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Government Spending and Cyclical Adjustment in a Small Open Economy

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Abstract

The paper studies the effects of expansionary fiscal policy in a general equilibrium model of a small open economy with a recursive time preference structure in which the intertemporal optimizing behavior of infinitely-lived agents endogenously determines employment, investment, and the current account. It is shown that the adjustment to the steady state may be cyclical and the long-run government spending multiplier may exceed one.

I. Introduction

The continuing process of international integration of capital markets has recently given rise to a growing literature studying its implications for small open economies. The issues analyzed range from the effects of supply and terms of trade shocks ⟨Sen and Turnovsky [1989], Mendoza [1991]⟩ to the implications of changes in various fiscal and tax policy instruments ⟨Obstfeld [1989], Sen and Turnovsky [1991], Frenkel and Razin [1992]⟩.

Motivated by questions raised in recent closed-economy macroeconomic

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literature,[1] this paper investigates the supply-side responses of capital and labor to changes in government spending and taxes. It, thus, differs from the existing open economy literature analyzing the effects of fiscal policy, which has largely ignored the dynamic interactions between capital and labor.[2] Turnovsky and Sen [1991] incorporates such interactions in a model with time-additive preferences. In a small open economy model such as theirs, the existence of a steady-state requires that the exogenous rate of time preference somehow be equal the parametric world rate of interest. One consequence of this equality requirement is that the steady state of the economy depends on initial conditions, making the effects of fiscal policy sensitive to these initial conditions. In addition, empirical studies of time-additive preferences have generated strong rejections, and led researchers to incorporate intertemporal dependence in tastes into their models.[3] This confirms the argument of Hicks [1965] that the independence of consumption levels between successive periods implied by time-additive preferences is counter-intuitive, and that one normally expects a strong complementarity between them. A simple solution to this problem is to endogenize the rate of time preference.

This paper adopts the recursive time preference structure proposed by Epstein and Hynes [1983] which is a more tractable extension of the Uzawa [1968] utility function.[4] It shows that government spending in a model that allows for intertemporal dependence in consumption and leisure between

1. See, for instance, Aschauer [1988], Barro [1989], and Baxter and King [1993].
2. Frenkel and Razin [1992] analyzes the effects of fiscal deficits comprehensively. However, this study mostly ignores the effects of government spending on output and capital accumulation. When such effects are considered, they are treated in a two-period setting. Another influential treatment of the issue – that of Obstfeld [1989] – introduces capital accumulation, but like Frenkel and Razin [1992] abstracts from the endogeneity of labor supply
3. See Obstfeld [1990] on the relevant literature.
4. Epstein [1987] and Obstfeld [1990] discuss the implications of the Uzawa-Epstein time preferences. Mendoza [1991], Devereux and Shi [1991] (which studies the effects of government spending while abstracting from the endogeneity of labor supply), Shi [1994] (which incorporates endogenous labor supply to discuss the effects of various tax instruments), and Karayalcin [1994] employ Uzawa-Epstein functions in open economy frameworks.

successive periods produces effects significantly different from models in which such dependence is ignored. The most important of these differences concern the adjustment path of the small open economy in response to an expansionary fiscal shock: While, for instance, in Obstfeld [1989], Turnovsky and Sen [1991] and Frenkel and Razin [1992] the economy adjusts monotonically after such a shock, when consumption and leisure are intertemporally dependent the transient path displays cyclical adjustment. This result is due to the two-way link between investment decisions of the firms and the saving and labor supply decisions of households established by the combination of endogenous labor supply and the intertemporal dependence of preferences here.[5] This link is ignored by models that impose intertemporal independence in preferences, and is reflected in the isolation of the price of equities and investment decisions of the firms, which form an independent dynamical subsystem, from the rest of the economy as in Turnovsky and Sen [1991].

Furthermore, unlike in models with intertemporally independent preferences, the present paper shows that the expansionary fiscal policy may have contractionary short-run effects, reducing labor supply and equity prices on impact and thereby giving rise to a period of depressed stock markets and reduced capital accumulation.

The model presented here also possesses a steady state that does not depend on initial conditions. This is in sharp contrast to models such as Turnovsky and Sen [1991] that use a fixed rate of time preference which has to be arbitrarily set equal to the exogenous world rate of interest to ensure the existence of long-run equilibrium.

Finally, three long-run results that emerge from the paper conform with the findings of closed-economy research ⟨Baxter and King [1993]⟩ and are supported by the empirical findings in small open economy setups ⟨Ahmed [1986]⟩. First, increases in government expenditures financed by non-distortionary taxes cause a negative wealth effect which induces a rise in labor supply. Second, government spending crowds out private consump-

5. Shi and Epstein [1993] also establish the possibility of cyclical adjustment in a model with recursive time preferences. However, their framework is based on habit formation and they assume exogenous labor supply.

tion. Third, fiscal expansions have strong positive effects on domestic output.

