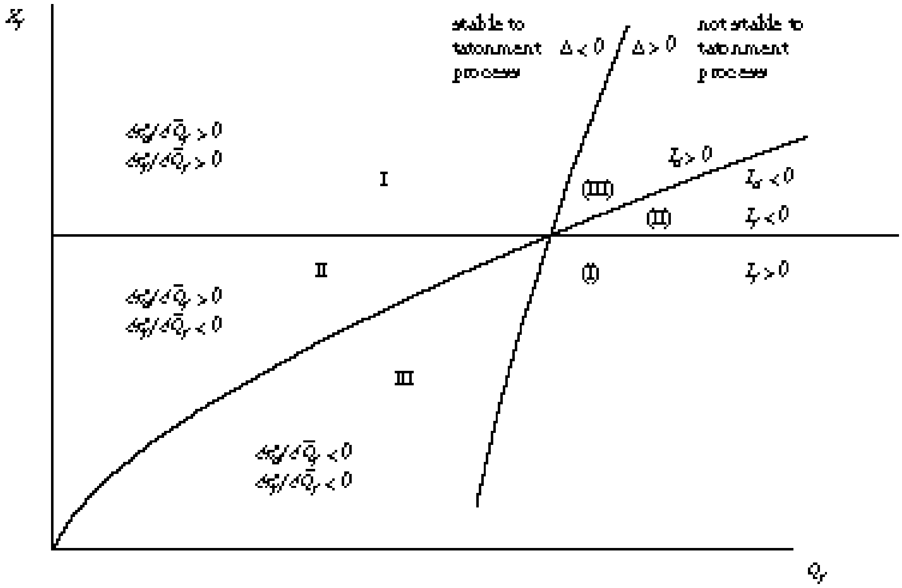
Figure 1 illustrates the three qualitatively different equilibria. Restricting attention to stable equilibria, regions I, II, and III correspond loosely to high, intermediate, and low import quality.[6] The three regions correspond directly to the sign patterns in Proposition 2. Sign patterns I, II, or III occur when the original combinations of Q_f and X_f are in respectively regions I, II, or III.

A tighter quota leads to import upgrading ($dX_f^*/dQ_f < 0$) when the equilib-

5. Employing the fact that $\partial^2\pi^f/\partial Q_f \partial X_f = \partial^2\pi^f/\partial Q_d \partial X_f$ in equilibrium, straightforward manipulation of (10) and the definition of Δ, implicit in (8), implies $[\partial^2\pi^d/\partial X_d^2]I_d + [\partial^2\pi^f/\partial X_f^2]I_f = \Delta$, regardless of the distribution of tastes. When any two of I_f, I_d, and Δ are zero, the third must also be zero.

6. Recall that the foreign product always has lower quality. Hence the terms "high", "intermediate", and "low" refer to some absolute measure.

Figure 1
Comparative Statics with Simultaneous Choice



rium occurs in regions II and III, and to domestic downgrading ($dX_d^*/dQ_f >$ 0) in regions I and II. When import quality is low, a tighter quota leads to upgrading by both firms. When import quality is high, tightening a quota leads to downgrading by both firms, a result not in the literature.

Das and Donnenfeld [1989] also analyze the effect of a quota change in the special case that willingness to pay is uniformly distributed. Employing a similar model, they derive expressions for dQ_d^*/dQ_f and dX_f^*/dQ_f. After accounting for their use of a physical measure of quality, their result is equivalent to (13i), (13iii), and (14). Das and Donnenfeld assume, however, that two expressions, α and β, which are respectively proportional to I_f and I_d, are positive. By footnote 5, their assumption implies $\Delta < 0$. Das and Donnenfeld only arrive at case II in Proposition 2.

Total sales by the domestic firm rise in response to a tighter quota ($dQ_d^*/dQ_f > 0$) if the equilibrium occurs in regions I and II, but fall in region III. Domestic sales fall when a quota is placed on a low quality import, but rise when a quota is placed on intermediate or high quality imports. Total (domestic plus foreign) sales fall in response to a tighter quota in regions II and III, but rise in region I.

While all three equilibria in Proposition 2 can be obtained with uniform willingness to pay, the range of potential qualitative responses to the quota is substantially circumscribed. For instance, a quota cannot promote domestic output without simultaneously reducing domestic quality. Further, these results rule out the possibility that a quota might promote domestic quality while discouraging foreign quality.

