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Welfare-Enhancing Import Subsidies

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Abstract

The concept of "comparative preference" is developed. It is shown using this concept in a model analogous to that of Itoh and Kiyono that a subsidy on imports of a good may be beneficial to the importing country. Two sets of models are developed to demonstrate when and why import subsidies may be welfare-enhancing to the importer. One set includes three goods and two countries and the other includes two goods and three countries. For each set, propositions are illustrated for a simple model with specific functional forms using the phenomenon of comparative preference. Propositions are also presented in more detail with general models. The roles of key parameters are explained and optimal subsidy expressions are derived. Relationships with alternative tariff policies are explained.

I. Introduction

In a recent paper, Itoh and Kiyono used a trade model with three goods and Ricardian production to demonstrate that the home country may exer-

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cise monopsony power by applying an export subsidy to one of the goods. If goods are ranked by degree of comparative advantage, then it may be optimal to subsidize export of the middle, or marginal, good. The present paper uses a three-good model analogous to theirs to demonstrate the possible optimality of an *import* subsidy on the marginal good. In the spirit of Jones [1961], it also extends the analysis to the case of two goods but three *countries*. There the home country may benefit from targeting subsidies on imports from one of the other two countries. Both models of the paper incorporate what might be called Ricardian preferences, i.e., preferences with perfect substitutability between goods. The concept of *comparative preference* (rather than comparative advantage) is developed and used to rank goods for import and export.

Itoh and Kiyono are far from alone in indicating the possible optimality of export subsidies. A recent set of papers which use a model with perfect competition include Abbott, Paarlberg, and Sharples; Feenstra; and Dutton. In addition, papers which use models with imperfect competition to justify export subsidies include Brander and Spencer [1985] and Tower.

The idea of welfare-enhancing import subsidies has also appeared from time to time in the literature. Several papers present analysis of beneficial import subsidies in the presence of imperfect competition. Corden, Brander and Spencer [1984], and Jones [1987] provide cases when there is domestic monopoly in the import-competing industry, and Katrak provides a case in the presence of foreign monopoly of a domestic import. Tower provides a case of bilateral monopoly in which an import subsidy may be beneficial.

There are also papers on subsidies in the case of perfect competition. Graaf, in an early work, discusses the possibilities in a general way. Kemp presents a more formal analysis, with an indication of several situations in which a subsidy might be an optimal policy. One case occurs when a subsidy is a local (rather than a global) optimum. Another occurs in the presence of an international transfer. Gruen and Corden provide an example of supply cross effects which might lead to negative terms of trade effects of a tariff (and implicitly positive effects of a subsidy). Horwell and Pearce supply a formal analysis of optimal tariff policy in an n-good model. They indicate that in a model with n goods, the vector of optimal trade taxes or subsidies is indeterminate without a normalization (the usual normalization is to set one tax or subsidy equal to zero). Signs of import and export measures are not invariant to the choice of normalization. Bond, in a recent paper, reiterates these points. He demonstrates that, when a vector of optimal trade measures is applied and when goods in the foreign country are all gross substitutes, the good which is most protected is an import good and that which is least protected is an export good. Also, under certain restrictions on the matrix of own and cross price effects abroad, trade measures cannot consist of all subsidies. In addition, he demonstrates conditions under which revenues from trade measures must be nonnegative. The present analysis using the three-good two-country model does not meet the conditions for Bond's propositions to hold. The most protected good is not necessarily an import and the least protected is not necessarily an export. Also, there is a first best optimum with only an import subsidy and with negative revenue. In addition, Bond's analysis does not apply to the two-good three-country model, in which measures may be targeted at one country only. Young, also in a recent paper, indicates several conditions for an n-good economy for which optimal tariffs must be positive on average. He also provides a two-country case in which optimal tariffs can be negative on average in the presence of several consumers in the foreign country. The income shifts among consumers in such a case may create the conditions for a negative tariff structure.

There is a long literature on tariff preferences. A recent paper by Gardner and Kimbrough summarizes that literature and describes the effects of country-specific tariffs. That paper does not deal with import subsidies.

The remainder of this paper is organized as follows. In section II. A a simple three-good two-country model with specific utility and production functions is used to present the concept of comparative preference and to use it to demonstrate the possible optimality of an import subsidy. In section II. B the possibility of an optimal import subsidy is indicated in a more general model with three goods and two countries. In section III. A, a simple model with two goods and three countries and specific functions is developed and used to demonstrate the possible optimality of a targeted or country-specific import subsidy. Section III. B contains a more general version of the two-good three-country model with a welfare-improving subsidy. Section IV is a summary and conclusion.

II. Situation One: Two Countries and Three Goods

A. Simple Model

Consider a model with two countries and three goods, similar to the initial intuitive model used by Itoh and Kiyono. The home country and the foreign country each produce fixed quantities of the three goods. Utility functions for the two countries are:

$$U = a_1C_1 + a_2C_2 + a_3C_3 \tag{1a}$$
$$U^* = a_1^*C_1^* + a_2^*C_2^* + a_3^*C_3^* \tag{1b}$$

where the C_i's are consumption levels, the a_i's are parameters, and asterisks indicate variables for the rest of the world (ROW). These functions imply perfect substitutability between any pair of goods. The utility parameters indicate the relative utility values placed by each country on the three goods. These parameters fit the relationships:

$$\frac{a_1}{a_1^*} > \frac{a_2}{a_2^*} > \frac{a_3}{a_3^*}$$

These ratios signify that the home country preference for good 1 relative to good 2 is stronger than that of the foreign country. The home country has a *comparative preference* for good 1 relative to good 2, and the foreign country has a comparative preference for good 2 relative to good 1. Also, the home country has a comparative preference for good 2 relative to good 3 and the foreign country has the opposite. Note the strong similarity between the comparative preference relationship here and the more typical *comparative advantage* relationship resulting from Ricardian production. In the same way that input coefficient ratios can be used to establish comparative advantage rankings, in this model the utility parameter ratios can be used to indicate comparative preference rankings.