The rest of the paper is organized as follows. The next section sets up the model which establishes a small open economy general equilibrium framework where investment, employment and the current account are endogenously determined via the intertemporal optimizing behavior of infinitely-lived agents. The economy is assumed to produce a single traded good, used for both consumption and investment which is subject to adjustment costs. The effects of changes in government spending financed by no n-distortionary taxes are studied in section III, where the time path of the economy in response to fiscal shocks is obtained through simulation analysis. The last section provides some concluding remarks.

II. The Model

Consider a small open economy with a constant number (normalized, without loss of generality, to one) of identical households that possess perfect foresight. Suppose, in addition, that perfectly competitive firms in the economy produce a single traded good that can be used for consumption and investment. I now turn to a detailed analysis of household and firm behavior.

A. Households

Household labor supply, l, is endogenous. Two assets, an internationally traded bond and equities issued by domestic firms, are held in the portfolios of households. Since the assets are perfect substitutes, they earn the same rate of return: the exogenously given world interest rate r. The time preference structure is recursive and of the Epstein and Hynes [1983] type.

The representative household maximizes lifetime welfare Ω over consumption path C and labor path L, that is it maximizes

$$\Omega(C_0, L_0) = \int_0^{\infty} -\exp(-z_t) \exp(-rt)\,dt \tag{1}$$

subject to

$$\dot{z}_t = U(c, l, g) - r \tag{2}$$

$$\dot{a}_t = ra_t + wl - c - \tau \tag{3}$$

$$z_0 = 0 \tag{4}$$

where a, w, c, τ, g denote the nonhuman wealth, the wage rate, the consumption level, the lump-sum tax, and government spending. The felicity function $U(c, l, g)$ is assumed to have the following properties:

$$U(c, l, g) = u(c, l) + v(g),\ u_c > 0,\ u_l < 0,\ v_g > 0,$$
$$u_{cc} < 0,\ u_{ll} < 0,\ u_{cl} < 0,\ v_{gg} < 0,\ u_{cc}u_{ll} - u_{cl}^2 > 0. \tag{5}$$

Thus, households derive positive marginal utility from the consumption of the traded good and from government services; they derive positive marginal disutility from providing labor services. The felicity function is additive in (c, l) and g. The functions $u(c, l)$ and $v(g)$ are strictly concave in their arguments. Following Sen and Turnovsky ([1989], I also adopt the "plausible assumption" $u_{cl} < 0$, that the marginal utility of consumption increases with leisure.[6]

Appendix A shows that the solution of the lifetime welfare maximization problem yields[7]

$$\lambda = -\phi u_c(c, l) \tag{6}$$

$$\frac{-u_l}{u_c} = w \tag{7}$$

$$\dot{\lambda} = \lambda\{U(c, l, g) - r\} \tag{8}$$

$$\dot{\phi} = 1 + U(c, l, g)\phi \tag{9}$$

6. This assumption is sufficient but not necessary for the conclusions that follow. Thus, with $u_{cl} = 0$, as would be the case if consumption and leisure enter additively into the felicity function, all the substantial results obtained here continue to hold. With $u_{cl} > 0$ appropriate restrictions (which can be easily seen from appendices B and C) have to be imposed for the same results.

7. Henceforth I drop the time subscripts except when there is risk of confusion.

where λ is the costate variable associated with the constraint (3) and as such has the conventional interpretation of denoting the marginal utility of wealth. ϕ is the costate variable associated with the auxiliary variable z [(2)], which is an indicator of the stock of accumulated impatience ⟨Obstfeld [1990]⟩. Put differently, $\phi_t = \Omega({}_tC, {}_tL)$, where ${}_tC$ and ${}_tL$ stand for the parts of the consumption and labor paths beyond time t. Thus, ϕ_t denotes the present discounted value of the future felicity stream at time t, or, in other words, the maximized value of lifetime welfare at time t. One can easily show, as in Epstein and Hynes [1983], that this setup gives rise to (i) a marginal rate of substitution between consumption and leisure levels at different dates that depends on future consumption and leisure through the utility index ϕ, thereby establishing the interdependence of consumption and leisure at different points in time; and (ii) an endogenous rate of time preference $\rho = -1/\phi$. Assuming $u(c, l)$ is increasing in consumption and leisure implies that ρ is increasing in consumption and leisure paths. This assumption, labeled "increasing marginal impatience" by Lucas and Stokey [1984], is necessary for dynamic stability, but has proven to be quite controversial. It is generally argued, for at least low consumption levels, that decreasing impatience carries a great deal of intuitive appeal: for poverty levels of consumption the importance one attaches to present as opposed to future consumption tends to be higher. However, Shi and Epstein [1993], Epstein [1987], and Obstfeld [1990] have cogently argued that increasing impatience has more intuitive appeal for higher consumption levels. One can, therefore, interpret the local dynamics studied below as pertaining to such relatively high consumption and leisure levels. Equation (7) describes the well-known condition that sets the marginal rate of substitution between consumption and leisure equal to the real wage rate.