V. Sequential Choice: Quality Before Quantity

When quality comes from characteristics achieved in design, a sequential representation is more realistic. Each firm must then select the quality of its product prior to production. Comparative static analysis of a sequential model requires evaluation of expressions involving second and third derivatives of demand and cost functions. Few plausible statements concerning characteristics of these high order derivatives can be made without specifying functional forms. Analysis is therefore restricted to the case of a uniform distribution of willingness to pay, as in the previous section.

The model is solved recursively, starting with the second period. Assume the two firms have chosen qualities X_d and X_f, and select outputs Q_d and Q_f in the second stage. Output of the foreign firm is subject to a quota $Q_f \le \bar{Q}_f$. Denote second stage choices by $Q_d^{**}(X_d, X_f, Q_f)$ and $Q_f^{**}(X_d, X_f, Q_f)$. Assume the foreign firm produces a product of lower quality than the domestic firm, and the quota strictly binds in the second stage equilibrium. Then $Q_f^{**} = \bar{Q}_f$ and Q_d^{**} solves

$$^{d}(Q_d, X_d, Q_f, X_f) / \quad Q_d = 0. \tag{15i}$$

Also,

$$^{f}(Q_d^{**}, X_d, \bar{Q}_f, X_f) / \quad Q_f > 0. \tag{15ii}$$

With $H(Q)$ specified by (12), solving (15) yields

$$Q_d^{**} = (\bar{\ } - c_d / X_d - hX_f \bar{Q}_f / X_d) / 2h \tag{16i}$$

$$\bar{Q}_f < (\bar{\ } - hQ_d - hQ_d - c_f / X_f) / 2h. \tag{16ii}$$

In the first stage, firms simultaneously select qualities X_d and X_f to maximize profits in the second stage quantity equilibrium. Assuming an interior solution, X_d and X_f are selected to satisfy

$$\frac{d}{dX_d}\{ \ {}^{d}[Q_d^{**}(X_d, X_f, \bar{Q}_f), X_d, Q_f^{**}(X_d, X_f, \bar{Q}_f), X_f]\} = 0 \qquad (17i)$$

$$\frac{d}{dX_f}\{ \ {}^{f}[Q_d^{**}(X_d, X_f, \bar{Q}_f), X_d, Q_f^{**}(X_d, X_f, \bar{Q}_f), X_f]\} = 0 \qquad (17ii)$$

Employ (16), rearrange, and simplify to find first order conditions for the first stage[7]

$$\bar{\ } + c_d / X_d - 2c_d' + hX_f\bar{Q}_f / X_d = 0 \qquad (18i)$$

$$\bar{\ } + c_d / X_d - 2c_f' - 2h\bar{Q}_f(1 - X_f / X_d) = 0. \qquad (18ii)$$

Second order conditions require

$$(c_d' - c_d / X_d - hX_f\bar{Q}_f / X_d)X_d - 2c_d'' < 0 \qquad (19i)$$

$$h\bar{Q}_f / X_d - c_f'' < 0. \qquad (19ii)$$

Denote solutions of (18) by $X_d^{**}(\bar{Q}_f)$ and $X_f^{**}(\bar{Q}_f)$. Interest focuses on the terms $dX_d^{**}/d\bar{Q}_f$ and $dX_f^{**}/d\bar{Q}_f$. Analysis is simplified by introducing

$$A \equiv \bar{\theta} + c_d / X_d, \qquad (20)$$

and $A' \equiv dA/dX_d$. Derivatives of the functions X_d^{**} and X_f^{**} must satisfy the linear system of equations derived by differentiating (18),

$$\begin{bmatrix} A' - 2c_d'' - hX_f\bar{Q}_f / X_d^2 & \dfrac{h\bar{Q}_f}{X_d} \\ A' - 2h\dfrac{X_f}{X_d^2}\bar{Q}_f & -2c_f'' + 2h\bar{Q}_f / X_d \end{bmatrix} \begin{bmatrix} \dfrac{dX_d^{**}}{d\bar{Q}_f} \\ \dfrac{dX_f^{**}}{d\bar{Q}_f} \end{bmatrix} = -\begin{bmatrix} h\dfrac{X_f}{X_d} \\ -2h(1 - \dfrac{X_f}{X_d}) \end{bmatrix} \qquad (21)$$

Let Δ^* denote the determinant of the matrix in (21). Solving this system implies:

