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Government Spending on Public Goods, International Factor Mobility and Trade

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Abstract

This paper examines the trade patterns and the direction of factor movements generated by an international difference in government spending on public goods. The model consists of one final good, one intermediate good, one public good, and two primary factors of production; capital and labour. The public good is supplied by the government, and the budget constraint of the government is satisfied through an adjustment of the income tax rate. Two cases have been examined in this paper. The first case is that the final good and the intermediate good are internationally traded without international factor movements. The second case is that the final good is internationally traded and one of the primary factors of production is internationally mobile.

I. Introduction

Public goods are collective consumption goods which enter into utility functions. Public inputs, on the other hand, are collective intermediate goods which enter into production functions.

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In the context of a two-good, one-primary factor model, Manning and McMillan [1979] have shown that the comparative advantage of an economy depends on the supply of a pure input. Abe [1990] has shown that the differences in the supply of a pure public input alone can determine the pattern of trade.[1] Clarida and Findlay [1991] have considered the role of public sector in the context of a specific factor model. Anwar [1992] has examined the role of the supply of a public input in determining the pattern of trade between underemployed economies.[2] Ishizawa [1988], Khan [1982], Tawada and Okamoto [1983] and Tawada and Abe [1984] have re-examined HOS Theorem in the presence of a public input. However, the relationship between the supply of public goods and the pattern of international trade has not received much attention in the available literature.[3] In addition, none of the available studies have examined the relationship between government spending (either on public goods or on public inputs) and the pattern of international factor mobility.

The purpose of the present study is to examine the relationship between government spending on a public good and the pattern of international trade and factor mobility. The paper develops a two-country, three-good,

1. Abe [1990] has incorporated a pure public input into a two-country, two-good, two-factor general equilibrium model. Abe has shown that when two countries have identical homothetic preferences, production technology, factor supplies, and the factor intensity of public sector is the same as that of private sectors, then the country that produces more public input exports (imports) the output of the industry which derives more (less) benefits from its supply.
2. Anwar [1992] has shown that the differences in the supply of a pure public input can influence the pattern of trade between underemployed countries even if both industries derive equal benefits from its supply.
3. Clarida and Findlay [1991] have utilized a specification of the Ricardo-Viner (specific factor) model where the private sector produces two goods and the government provides a public input and a public good. The production functions utilized by Clarida and Findlay are Cobb-Douglas and the share of labor in the production of private goods is identical. It is shown that comparative advantage cannot be determined simply by specifying, factor endowments, technology and preferences of each economy's households alone. Clarida and Findlay have not considered the relationship between government activity and international factor mobility. On the other hand, the present study utilizes a specification of the Heckscher-Ohlin-Samuelson (HOS) model. A very general specification of the production functions is used to consider the impact of government activity on the pattern of trade and factor mobility.

and two-factor general equilibrium model where all resources are fully utilized. The private sector produces two goods: an intermediate good which is produced by means of capital and labor; and a final good which is produced by means of capital, labor and the intermediate good. The public sector produces a non-traded public good by means of capital and labor. The present study shows that the differences in the supply of a public good alone can also determine the pattern of international trade and factor mobility.

The paper is organized as follows. A simple general equilibrium model is developed in the next section. The model is used to study the relationship between the supply of a public good and the pattern of trade in goods and factors in section three, whereas the last section contains a summary and concluding remarks.

II. A Simple General Equilibrium Model

In order to determine the relationship between the supply of a public good and the pattern of trade in goods and factors, the autarky equilibrium of a representative economy is characterized in this section. In other words, there are two countries, but both are assumed to be identical in every respect in the initial equilibrium. Consequently there is no trade in the initial equilibrium and the autarky equilibrium of a representative country can be considered.

The next section assumes that the countries are identical in every respect except for differences in the supply of a public good. The impact of differences in the supply of the public good on autarky relative prices is examined in the next section by means of a comparative statics exercise. It is shown that differences in the supply of the public good alone can lead to differences in autarky relative prices. The resulting pattern of international trade and factor mobility can therefore be attributed to differences in the supply of the public good.