B. Firms

Perfectly competitive firms produce the single good by employing capital k and labor l under constant returns to scale. The production function $f(k, l)$ possesses the following properties:

$$f_k > 0,\ f_l > 0,\ f_{kk} < 0,\ f_{ll} < 0,\ (f_{kk}f_{ll} - f_{kl}^2) \geq 0, \tag{10}$$

Since installing investment goods is costly, it takes $i[1 + T(i/k)]$ units of output to increase the capital stock by i units.[8] The installation cost function T obeys

$$T(0) = 0,\ T^{I}(i/k) > 0,\ 2T^{I} + (i/k)\,T^{II} > 0. \tag{11}$$

Firms choose the time path of investment to maximize the present discounted value of net profits $\pi = f(k, l) - i(1 + T) - wl$ subject to the constraint $i = dk/dt$.[9] The solution of this problem yields

$$1 + T(i/k) + (i/k)\,T^{I}\,(i/k) = q \tag{12}$$

$$\dot{q} = rq - f_{f}(k, l) - (i/k)^{2}\,T^{I}\,(i/k) \tag{13}$$

$$w = f_{l}(k, l) \tag{14}$$

where q denotes the shadow price of capital. Under our assumptions Tobin's average q and Tobin's marginal q ⟨Hayashi [1982]⟩ coincide. Thus q also stands for the stock market price of equity relative to replacement cost.

Equation (12) implies that the rate of investment i/k is the following function of q

$$\frac{i}{k} = \frac{\dot{k}}{k} = \varphi(q),\ \varphi^{I} \equiv s > 0,\ \varphi(1) = 0. \tag{15}$$

That is, investment is an increasing function of the price of equity; and when the price of equity equals the unitary replacement cost of capital ($q = 1$) investment is zero.

C. The Current Account

To obtain the dynamics of the current account use $a = b + qk$ (where b denotes the representative household's holdings of the internationally traded bond), (3) and (13)–(15) which yield

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8. I follow Blanchard [1983] in the formulation of installation costs.

9. For notational ease I assume that investment is exclusively financed by retained earnings. It is well known that in models such as this different forms of financing are equivalent.

$$\dot{b} = rb + f(k, l) - j(1 + T) - c - \tau \tag{16}$$

that is, the current account is equal to national income, $rb + f(k, l)$ less the sum of investment, consumption and lump-sum taxes.

D. Equilibrium

Equations (6), (7), and (14) can be used to solve for c and l as[10]

$$c = c(k, \lambda, \phi), \quad c_j < 0, \quad j = 1, 2, 3 \tag{17}$$

$$l = l(k, \lambda, \phi), \quad l_j > 0, \quad j = 1, 2, 3 \tag{18}$$

The intuition underlying the partial derivatives is easy to grasp. A rise in the capital stock, k, increases the real wage rate and, thus, causes a substitution of labor for consumption. An increase in the marginal utility of consumption, $\lambda/(-\phi)$ [(6)], similarly changes the consumption-leisure trade-off against consumption and in favor of labor services.

Substitution of (17) and (18) into (8), (9), (13), (15), and (16) yields the five differential equations of the system in the variables λ, ϕ, q, k, and b. Setting the last five equations equal to zero and using (7), one obtains the steady-state level of $\xi = (q, k, \lambda, \phi, b)$ as

$$\begin{aligned} &q^* = 1, \quad f_k(k^*, l^*) = r = U(c^*, l^*, g), \\ &rb^* = c^* + \tau - f(k^*, l^*) \end{aligned} \tag{19}$$

where asterisks denote the steady-state values of the variables. Some aspects of this steady-state merit comment. First, (19) implies well-defined long-run target utility and wealth levels, the empirical plausibility and analytical advantage of which have been emphasized by Obstfeld [1990]. Second, $r = f_k(k^*, l^*)$ indicates that long-run arbitrage (or, stock market equilibrium) requires that the marginal productivity of capital equal the world interest rate.

10. Throughout partial derivatives are evaluated near a steady state. They are given in explicit form in appendix B.

Figure 1 shows the determination of c^* and l^*. The upward-sloping *RR* line depicts $r = U(c^*, l^*, g)$; the downward-sloping *MM* line is obtained by solving for k^* from $\theta f_k(k^*, l^*) = r$ and substituting this into $-u_l/u_c = w$.

I now turn to local dynamics represented by the linearization of the system [(8), (9), (13), (15), (16)]. Since it possesses two predetermined variables, k and b, local stability of the system requires that the matrix A obtained from its linearization have two eigenvalues with negative real parts.[11] The convergent path will be cyclical if these two eigenvalues are complex.

The following theorem, proved in appendix C, summarizes the local dynamics of the economy.