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7. For the foreign firm, $d\pi^f / dX_f = \partial\pi^f/\partial X_f + (\partial\pi^f/\partial Q_d) / (\partial Q_d^{**}/\partial X_f) + (\partial\pi^f/\partial Q_f)(\partial Q_f^{**}/\partial X_f)$. The second term is a strategic term which takes into account the effect of quality choice on the location of the second stage equilibrium. The last term is zero due to the assumption of a binding quota. For the domestic firm, $d\pi^d / dX_d = \partial\pi^d / \partial X_d$ + $(\partial\pi^d/\partial Q_d)(\partial Q_d^{**}/\partial X_d)$+$(\partial\pi^d/\partial Q_f)(\partial Q_f^{**}/\partial X_d)$. This second term is zero by (15i), and the strategic effect in the last term is zero.

$$dX_d^{**} / d\overline{Q}_f = -2h\, I_d^* / (\Delta^* X_d) \tag{22i}$$

$$dX_f^{**} / d\overline{Q}_f = -h\, I_f^* / \Delta^* \tag{22ii}$$

where:

$$I_d^* = h\overline{Q}_f - X_f c_f'' \tag{23i}$$

$$I_f^* = -2(A' - 2c_d'') + (A' - 4c_d'')(X_f / X_d) + 2hX_f\overline{Q}_f / X_d^2 \tag{23ii}$$

$$\Delta^* = -2c_f''(A' - 2c_d'') + h\overline{Q}_f(A' - 4c_d'') / X_d + 2hX_f\overline{Q}_f c_f'' / X_d^2. \tag{23iii}$$

Straightforward manipulation of (23) implies that I_d^*, I_f^*, and Δ^* obey

$$X_f\Delta^* = h\overline{Q}_f I_f^* + 2(A' - 2c_d'' - hX_f\overline{Q}_f / X_d^2)I_d^*. \tag{24}$$

Note that second order condition (19i) can be written as $A' - 2c_d'' - hX_f\overline{Q}_f/X_d < 0$. The coefficient of on the right side of (24) is thus negative. Only three possible combinations of I_d^*/Δ^* and I_f^*/Δ^* can arise in an equilibrium,

I_d^*/Δ^*	I_f^*/Δ^*
–	–
–	+
+	+

Proposition 3: Suppose the distribution of willingness to pay is uniform. Let $\overline{Q}_f$ be a strictly binding quota in the second stage of a sequential game, with $X_f^{**} < X_d^{**}$. Then the comparative statics of a Nash equilibrium must follow one of three possible patterns:

	$dX_d^{**}/d\overline{Q}_f$	$dX_f^{**}/d\overline{Q}_f$
I	+	+
II	+	–
III	–	–

Proposition 3 indicates that a tighter quota can lead both to uniform quality upgrading (III) and uniform downgrading (I). Domestic upgrading can occur only when accompanied by foreign upgrading. Similarly, foreign downgrading can occur only when accompanied by domestic downgrading. These results are strictly analogous to Proposition 2 in the simultaneous choice model.

Further light can be shed on the comparative static results by substituting the expression

$$A' = [c_d' - c_d / X_d] / X_d \tag{25}$$

into (23ii) and rewriting (22) as

$$dX_d^{**} / d\bar{Q}_f = -2h[h\bar{Q}_f - X_f c_f''] / \Delta^* X_d \tag{26i}$$

$$dX_f^{**} / d\bar{Q}_f = -h\{-(c_d - c_d / X_d)(2 - X_f / X_d) / X_d + 4c_d(1 - X_f / X_d) + 2hX_f\bar{Q}_f / X_d^2\} / \Delta^*. \tag{26ii}$$

As in the simultaneous model, restrict attention to equilibria stable to a tatonment process. Requiring the first stage Nash equilibrium in qualities (X_d^{**}, X_f^{**}) to be stable in a neighborhood of the equilibrium imposes the condition $\Delta^* > 0$.[8] Since c_d'' is positive and $X_f < X_d$, (26ii) implies that a condition sufficient for a tighter quota to lead to foreign upgrading is $c_d' < c_d / X_d$. Let e_x^d denote the elasticity of the domestic firm's cost with respect to quality X_d,