Consider a self sufficient economy that produces one final good, one intermediate good, and one public good (X, Y, and G respectively). The public good and the intermediate goods are produced by means of capital and labor, whereas the final good is produced by means of capital, labor and the intermediate good.[4] The supply of labor and capital in the economy is fixed. The public good is provided to the households free of charge by the government. The cost of public production is financed by means of a proportional income tax. The production technologies are linearly homogeneous. The final good X is the numéraire and all markets are competitive. The production functions are given below:

$$X = f(K_x, L_x, Y_x)$$

$$Y = h(K_y, L_y)$$

$$G = g(K_g, L_g)$$

K_i, and L_i are respectively capital and labor used in the production of good i ($i = x, y, g$) whereas Y_x is the amount of intermediate good used in the production of the final good. $Xc^x(r, w, p)$, $Yc^y(r, w)$ and $Gc^g(r, w)$ respectively are the cost functions for X, Y and G. w and r are respectively the wage rate and the rate of return on capital whereas p is the price of intermediate good.

By using the properties of cost functions, the factor market clearing conditions can be written as follows:

$$L_o = Xc^x_w(\cdot) + Yc^y_w(\cdot) + Gc^g w(\cdot) \tag{1}$$

$$K_o = Xc^x_r(\cdot) + Yc^y_r(\cdot) + Gc^g r(\cdot) \tag{2}$$

$$Y = Xc^x_p(\cdot) \tag{3}$$

where $Xc^x_w(\cdot)$ and $Xc^x_r(\cdot)$ are labor and capital used in the production of X; $Yc^y_w(\cdot)$ and $Yc^y_r(\cdot)$ are labor and capital used in the production of Y; $Gc^g_w(\cdot)$ and $Gc^g_r(\cdot)$ are labor and capital used in the production of G; $Xc_p(\cdot)$ is the amount of intermediate good used in the production of the final good; K_o is supply of capital; and L_o is supply of labor.

Due to unrestricted factor mobility within the country, the wage rate and the rate of return on capital is identical in all industries. Equation (3) indicates that the intermediate good is not available for direct consumption. The zero profit conditions for industry X and Y are the following:

4. The set-up of the paper is similar to Kemp and Ohyama [1978] and Anwar [1993]. These studies however consider entirely different issues.

$$c^x(r, w, p) = 1 \qquad (4)$$

$$c^y(r, w) = p \qquad (5)$$

For a given supply of the public good (G), equations (1) to (5) determine the equilibrium w, r, p, X and Y. The utility function of the representative household is the following:

$$U = u(X, G)$$

$u(\cdot)$ is strictly concave and the households take the supply of public good as given. The budget constraint of the aggregate household is the following, where t is the proportional income tax rate:

$$X = (1-t)[wL_o + rK_o]$$

The budget constraint of the government is the following:

$$G\, c^g(\cdot) = t[wL_o + rK_o] \qquad (6)$$

Left hand side of the above equation is the total cost of the public good; whereas the right hand side is government tax receipts. Equation (6) shows that the cost of public production is financed by the proportional income tax. The budget constraints of the public and the private sectors can be combined into the following equation:

$$X = [wL_o + rK_o] - G$$

Accordingly, indirect utility function of the aggregate household can be written as

$$U = u(wL_o + rK_o - G, G)$$

The above indirect utility function can be used to derive the optimal supply of public good in the representative economy.[5] The full equilibrium of the closed economy under consideration is given by equations (1) to (6).

5. Optimal supply of public good can be determined by maximizing the indirect utility with respect to G. The first-order condition which determines the optimal supply of the public good is the following; $[U_g(\cdot)/U_x(\cdot)] = L_o[\{(K_x+K_y)/(L_x+L_y)\} - (K_o/L_o)][\partial r/\partial G]$.

These are six equations in six endogenous variables: w, r, p, X, Y, and t. This completes the description of a representative closed economy. The relationship between the supply of a public good and relative prices is examined in the next section.

III. Pattern of Trade and Factor Mobility

For a given supply of the public good, equations (1) to (5) describe an autarky equilibrium of the private sector of a representative country. Consider two such countries. If the two countries are identical in every respect, then there is no basis for international trade or factor mobility, since the autarky prices in the two countries will be identical. However, differences in the supply of the public good can lead to differences in the autarky relative prices.