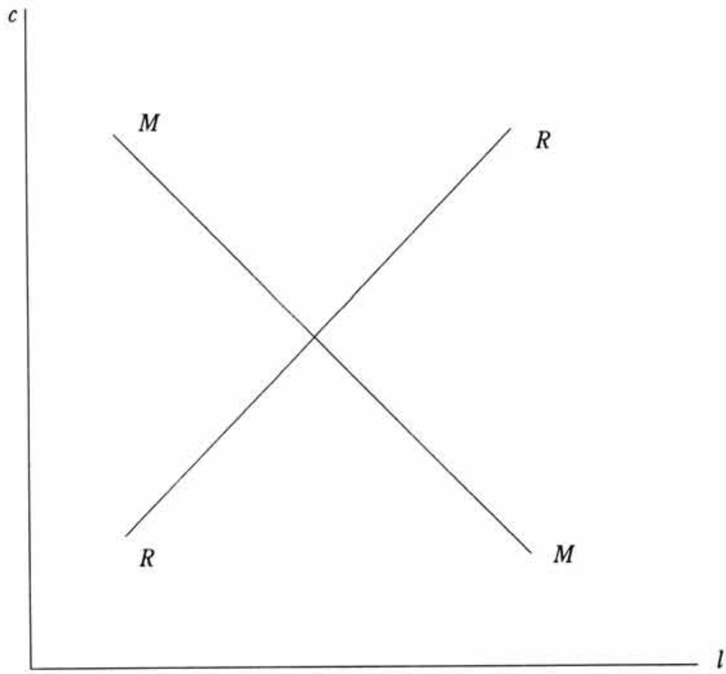

Figure 1
The Long Run Equilibrium

11. If the regularity condition discussed in appendix C holds, this is also a sufficient condition for local stability.

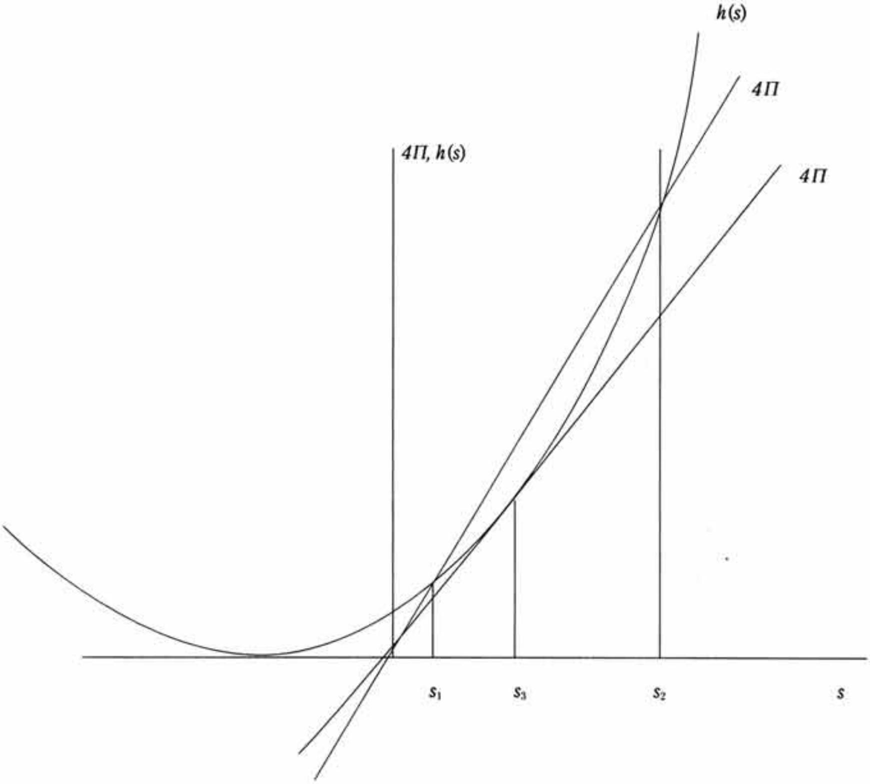

Figure 2
Adjustment Costs and Eigenvalues

Theorem 1: The system [(8), (9), (13), (15), (16)] is locally stable for all $0 < s$ except (s_1, s_2, s_3).[12]

The convergent path is cyclical if and only if $s_1 < s < s_2$.

The result concerning the possibility of cyclical adjustment merits further comment. Such adjustment here is due to the combination of three ingredients of the model: (i) endogenous time preference, (ii) endogenous labor supply, (iii) adjustment costs in investment. Karayalcin [1994] has established that the combination (i) and (iii) with exogenous labor supply gives rise to monotonic and nonmonotonic convergent paths for the predetermined variables *k* and *b* respectively. Endogenizing the labor supply preserves, as expected, the stability properties of the system [the steady state

12. Recall (15) for the definition of *s*.

here is locally stable for all s_0, except for s_1, s_2, s_3 at which points the regularity condition (see appendix C and Figure 2) fails to hold. Sen and Turnovsky [1991] combine (ii) and (iii) with time-additive preferences and obtain a monotonic transient path for the economy. Finally, Shi [1994] develops a model that combines (i) and (ii) and shows again that the economy adjusts monotonically towards its steady state.