$$e_x^d \equiv (dc_d / dX_d)(X_d / c_d). \tag{27}$$

Then

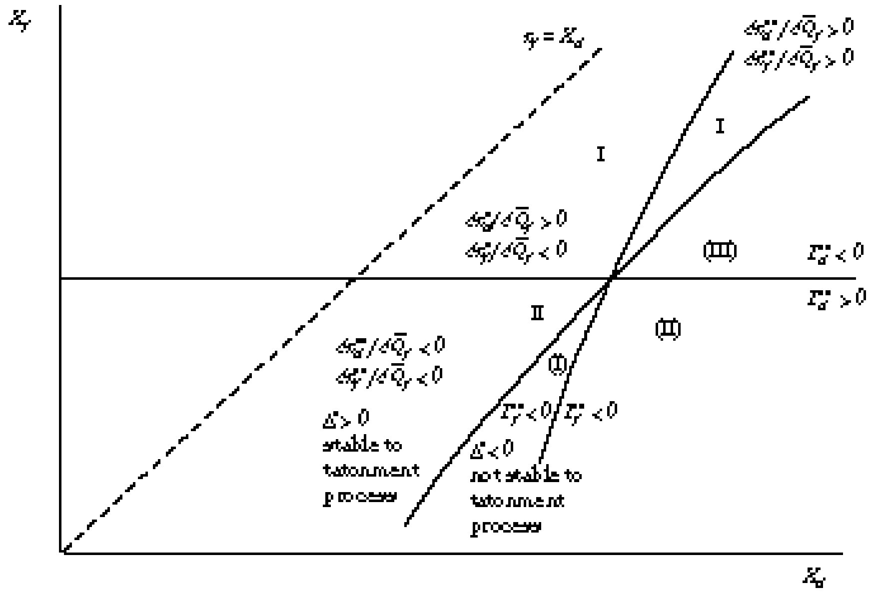
$$dc_d / dX_d - c_d / X_d = c_d(e_x^d - 1) / X_d. \tag{28}$$

A sufficient condition for a tighter quota to lead to foreign upgrading is that the elasticity of the domestic firm's cost with respect to product quality is less than one.

The magnitude of e_x^d can only be determined by specifying a cost function $c_d(X_d)$. It is plausible that e_x^d would increase with X_d, and be greater than 1 for relatively large values of X_d. This assumption is adopted in Figure 2, where relative locations of the three equilibria in (X_d, X_f) space are sketched. Boundaries between regions of stable and unstable equilibria, positive and negative values of I_d^*, and positive and negative values of I_f^* all

8. In each round of the tatonnment process, each firm chooses its optimal quality given its rival's quality in the preceding round and taking into account the effect of quality choice on the second stage equilibrium. The requirement that Δ^* be positive (rather than negative as in the previous section) reflects the even number of variables (X_d, X_f) rather than the odd number in the previous section (Q_d, X_d, X_f).

Figure 2
Comparative Statics with Sequential Choice



intersect at the equilibrium point, a consequence of (24). By (26i), the $I_d^* = 0$ boundary is horizontal. Assuming that c_f'' is an increasing function of X_f, I_d^* is negative above this line and positive below it.[9]

The most striking characteristic of the comparative statics in Figure 2 is their remarkable similarity to the simultaneous model in Figure 1. Regions I, II, and III of the sequential model imply responses that correspond exactly to regions I, II, and III in the simultaneous model. Indeed, the dX_d/dQ_f boundary is identical in both models. While the three stable regions of the simultaneous model loosely correspond to equilibria of high, intermediate, and low foreign quality, the three stable regions of the sequential model correspond loosely to the average quality of the two firms.[10]

Similarity of results across models is surprising in light of the lesson from game theory that the temporal structure is often critically important. Of

9. The $\Delta^* = 0$ and $I_f^* = 0$ loci do not necessarily have positive slopes everywhere as in Figure 2.
10. This classification should not be taken too literally, since casual inspection of Figure 2 indicates that the regions are not differentiated by average quality alone.

course, the demand structure is the same in both models, which leads to a strict relationship between quantities and the marginal value of quality. In the sequential model, however, the benefit to the firm of enhancing product quality includes its strategic effect on the second stage equilibrium. The direct link between a firm's profit maximizing quality and quantity is relaxed.

When the quantity produced by the foreign firm is restricted by the quota rather than by profit motives, domestic quality has no strategic impact on the second stage. The domestic firm finds it optimal to equate marginal cost of quality to marginal benefit for its marginal customer. As in the simultaneous model, there is a direct link between domestic quality and characteristics of marginal consumers. For this reason, the boundary between regions of positive and negative values of dX_d^*/dQ_f are the same in both models.

Interest in evaluating both simultaneous and sequential models stems from recognizing that quality can be analyzed in different ways. The correspondence between equilibria across models provides the tantalizing suggestion that the interpretation of quality is not critical when studying quotas. This is a potentially important result in the analysis of the costs and effects of quotas.