The supply of the public good influences the autarky prices (p, w, and r) through its impact on the availability of primary factors to the private sector. In the following three cases are considered: (i) the final and the intermediate goods are traded, but there is no factor mobility across international boundaries; (ii) only labor is internationally mobile and either the final or the intermediate good is traded; and (iii) only capital is internationally mobile and either the final or the intermediate good is traded.

The relationship between the supply of a public good and the pattern of trade when capital and labor are internationally immobile can be examined by means of the following derivative, which describes the impact on the autarky price ratio (p) of a country when its supply of the public good increases by a small amount[7]

$$\partial p/\partial G = A_1[\{(K_x+K_y)/(L_x+L_y)\}-(K_g/L_g)][(K_y/L_y)-(K_x/L_x)]/B \quad (7)$$

$$A_1 = c_w^x(\cdot)c_w^g(\cdot)c_w^y(\cdot)[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)] > 0$$

$$B = Xc_{pp}^x(\cdot)[c_w^x(\cdot)c_r^y(\cdot)-c_r^x(\cdot)c_w^y(\cdot)]^2 +$$

6. From equations (4) and (5), factor prices will be equalized if the final good and the intermediate good are under free trade. Consequently, if the final good and the intermediate good are traded then there will be no international factor movement.

7. The properties of the cost functions are used in the derivation of equation (7).

$$[c_r^x(\cdot)+c_p^x(\cdot)c_r^y(\cdot)]^2[Xc_{ww}^x(\cdot)+Yc_{ww}^y(\cdot)+Gc_{ww}^g(\cdot)]+$$

$$[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)]^2[Xc_{rr}^x(\cdot)+Yc_{rr}^y(\cdot)+Gc_{rr}^g(\cdot)]-$$

$$[c_r^x(\cdot)+c_p^x(\cdot)c_r^y(\cdot)]\,[c_w^x(\cdot)+c_p^y(\cdot)c_w^y(\cdot)]$$

$$[X\{c_{rw}^x(\cdot)+c_{wr}^x(\cdot)\}+Y\{c_{rw}^y(\cdot)+c_{wr}^y(\cdot)\}+G\{c_{rw}^g(\cdot)+c_{wr}^g(\cdot)\}]+$$

$$X[c_w^x(\cdot)c_r^y(\cdot)-c_r^x(\cdot)c_w^y(\cdot)]\,\{[c_{rp}^x(\cdot)+c_{pr}^x(\cdot)]\,[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)]-$$

$$[c_{wp}^x(\cdot)+c_{pw}^x(\cdot)]\,[c_r^x(\cdot)+c_p^x(\cdot)c_r^x(\cdot)]$$

B is negative as long as the equilibrium is stable. The stability condition is derived in the appendix. Equation (7) is derived by using equations (1) to (5). The sign of the above derivative depends on the sign of: (i) $[\{(K_x+K_y)/(L_x+L_y)\}-(K_g/L_g)]$, and (ii) $[(K_y/L_y)-(K_x/L_x)]$. If $\{(K_x+K_y)/(L_x+L_y)\}$ is equal to (K_g/L_g), then the supply of public good cannot influence the autarky price ratio. Consequently, the differences in the supply of a public good have no influence on the pattern of international trade. Because, an increase in the supply of the public good leads to a decrease in the production of both X and Y by the same proportion, hence the autarky relative price ratio (p) is unchanged.

On the other hand, if both (i) and (ii) are either positive or negative, then (7) is negative which implies that the country which produces more public good will export the intermediate good and import the final good. On the other hand if the signs of (i) and (ii) are not identical then the country which produces more public good imports the intermediate good and exports the final good. This follows from the fact that the autarky relative price of the intermediate good in a country which produces more public good is higher.