In both Karayalcin [1994] and Sen and Turnovsky [1991] the presence of adjustment costs implies that the stock market price of domestic equity and the domestic capital stock form a dynamic subsystem which determines their adjustment independently of the rest of the system. However, the combination of (i), (ii), and (iii) abolishes this independence of investment and establishes additional linkages between investment decisions of the firms and savings and labor supply decisions of the households which gives rise to the possibility of cyclical adjustment. Such cyclical adjustment occurs if $s_1 < s < s_2$. To interpret this condition, note from (12) and the assumed properties of the installation cost function T in (11) that s is a decreasing function of installation costs. Thus, the condition $s_1 < s < s_2$ states that convergence to the steady-state will be cyclical for a well-defined medium range of adjustment costs. To see why, note that with relatively high or relatively low adjustment costs the capital stock of the economy will tend to adjust either quite slowly or quite fast dominating the convergence path of the economy to its steady state, and giving rise to noncyclical transient paths. It is only when adjustment costs are in the middle range that the possibility of cyclical adjustment arises.[13]

III. The Effect of an Increase in Government Spending

I now put the model to work by considering the effects of a permanent fiscal expansion financed by non-distortionary taxes, $d\tau = dg > 0$.

In figure 1 this demand shock shifts the *RR* curve downwards, showing a long-run fall in consumption and rise in employment.[14] Intuitively, the

13. As the simulation discussed below shows adjustment costs as low as 0.0005 per unit of output are sufficient to yield cyclical adjustment.

14. See appendix B for the derivatives.

increase in public services, $dg > 0$, raises the utility level $U(c, l, g)$ above its long-run target, prompting a decrease in consumption and leisure (thus, an increase in labor services). Hence, households substitute government services for consumption and leisure in the long run. Viewed differently, the higher level of non-distortionary taxes required to finance increased government spending causes a negative wealth effect reducing consumption and leisure. The rise in employment increases the marginal productivity of capital above the world interest rate; the long-run capital stock must, therefore, rise to ensure stock market equilibrium. These results conform with the findings of recent empirical research concerning the effects of fiscal expansions in open economies ⟨Ahmed [1986]⟩. The effect of the fiscal expansion on the long-run holdings of traded bonds depends on the relative strength of two opposing forces. The increase in consumption and lump-sum taxes requires a higher b^* to support the increased domestic absorption. The rise in domestic output, on the other hand, tends to reduce b^*, by meeting this demand. The net effect on b^* remains ambiguous.

How does the economy adjust at a point in time in response to the fiscal shock? As mentioned earlier, the transient path may be cyclical if adjustment costs fall in a medium range. Since such cyclical adjustment stands in contrast to results obtained in the existing literature, the rest of the paper will concentrate on the analysis of the transient time path in this case.

As the determination of the time path of the economy under cyclical adjustment does not yield closed-form solutions, I solve the model through simulation, using its linearized version. The parameters are chosen to fit available evidence and yield a plausible long-run equilibrium.[15]

Figure 3 depicts the transient paths of the capital stock, holdings of traded bonds, Tobin's q, λ, ϕ, employment l, consumption c, and output. On impact, the marginal utility of wealth λ drops below its initial level as taxes are permanently raised. Simulation shows lifetime welfare ϕ jumping on impact, implying that the welfare-improving effect of the increase in government spending outweighs the opposite effect of the rise in taxes. The drop in the marginal utility of wealth and jump in lifetime welfare have opposing