VI. Welfare Analysis

A brief analysis of the impact of a quota on domestic welfare follows. Let W denote welfare, defined as the sum of domestic producer surplus and consumer surplus,

$$W = \int_{H(Q_d+Q_f)}^{H(Q_f)} [\theta X_f - P_f] f(\theta) d\theta + \int_{H(Q_d)}^{\bar{\theta}} [\theta X_d - c_d] f(\theta) d\theta . \qquad (29)$$

The effect of a quota change on welfare may be derived by differentiating (29) with respect to Q_f. Let θ_f^a and θ_d^a denote the average willingness to pay for quality among those who purchase the foreign and domestic product,

$$\theta_f^a \equiv \int_{H(Q_d+Q_f)}^{H(Q_d)} \theta f(\theta) d\theta \Big/ \int_{H(Q_d+Q_f)}^{H(Q_d)} f(\theta) d\theta \qquad (30i)$$

$$\theta_d^a \equiv \int_{H(Q_d)}^{\bar{\theta}} \theta f(\theta) d\theta \Big/ \int_{H(Q_d)}^{\bar{\theta}} f(\theta) d\theta . \qquad (30ii)$$

It follows that[11]

$$dW / d\bar{Q}_f = (P_d - c_d)(dQ_d^* / d\bar{Q}_f) - X_f \bar{Q}_f H'(Q_d + Q_f)(dQ_d^* / d\bar{Q}_f + 1)$$
$$+ ({}_d^a - c_d')Q_d(dX_d^* / d\bar{Q}_f) + [{}_f^a - H(Q_f + Q_d)]Q_f(dX_f^* / d\bar{Q}_f). \quad (31)$$

The welfare effect is the sum of

- (i) the effect of the change in domestic quantity on producer and consumer surplus
- (ii) the effect of the change in foreign price on consumer surplus
- (iii) the effect of the change in domestic quality on producer and consumer surplus
- (iv) the effect of the change in foreign quality on consumer surplus.

Profit maximization by the domestic firm requires that $c_d = H(Q_d)$ and $dQ_d / dQ_f = [c_d / H'(Q_d)](dX_d / dQ_f)$. With simultaneous choice, profit maximization by the foreign firm requires that $dQ_d^* / d\bar{Q}_f = [c_f / H'(Q_d + Q_f)]$ $(dX_f / dQ_f) - 1$. Hence, (31) can be rewritten as

$$dW / d\bar{Q}_f = \{[\theta_d^a - H(Q_d)]Q_d + (P_d - c_d)c_d'' / H'(Q_d)\} dX_d^* / dQ_f$$
$$+ \{\theta_f^a - H(Q_d + Q_f) - X_f c_f''\} Q_f \, dX_f^* / d\bar{Q}_f. \quad (32)$$

In the special case of uniform willingness to pay, (31) reduces to

$$dW/dQ_f = (Q_d/2)(hQ_d - 2X_d c_d) dX_d / dQ_f + (Q_f/2)(hQ_f - 2X_f c_f)\, dX_f / dQ_f. \quad (33)$$

Second order conditions require only that the foreign profit function be locally concave in X_f. If the quota is in the neighborhood of the unconstrainted equilibrium, the foreign profit function would be locally concave in both X_f and Q_f. The term multiplying $dX_f^* / d\bar{Q}_f$ would then be negative, and the welfare effect of a quota change would be summarized

	$dW / d\bar{Q}_f$
I	–
II	?
III	+

11. Equation (31) applies with both simultaneous and sequential choice. Derivatives of equilibrium choices would be denoted with double asterisks under sequential choice.

Because of the special assumption of uniform willingness, it is hazardous to draw general conclusions. This analysis is, however, at least suggestive. As indicated by (32), an increase in domestic quality involves two effects. First, there is a welfare enhancing effect proportional to the product of domestic output and the difference between average and marginal willingness to pay. Second, there is a welfare reducing effect proportional to the product of the lost domestic output times the difference between price and marginal cost.

Increased foreign quality has two effects. First, there is a welfare enhancing effect proportional to the product of foreign output and the difference between average and marginal willingness to pay. Second, there is a welfare reducing effect that is proportional to the product of output and the price change.

Which welfare effect dominates depends on the relationship between marginal and average willingness to pay for quality. In the special case of uniform distribution, there is an exact linear relationship between average willingness to pay, marginal willingness to pay, and quantity: $\theta_d^a - H(Q_d) = (1/2)Q_d$ and $Q_f^a - H(Q_d+Q_f) = (1/2)Q_f$. Each welfare enhancing quality effect is smaller than the welfare reducing quantity effect or the foreign price effect when second order necessary conditions for concavity of the foreign profit function are satisfied. With uniform distribution, a quota reduces welfare.