The following propositions can be derived from (7):

Proposition 1: *When two countries have (i) identical technology, factor supplies, the primary factors of production are internationally immobile, (ii) the production of the public good is less (more) capital intensive as compared to the overall capital intensity of the private sector, and (iii) the production of intermediate good is more (less) capital intensive as compared to the final* *good; then the country which produces more public good exports the intermediate good and imports the final good.*

Trade pattern shown in proposition one can be explained by the indirect effect defined in Abe [1990]. If the production of the public good is less capital intensive as compared to the overall capital intensity of the private sector, an increase in the supply of the public good will create more demand for labor compared to that for capital. Thus, the wage rate will be higher and the rental will be lower. From the zero profit conditions, the relative price of the intermediate good is more capital intensive as compared to the final good. This effect corresponds to the "indirect effect" in Abe [1990].[8]

Proposition 2: *When two countries have (i) identical technology, factor supplies, the primary factors of production are internationally immobile, (ii) the production of the public good is less (more) capital intensive as compared to the overall capital intensity of the private sector, and (iii) the production of intermediate good is less (more) capital intensive as compared to the final good then the country which produces more public good imports the intermediate good and exports the final good.*

In the above case, the primary factors of production are internationally immobile. Suppose that there are no restrictions on international labor mobility and the intermediate good is not traded.[9] In such a case, differences in the supply of the public good alone can lead to international trade and factor mobility. The following equation can be derived from equations (1) to (5):

$$\partial w/\partial G = A_2\,[\{(K_x+K_y)/(L_x+L_y)\} - (K_g/L_g)]/B \qquad (8)$$

$$B<0;\ A_2 = -c_w^g(\cdot)\,[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)]\,[c_r^x(\cdot)+c_p^x(\cdot)c_r^y(\cdot)] < 0$$

The following proposition follows from (8):

Proposition 3: *When two countries have (i) identical technology, factor supplies, capital is internationally immobile and the intermediate good is not*

8. I am extremely thankful to an anonymous referee for bringing this point to my attention.

9. The results of this paper can easily be extended to the case where the intermediate good is traded but the final good is non-traded.

traded, (ii) the production of the public good is less (more) capital intensive as compared to the overall capital intensity of the private sector, then the country which produces more public good exports (imports) the final good and experiences labor inflow (outflow).

Equation (8) shows that the wage rate is higher in the country which produces more public good, if the private sector as a whole is relatively more capital intensive as compared to the public sector. Consequently, labor migrates to the country which produces more public good.

Suppose that there are no restrictions on international capital mobility, labor is internationally immobile and the intermediate good is not traded. In such a case differences in the supply of the public good can also lead to trade in the final good and international capital mobility. The following equation can be derived from equations (1) to (5):

$$\partial r/\partial G = A_3\ [\{(K_x+K_y)/(L_x+L_y)\} - (K_g/L_g)]/B \tag{9}$$

$$B<0; A_3 = c_w^g(\cdot)\,[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)]\,[c_w^x(\cdot)+c_p^x(\cdot)c_w^y(\cdot)] > 0$$

The following proposition follows from (9):

Proposition 4: *When two countries have (i) identical technology, factor supplies, labor is internationally immobile and the intermediate good is not traded, (ii) the production of the public good is less (more) capital intensive as compared to the overall capital intensity of the private sector, then the country which produces more public good imports (exports) the final good and experiences capital outflow (inflow).*

Equation (9) shows that the rate of return on capital is lower in the country which produces more public good, if the private sector as a whole is relatively more capital intensive as compared to the public sector. Consequently, capital moves to the country which produces less public good.

In (one interpretation of) the Clarida and Findlay [1991] model, differences in provision of the public consumption good stem from differences in preferences of the public. Thus, differences in preferences over public goods can provide a basis for international trade as they do in this paper. The results would be analytically similar if there were symmetries across countries over preferences for privately produced non-traded goods. Differences in preferences over traded goods will affect imports and exports, but not the pattern of production, while differences in preferences over non-traded goods (be they publicly or privately provided) can affect traded good production through the factor intensity effects shown in this paper.[10]

The empirical relevance of the results clearly depends on the extent to which the factor intensity of publicly provided goods differ from the average factor intensity of the traded good sector. The existing literature however does not provide any clear cut answers in this regard. Most studies which consider issues involving international trade do not explicitly include the public sector. On the other hand, most studies which explicitly consider the provision of public goods or the public inputs assume that the economy is closed. Abe [1990], Okamoto [1985], Tawada [1983] and Tawda [1984] among others have utilized the assumption that the factor intensity of publicly provided goods differ from the average factor intensity of the traded good sector however they have not discussed the empirical relevance of this assumption.