15. Many of the parameter values, which are reported in appendix D, coincide with those used in Stoker [1994].

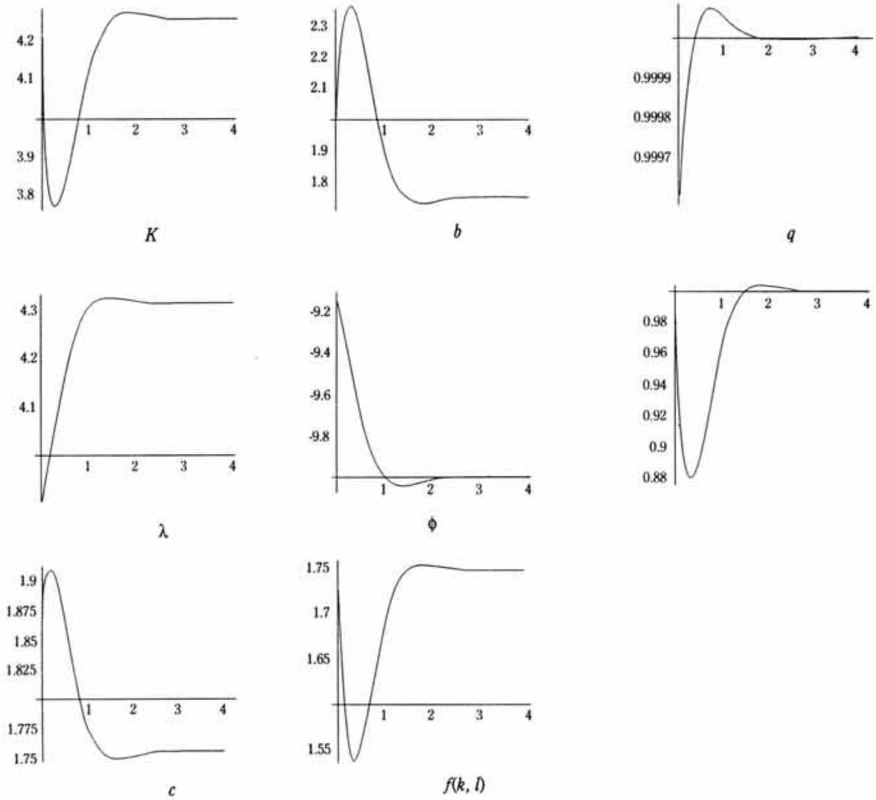

Figure 3
The Adjustment of the Variables

effects on the marginal utility of consumption [(6)] and, thus, on consumption and leisure. The parameter values used in the simulation imply that the former outweighs the latter, yielding a jump in consumption and leisure (or a drop in employment) on impact. The drop in labor supply increases wages and reduces net profits, prompting a drop in the stock market price of equity that increases the current yield on domestic equity and gives rise to expected capital gains, thereby restoring the equality between the rates of return on domestic and foreign equities. Since the domestic capital stock is predetermined, the decline in employment reduces GDP on impact.

Initially, the domestic capital stock declines along the transient path, following the drop in the stock market price of domestic equity, which causes disinvestment. Simulation also shows that the economy runs a current account surplus in the initial stages of its adjustment to the new steady state, implying that the disinvestment taking place outweighs the effects of the jump in consumption and rise in taxes. At the same time, the decline in capital stock increases the marginal utility of wealth. This raises the marginal utility of consumption and tends to decrease both consumption and leisure. On the other hand, two factors tend to raise consumption and leisure. First, the decline in the capital stock lowers the marginal productivity of labor and, thus, the wage rate leading to the substitution of consumption for labor. Second, the simultaneous fall in lifetime welfare (caused by the initial jump in consumption and leisure following the rise in g [8]) reduces the marginal utility of consumption, thereby tilting the consumption-leisure trade-off in favor of consumption and leisure. In the simulation these positive effects on consumption and leisure outweigh the negative effect, causing a rise in c and a fall in employment in the initial stage of the adjustment process. Further, the decline in the domestic capital stock and employment reduce output.

As the stock market price of domestic equity continues to rise at some point it equals and then exceeds the replacement cost of capital (which is unity), inducing firms to invest. Thus, at this point, the decline in the domestic capital stock comes to an end, and k starts rising. The increase in the domestic capital stock then changes the balance of two opposing forces on consumption and leisure, namely, the ones associated with the marginal utility of wealth and lifetime welfare. As a result, the negative effect of the former on consumption and leisure outweighs the positive effect of the latter. Consequently, consumption starts to decline, whereas employment starts to rise. With both the capital stock and employment increasing, domestic output follows suit. At the same time, the strong negative effect of investment on the current account outweighs the positive effects of declining consumption and rising output, causing the economy to run a current account deficit.

To see how the cyclical adjustment of the economy continues, it is now sufficient to discuss briefly the change in the domestic capital stock. As the economy continues to accumulate capital the positive effect of this on wage rates comes to dominate the negative effect of higher labor supply on the same, increasing the wage rate and reducing the net profits of the firms. Thus, the stock market price of domestic equities q has to fall to raise the current yield on them and to ensure international arbitrage. But, once the decline in q lowers it below the replacement cost of capital, firms start decumulating capital, and the forces discussed above give rise to a new dampened cycle. The simulation depicted in figure 3 suggests that after four periods the cyclical adjustment of the variables brings the economy arbitrarily close to its steady state.

As we saw, across steady states the negative wealth of the fiscal expansion increases labor supply which reduces the wage rate. The ensuing rise in net profits of the firms that increases the rate of return on domestic equity above the parametric world rate of interest requires a rise in domestic capital stock to lower its marginal productivity and to ensure international yield arbitrage. The rise in both employment and the domestic capital stock increases domestic output. One interesting issue is the value of the long-run multiplier. Since the model explicitly considers the interaction between capital and labor and their responses to changes in government spending as in the closed economy model of Baxter and King [1993] one would expect the m ultiplier to be considerably higher than 1. This expectation is fulfilled by the results from simulation analysis which yields a long-run multiplier of 11.[16] However, unlike the closed economy model, the small open economy model presented here shows, for reasons discussed above, that a balanced fiscal expansion reduces domestic output on impact.

IV. Conclusion

The paper has studied the effects of a balanced fiscal expansion in a general equilibrium model of a small open economy with a recursive time preference structure, in which employment, investment and the current account are endogenously determined by the intertemporal optimizing behavior of firms, whose investment is subject to adjustment costs, and infinitely-lived households. It has been shown that since the endogeneity of labor supply establishes an additional link between investment decisions of the firms and consumption, saving, and labor supply decisions of households that is absent in the exogenous labor supply models, the economy may adjust cycli-

16. Baxter and King [1993] also get multipliers as high as 13 in a closed economy model.

cally towards its steady state. Moreover, the paper demonstrates that the long-run government spending multiplier may exceed unity.