It is important to note, however, that there are any number of nonuniform distributions which would lead to identical equilibrium choices but opposite welfare results. Simply perturb the marginal willingness function $H(Q)$ in neighborhoods away from the equilibrium quantities Q_d^* and Q_f^* to make the average willingness to pay of each firm sufficiently larger than the marginal willingness.

Despite this potential ambiguity, (32) suggests circumstances under which an induced quality change is likely to have greater impact on welfare than the corresponding quantity change. The quality effect is likely to outweigh the quantity effect only when the difference between average and marginal willingness is large. When customers are fairly similar or differences among customers are spread evenly, the quantity effect can be expected to dominate.[12]

VII. Conclusion

A quota on imports can have unwelcomed effects on the quality of both domestic output and imports. The purpose of this paper is to examine the range of effects in an international duopoly with the home and foreign firms competing in both qualities and quantities.

Using the demand framework of Mussa and Rosen [1979], the possible effects of a change in a binding quota on the qualities and quantities of both domestic and foreign goods are determined. Quality and quantity are chosen simultaneously in one model, with quality interpreted as options or workmanship. In a sequential model, quality is interpreted as a design characteristic selected prior to quantity. Simultaneous and sequential models of product quality are expected to lead different strategic outcomes. This paper suggests, however, that there is little analytical difference between the two concepts of product quality when studying quotas.

Three sets of responses to a change in a binding quota are possible, and a number of potential adjustments are eliminated. A quota may cause import quality downgrading, a conclusion not found in most theoretical results. Foreign downgrading is more likely in a market initially characterized by low levels of high quality import, and must be accompanied by domestic downgrading. Changes in domestic quality and quantity must move in opposite directions. A quota which increases domestic output would also lower domestic quality.

While the implications of a quota for domestic welfare are unclear without specifying the distribution of tastes, the effects of induced output changes are likely to dominate quality effects when customers are similar or uniformly diverse. Quality effects, on the other hand, are more likely to dominate when marginal customers differ substantially from average customers. Welfare consequences of quotas might be expected to vary substantially across markets.

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12. When quality and quantity are chosen sequentially, it remains true that $c_d' = H(Q_d)$, and consequently the exact relation between dQ_d^{**}/dQ_f and dX_d^{**}/dQ_f is similar to (31). However, the lack of an exact relation between dX_f^{**}/dQ_f and dQ_d^{**}/dQ_f +1, as well as the lack of similarity between terms in the second order conditions and those in the difference between average and marginal quality, prevent a straightforward conclusion about which effects dominate.

Appendix A:
Derivatives of Profit Functions

Let $c_i' \equiv dc_i(\cdot)/dX_i$ and $c_i'' \equiv d^2c_i(\cdot)/dX_i^2, \quad i = d, f.$

$$\partial\pi^d/\partial Q_d = X_f H(Q_d+Q_f)+(X_d-X_f)H(Q_d)-c_d+Q_d[X_f H'(Q_d+Q_f)+(X_d-X_f)H'(Q_d)] \quad \text{(A1)}$$

$$\partial\pi^d/\partial X_d = [H(Q_d)-c_d']Q_d \quad \text{(A2)}$$

$$\partial\pi^d/\partial Q_f = X_f Q_d H'(Q_d+Q_f) \quad \text{(A3)}$$

$$\partial\pi^d/\partial X_f = [H(Q_d+Q_f)-H(Q_d)]Q_d \quad \text{(A4)}$$

$$\partial^2\pi^d/\partial Q_d^2 = 2[X_f H'(Q_d+Q_f)+(X_d-X_f)H'(Q_d)]+Q_d[X_f H''(Q_d+Q_f)+(X_d-X_f)H''(Q_d)] \quad \text{(A5)}$$

$$\partial^2\pi^d/\partial Q_d\partial X_d = H(Q_d)-c_d'+Q_d H'(Q_d) \quad \text{(A6)}$$

$$\partial^2\pi^d/\partial Q_d\partial Q_f = X_f[H'(Q_d+Q_f)+Q_d H''(Q_d+Q_f)] \quad \text{(A7)}$$

$$\partial^2\pi^d/\partial Q_d\partial X_f = H(Q_d+Q_f)-H(Q_d)+Q_d[H'(Q_d+Q_f)-H'(Q_d)] \quad \text{(A8)}$$