IV. Concluding Remarks

The present study has attempted to examine the relationship between government spending on households and the pattern of international trade and factor mobility. Government spending on households is included into utility functions in terms of a public good. A two-country general equilibrium model is utilized where each country produces a final good by means of capital, labor, and an intermediate good. The intermediate good which is not available for direct consumption is produced by means of capital and labor. The paper shows that the differences in the supply of a public good alone can explain the pattern of international trade and factor mobility between otherwise similar economies.

Three cases are considered: (i) the primary factors are immobile across international boundaries, it is shown that the differences in the supply of the public good alone can result is international trade in the final good and the intermediate good; (ii) the intermediate good is not traded and capital is

10. I am extremely thankful to an another anonymous referee for this point.

internationally immobile, it is shown that the differences in the supply of the public good alone can result in international trade in the final good and international labor mobility; and (iii) the intermediate good is not traded and labor is not mobile across international boundaries, it is shown that the differences in the supply of the public good alone can result in international trade in the final good and international capital mobility.

Appendix

Equations (1) to (5) can also be used to derive the Routh-Hurwitz stability conditions. Chang [1981] and Okamoto [1985] have used a similar stability condition. The postulated dynamic adjustment process is described by means of the following equations, where the left hand side is the time derivative of the relevant variable:

$$dw/dt = a_w[Xc^x_w(\cdot) + Yc^y_w(\cdot) + Gc^g_w(\cdot) - L_o]$$

$$dr/dt = a_r[Xc^x_r(\cdot) + Yc^y_r(\cdot) + Gc^g_r(\cdot) - K_o]$$

$$dp/dt = a_p[Xc_p(\cdot) - Y]$$

$$dX/dt = a_x[1 - c^x(r, w, p)]$$

$$dY/dt = a_y[p - c^y\ (r, w)]$$

where the relevant speeds of adjustment (a_w, a_r, a_p, a_y, a_x) are assumed to be positive constants.

The economic meanings of the above equations are obvious, therefore the interpretation is not included. The relevant Jacobian matrix, denoted by J is the following:

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

where

$$a_{11} = [Xc^x_{ww}(\cdot) + Yc^y_{ww}(\cdot) + Gc^g{}_{ww}(\cdot)]$$

$$a_{12} = [Xc^x_{wr}(\cdot) + Yc^y_{wr}(\cdot) + Gc^g_{wr}(\cdot)]$$

$$a_{13} = Xc^x_{wp}(\cdot),\ a_{14} = c^x_w(\cdot),\ a_{15} = c^y_w(\cdot)$$

$$a_{21} = [Xc^x_{rw}(\cdot) + Yc^y_{rw}(\cdot) + Gc^g_{rw}(\cdot)]$$

$$a_{22} = [Xc^x_{rr}(\cdot) + Yc^y_{rr}(\cdot) + Gc^g_{rr}(\cdot)]$$

$$a_{23} = Xc^x_{rp}, \qquad a_{24} = c^x_r(\cdot), \qquad a_{25} = c^y_r(\cdot)$$

$$a_{31} = Xc^x_{wp}(\cdot), \qquad a_{32} = Xc^x_{rp}(\cdot), \qquad a_{33} = Xc^x_{pp}(\cdot)$$

$$a_{34} = c^x_p(\cdot), \qquad a_{35} = -1 \qquad a_{41} = -c^x_w(\cdot)$$

$$a_{42} = -c^x_r(\cdot), \qquad a_{43} = -c^x_p(\cdot), \qquad a_{44} = 0$$

$$a_{45} = 0, \qquad a_{51} = -c^y_w(\cdot), \qquad a_{52} = -c^y_r(\cdot)$$

$$a_{53} = 1, \qquad a_{54} = 0, \qquad a_{55} = 0$$

One of the Routh-Hurwitz stability condition requires that

$$(-1)^5 |J| > 0$$

where "| |" stands for the determinant.

The determinant of the above Jacobian matrix is except for a positive scalar, identical to B. Clearly, the determinant condition is satisfied if B is negative. Furthermore, another Routh-Hurwitz condition requires that trace of J be negative. Trace of J is equal to $a_{11} + a_{22} + a_{33} + a_{44} + a_{55}$. This condition is satisfied as long as the unit cost functions are concave in relevant factor prices.

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