The model can be generalized in a number of ways. For instance, introducing a second good will enable one to study the effects of shocks in the allocation of resources across different sectors. Or, extending the model to a two-country framework will make it possible to analyze the international transmission of shocks.

Appendix A

The current value Hamiltonian for the lifetime welfare maximization problem of section II can be written as

$$H = -\exp(-z) + \lambda'(ra + wl - c - \tau) - \phi\,[U(c, l, g) - r] \tag{A1}$$

where λ' and ϕ' are the costate variables associated with a and z respectively. First-order conditions are (3), (6), (8), (9), and

$$w\lambda - \phi u_l = 0 \tag{A2}$$

where I have used the normalizations $\lambda = \lambda' \exp[z(t)]$ and $\phi = \phi' \exp[z(t)]$. Equation (7) of the text is obtained from (6) and (A2). Note that $-\exp(-z)$ is strictly concave in z, while (3) is linear in a. Similarly $u(c, l)$ is strictly concave in (c, l). Thus, the maximized Hamiltonian is strictly concave in c, l, and z. Impose the conditions

$$\lim_{t\to\infty} \lambda'(t)\exp(-rt) \geq 0, \quad \lim_{t\to\infty} -\phi(t)\exp(-rt) \geq 0,$$

$$\lim_{t\to\infty} \lambda'(t)a(t)\exp(-rt) = \lim_{t\to\infty} -\phi(t)\,z(t)\exp(-rt) = 0 \tag{A3}$$

That the convergent path is optimal then follows from the sufficiency theorem for optimal controls ⟨Arrow and Kurz [1970], Obstfeld [1990]⟩.

Appendix B

The derivatives for (17), (18) and comparative statics exercises are as follows.

$$c_1 = (\Delta r)^{-1}\,\lambda u_{cl} f_{kl} < 0,\;\; c_2 = (\Delta r)^{-1}\,(u_{ll} + \lambda f_{ll} + u_{cl} w) < 0,$$

$$c_3 = u_c c_2 < 0, \tag{17}$$

$$l_1 = -(\Delta r)^{-1} \lambda u_{cc} f_{kl} > 0,\;\; l_2 = -(\Delta r)^{-1}(u_{cc} w + u_{cl}) > 0,\;\; l_3 = u_c l_2 > 0, \tag{18}$$

$$\Delta \equiv (r)^{-2}(u_{cc} u_{ll} - u_{cl}^2) + r^{-1} \lambda u_{cc} f_{ll} > 0,$$

$$\frac{dk^*}{d\theta} = \Lambda^{-1}\{\{-u_c[u_{ll} + u_c f_{ll} + u_{cl} f_l] + u_l(u_{lc} + u_{cc} f_l) + \theta u_c^2 f_l f_{kl}\} > 0,$$

$$\frac{dl^*}{d\theta} = \Lambda^{-1} u_c^2 f_k f_{kl} > 0,\;\; \frac{dc^*}{d\theta} = \Lambda^{-1} u_l u_c (\theta f_l f_{kk} - f_k f_{kl}) > 0,$$

$$\frac{db^*}{d\theta} = (f_{kk})^{-1}(r + f_{kl}\frac{dl^*}{d\theta}) - r^{-1} f(k, l) < 0$$

$$\frac{dk^*}{dg} = -\frac{f_{kl}}{f_{kk}}\,\frac{dl^*}{dg} > 0,\;\; \frac{dl^*}{dg} = \Lambda^{-1} v(g) f_{kk}(u_{cl} + w u_{cc}) > 0,$$

$$\frac{dc^*}{dg} = -\Lambda^{-1} v'(g)\,[u_c(f_{ll} f_{kk} - f_{kl}^2) + f_{kk}(u_{ll} + w u_{cl})] < 0,$$

$$\frac{db^*}{dg} = -(r u_c)^{-1}[u_c - v'(g)] + \frac{f_{kl}}{f_{kk}}\,\frac{dl^*}{dg}$$

$$\Lambda \equiv \theta u_c^2(f_{kk} f_{ll} - f_{kl}^2) - \theta f_{kk}[-u_c(u_{ll} + w u_{cl}) + u_l(u_{cl} + w u_{cc})] > 0.$$

Appendix C

Linearization of the system [(8), (9), (13), (15), (16)] yields

$$\xi = A(\xi - \xi^*),\;\; \xi \equiv (k, q, \lambda, \phi, b) \tag{A4}$$

vhere

$$A=\begin{bmatrix} 0 & sk & 0 & 0 & 0 \\ \eta_k & r & \eta_\lambda & u_c\eta_\lambda & 0 \\ -u_c v_k & 0 & -u_c v_\lambda & -u_c^2 v_\lambda & 0 \\ v_k & 0 & v_\lambda & r+u_c v_\lambda & 0 \\ r(1+v_k u_c^{-1}) & -sk & r v_\lambda u_c^{-1} & r v_\lambda & r \end{bmatrix} \tag{A5}$$

$$\eta_k \equiv \frac{\partial \dot{q}}{\partial k} = -r^{-2}\Delta^{-1}\left[f_{kk}\,(u_{cc}u_{ll}-u_{cl}^2)+u_{cc}u_c\,(f_{kk}f_{ll}-f_{kl}^2)\right]>0,$$

$$\eta_\lambda \equiv \frac{\partial \dot{q}}{\partial \lambda} = -f_{kl}l_2<0,\; v_k \equiv \frac{\partial \dot{\phi}}{\partial k} = -r^{-1}(u_c c_1+u_l l_1)>0,$$

$$v_\lambda \equiv \frac{\partial \dot{\phi}}{\partial \lambda} = -r^{-1}(u_c c_2+u_l l_2)>0.$$