$$\partial^2\pi^d/\partial X_d^2 = -c_d'' Q_d \quad \text{(A9)}$$

$$\partial^2\pi^d/\partial X_d\partial Q_f = 0 \quad \text{(A10)}$$

$$\partial^2\pi^d/\partial X_d\partial X_f = 0 \quad \text{(A11)}$$

$$\partial^2\pi^d/\partial Q_f^2 = Q_d X_f H''(Q_d+Q_f) \quad \text{(A12)}$$

$$\partial^2\pi^d/\partial Q_f\partial X_f = Q_d H'(Q_d+Q_f) \quad \text{(A13)}$$

$$\partial^2\pi^d/\partial X_f^2 = 0 \quad \text{(A14)}$$

$$\partial\pi^f/\partial Q_d = X_f Q_f H'(Q_d+Q_f) \quad \text{(A15)}$$

$$\partial\pi^f/\partial X_d = 0 \quad \text{(A16)}$$

$$\partial\pi^f/Q_f = X_f[H(Q_d+Q_f)+Q_f H'(Q_d+Q_f)]-c_f \quad \text{(A17)}$$

$$\partial\pi^f/\partial X_f = [H(Q_d+Q_f)-c_f']Q_f \quad \text{(A18)}$$

$$\partial^2\pi^f/\partial Q_d^2 = X_f Q_f H''(Q_d+Q_f) \quad \text{(A19)}$$

$$\partial^2\pi^f/\partial Q_d\partial X_d = 0 \quad \text{(A20)}$$

$$\partial^2\pi^f/\partial Q_d\partial Q_f = X_f[H'(Q_d+Q_f)+Q_f H''(Q_d+Q_f)] \quad \text{(A21)}$$

$$\partial^2 \pi^f / \partial Q_d \partial X_f = Q_f H'(Q_d + Q_f) \quad \text{(A22)}$$

$$\partial^2 \pi^f / \partial X_d^2 = 0 \quad \text{(A23)}$$

$$\partial^2 \pi^f / \partial X_d \partial Q_f = 0 \quad \text{(A24)}$$

$$\partial^2 \pi^f / \partial X_d \partial X_f = 0 \quad \text{(A25)}$$

$$\partial^2 \pi^f / \partial Q_f^2 = X_f [2H'(Q_d + Q_f) + Q_f H''(Q_d + Q_f)] \quad \text{(A26)}$$

$$\partial^2 \pi^f / \partial Q_f \partial X_f = H(Q_d + Q_f) - c_f' + Q_f H'(Q_d + Q_f)] \quad \text{(A27)}$$

$$\partial^2 \pi^f / \partial X_f^2 = -c_f'' Q_f \quad \text{(A28)}$$

Appendix B:
Requirements for Stability of a Tatonment Process

Derivatives of the profit functions are denoted by subscripts, where:

1 denotes differentiation w.r.t. Q_d
2 denotes differentiation w.r.t. X_d
3 denotes differentiation w.r.t. Q_f
4 denotes differentiation w.r.t. X_f .

Consider a tatonment process in which each firm in each round makes choices to maximize profit given its rival's choice in the previous round. Let superscripts denote the round and presume that the foreign firm sells as much as the quota allows. The dynamic process is captured by

$$Q_d^t = g(X_f^{t-1}) \quad \text{(B1i)}$$

$$X_d^t = h(X_f^{t-1}) \quad \text{(B1ii)}$$

$$X_f^t = s(Q_d^{t-1}, X_d^{t-1}). \quad \text{(B1iii)}$$

The choice of Q_d^t and X_d^t are not independent, since both must satisfy the domestic firm's first order conditions in (6). Treat X_d^t as a function of Q_d^t and X_f^{t-1}. Since Q_f^t equals $\bar{Q}_f$ for all t, X_d^t does not depend on Q_f^{t-1}. Specifically, let $\tilde{X}_d$ (Q_d^t, X_f^{t-1}) be defined by

$$\pi_2^d(Q_d^t, \tilde{X}_d(X_d^t, X_f^{t-1}), \bar{Q}_f, X_f^{t-1}) = 0.$$

The function $\tilde{X}_d(\cdot)$ has derivatives

$$\partial \tilde{X}_d / \partial Q_d^t - \pi_{21}^d / \pi_{22}^d \tag{B3i}$$

$$\partial \tilde{X}_d / \partial X_f^{t-1} = -\pi_{24}^d / \pi_{22}^d. \tag{B3ii}$$

The system of three variables in (B1) may be replaced by the system of two variables

$$Q_d^t = \tilde{Q}_d(X_f^{t-1}) \tag{B4i}$$

$$X_f^t = \tilde{X}_f(Q_d^{t-1}). \tag{B4ii}$$

Let (Q_d^e, X_f^e) denote an equilibrium of (B4). A linear expansion of this system in the neighborhood of the equilibrium point implies

$$\begin{bmatrix} Q_d^{t+1} - Q_d^e \\ X_f^{t+1} - X_f^e \end{bmatrix} \approx \begin{bmatrix} 0 & \tilde{Q}_d'(X_f^e) \\ \tilde{X}_f'(Q_d^e) & 0 \end{bmatrix} \begin{bmatrix} Q_d^t - Q_d^e \\ X_f^t - X_f^e \end{bmatrix} \tag{B5}$$

A necessary condition for stability of (B4) in the neighborhood of the equilibrium is that the determinant of the matrix in (B5) be less than one in absolute value. See Chapter 8 of Gandolfo [1980] for a discussion of stability conditions. Stability requires that

$$\left|\tilde{X}_f'(Q_d^e)\right| \left|\tilde{Q}_d'(X_f^e)\right| < 1.$$

The functions $Q_d(\cdot)$ and $X_f(\cdot)$ satisfy the first order necessary conditions (6i) and (6iii):

$$\pi_1^d\{\tilde{Q}_d(X_f), \tilde{X}_d[\tilde{Q}_d(X_f), X_f], \bar{Q}_f, X_f\} = 0 \tag{B7i}$$

$$\pi_4^f\{Q_d, \tilde{X}_d[Q_d, \tilde{X}_f(Q_d)], \bar{Q}_f, \tilde{X}_f(Q_d)\} = 0. \tag{B7ii}$$

The derivatives Q_d and X_f may be calculated by differentiating (B7) and substituting (B3),

$$\tilde{Q}_d'(X_f) = -\frac{\pi_{22}^d \pi_{14}^d - \pi_{12}^d \pi_{24}^d}{\pi_{11}^d \pi_{22}^d - (\pi_{12}^d)^2} \tag{B8i}$$

$$\tilde{X}'_f(Q_d) = \frac{{}^{f}_{41}\,{}^{d}_{22} - {}^{f}_{42}\,{}^{d}_{21}}{{}^{f}_{44}}\;{}^{d}_{22}. \qquad \text{(B8ii)}$$

By equations (A11) and (A25) of Appendix A, $\pi^d_{24} = \pi^f_{42} = 0$. Hence, (B8) and (B6) imply that for stability

$$\left| \frac{{}^{d}_{14}\,{}^{f}_{41}\,{}^{d}_{22}}{[\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2]\,{}^{f}_{44}} \right| < 1,$$

which may be rewritten as

$$\left| {}^{d}_{14}\,{}^{f}_{41}\,{}^{d}_{22} \right| < \left| [\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2]\,{}^{f}_{44} \right|.$$

By second order conditions (7i) and (7iv), the product inside the absolute value signs in (B10) must be negative at the equilibrium. Similarly, second order condition (7iii) and the fact from (A22) that $\pi^f_{41} = Q_f\,H'(Q_d+Q_f) < 0$ imply that $\pi^f_{41}\pi^d_{22} < 0$. Hence, (B10) requires

$$\left| {}^{d}_{14} \right| {}^{f}_{41}\,{}^{d}_{22} < -[\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2]\,{}^{f}_{44}, \qquad \text{(B11)}$$

which means the stability condition can be written

$$\left| {}^{d}_{14} \right| {}^{f}_{41}\,{}^{d}_{22} + {}^{f}_{44}\,[\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2] < 0. \qquad \text{(B12)}$$

From (8), note that

$$\Delta = {}^{d}_{14}\,{}^{f}_{41}\,{}^{d}_{22} + {}^{f}_{44}\,[\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2]. \qquad \text{(B13)}$$

The first product has the same sign as π^d_{14}, and the second product is negative. If $\pi^d_{14} < 0$, both products on the right of (B13) are negative, and Δ must be negative. If $\pi^d_{14} > 0$, stability condition (B12) would require

$${}^{d}_{14}\,{}^{f}_{41}\,{}^{d}_{22} + {}^{f}_{44}\,[\,{}^{d}_{11}\,{}^{d}_{22} - ({}^{d}_{12})^2] < 0, \qquad \text{(B14)}$$

which would imply $\Delta < 0$.

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