Manipulation of A yields the characteristic equation

$$(\mu-r)\left\{[\mu\,(\mu-r)]^2-\Gamma\mu\,(\mu-r)+\Pi\right\}=0 \tag{A6}$$

where

$$\Gamma \equiv sk\,\eta_k + r u_c v_\lambda > 0,\; \Pi \equiv skru_c(\eta_k v_\lambda - \eta_\lambda v_k)>0.$$

Clearly there is one positive eigenvalue $\mu = r$. The other eigenvalues are given by

$$\mu_i = \frac{r}{2} \pm \sqrt{\frac{\Gamma}{4}+\frac{r_2}{8}+\frac{\sqrt{16\Pi+4r^2\Gamma+r^4}}{8}} \pm \sqrt{\frac{\Gamma}{4}+\frac{r^2}{8}-\frac{\sqrt{16\Pi+4r^2\Gamma+r^4}}{8}} \tag{A7}$$

with $i = 2, \ldots, 5$.

It can easily be ascertained [Dockner and Feichtinger, 1991] that given $\Gamma > 0$ and $\Pi > 0$ two possibilities exist. If $\Gamma^2 > 4\Pi$ there are four real eigenvalues, two of them positive and the other two negative, implying noncyclical convergence to the steady state. On the other hand, if $\Gamma^2 < 4\Pi$ one obtains four complex eigenvalues, two with negative and two with positive real parts. In this case convergence to the steady state will be cyclical.

In a formally similar model, Shi and Epstein [1993] show that for stability to obtain a regularity condition, which requires that the projection of the stable manifold of the linearized system onto the (k, b) plane coincide with that plane, must hold. Following the procedure outlined in Shi and Epstein

[1993], it can easily be shown that the condition holds as long as

$$\beta_1 \neq \beta_2 \Leftrightarrow \Gamma^2 - 4\Pi \neq 0. \tag{A8}$$

To see what this implies, note that Γ^2 is a quadratic equation in s with

$$h(s) \equiv \Gamma^2,\ h'(s) = 2\Gamma k \eta_k > 0,\ h''(s) = 2k^2\eta_k^2 > 0$$
$$h'(s_{\min}) = 0 \Rightarrow s_{\min} = -ru_c v_\lambda (k\eta_k)^{-1} < 0, \tag{A9}$$

while Π is a positive linear function of s. Thus, Γ^2 is a strictly convex function of s, positive in the positive quadrant. We then have three possibilities depending on the values of partials given in (A5): (1) $\Gamma^2 > 4\Pi$ for all s, with the graph of Γ^2 lying strictly above the graph of 4Π, implying noncyclical adjustment; (2) $\Gamma^2 < 4\Pi$, for $s_1 < s < s_2$ (which gives rise to cyclical adjustment) and $\Gamma^2 > 4\Pi$ for $s_1 > s_2$ and $s > s_2$, such that the line $4P$ intersects the curve Γ^2 twice in figure 3; (3) there exists a unique s (denoted by s_3) with $\Gamma^2 = 4\Pi$ such that the line 4Π is tangent to the curve Γ^2 at s_3 in figure 3 (implying noncyclical convergence for all $s > 0$ other than s_3). At s_1, s_2, s_3 the regularity condition and, thus, stability fails to hold.

Appendix D

The functional forms used in the simulation are:[17]

$$f(k, l) = A[\delta K^{-\eta} + (1-\delta)l^{-\eta}]^{-1/\eta}$$

$$u(c, l) = c^{\alpha}(M - l)^{\beta},\ v(g) = \ln g$$

$$T = \frac{b}{2}\frac{i}{k}$$

The parameter values assumed are:

r=0.1, A=1.1, δ=0.4, η=0.5, α =0.399, β=0.6, M=3, g=0.166, b=1/s=0.0005

These parameter values yield the following steady-state solutions for the variables

17. The functional forms and parameter values used in the simulation follow those adopted by Baxter and King [1993], Lipton and Sachs [1983], and Stokey [1994],

$k^* = 4.26, b^* = 1.75, l^* = 1, c^* = 1.755, f(k^*, l^*) = 1.746, \lambda^* = 4.327, \phi^* = -10$

The steady-state levels of the same variables with the lower $g = 0.164$ are

$k^* = 4.22, b^* = 1.998, l^* = 0.99, c^* = 1.764, f(k^*, l^*) = 1.728, \lambda^* = 4.313, \phi^* = -10$

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