Because of the perfect substitutability between goods in each country, the utility function of each country completely determines relative prices of any goods consumed within that country. Within the home country, for example, if goods 1 and 2 are both consumed, relative prices must be $P_2/P_1 = a_2/a_1$. If the relative price ratio in the equation were too high (low), then the home country would not consume good 2 (1). Since the utility parameter ratios differ across countries, it cannot be true under a free trade regime that both countries consume all three goods. Some specialization in consumption will occur.[1]

Assume that in the current equilibrium situation the home country consumes only goods 1 and 2 and ROW consumes only goods 2 and 3. Resource endowments are such that the home country imports all of ROW's production of good 1 and exports all its own production of good 3. The home country consumes all of its own production of good 2 and also imports part of ROW's production. Because it is the only good consumed by both countries, good 2 is a "marginal" good in a sense analogous to that of Itoh and Kiyono. Table 1 summarizes the situation.

Table 1

	Home Country	ROW
Goods Consumed	1, 2	2, 3
Goods Produced	1, 2, 3	1, 2, 3
Goods Imported	1, 2	3

Relative prices at home and abroad under free trade are:

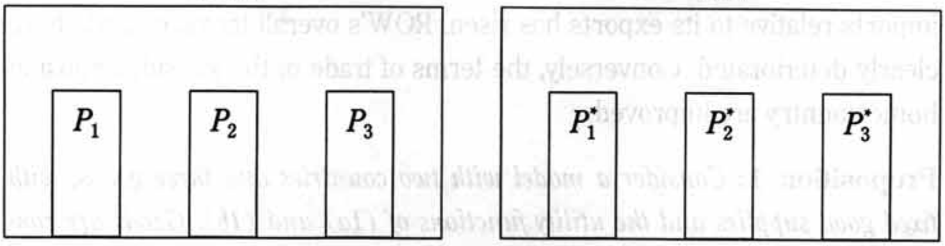
$$\underline{\text{Home}} \qquad\qquad \underline{\text{ROW}}$$

$$\frac{P_2}{P_1}=\frac{a_2}{a_1},\ \frac{P_3}{P_1}=\frac{a_2a_3^*}{a_1a_2^*} \qquad \frac{P_2^*}{P_1^*}=\frac{a_2}{a_1},\ \frac{P_3^*}{P_1^*}=\frac{a_2a_3^*}{a_1a_2^*}$$

These prices are set by the utility parameters. For simplicity and without loss of generality, choose units of measurement of the goods so that $a_1 = a_2$ and $a_2^* = a_3^*$.[2] Then the relative price levels at home and abroad can be

1. Again the analogy to comparative advantage relationships holds. In Ricardian comparative advantage models, relative prices are set by labor input requirements. If a country produces any two goods, then the relative prices of those goods are set by relative labor coefficients. Under free trade, two countries cannot produce the same two goods unless their labor input ratios are identical for those goods. In the present instance, two countries cannot consume the same two goods unless their utility parameter ratios are identical for those goods.
2. There are sufficient degrees of freedom to choose such a normalization.

Home ROW



graphed (with relative heights of boxes representing relative prices): These prices, determined entirely on the demand side, also set the international terms of trade.

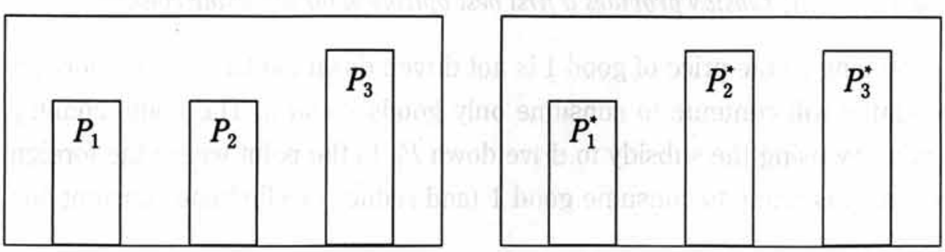
This model can be thought of as representing situations of goods with relatively fixed short-run supply, such as minerals. To fit, they must also consist of several different types which are closely substitutable. Coal might be an example. There are several types of coal which are close substitutes. A third requirement is that countries do not value the various good types equally. This might be the case with coal if two countries have different industrial structures, implying different valuations for the several coal varieties. A similar example could be developed with wheat.

Now consider the effect of an ad valorem subsidy s granted by the home country on imports of good 2, so that $P_2 = P_2^*(1-s)$. (The subsidy would be financed with lump-sum taxes.) Relative prices in the two countries would become:

<u>Home</u> <u>ROW</u>

$$\frac{P_2}{P_1} = \frac{a_2}{a_1}, \quad \frac{P_3}{P_1} = \left[\frac{a_2 a_3^*}{a_1 a_2^*}\right] / (1-s), \quad \frac{P_2^*}{P_1^*} = \left[\frac{a_2}{a_1}\right] / (1-s), \quad \frac{P_3^*}{P_1^*} = \left[\frac{a_2 a_3^*}{a_1 a_2^*}\right] / (1-s)$$

Home ROW



For ROW, the price of its import, good 3, has been raised relative to the price of one of its exports, good 1. The price it receives for good 2 remains the same relative to the price of its import. Since the average price of its imports relative to its exports has risen, ROW's overall terms of trade have clearly deteriorated. Conversely, the terms of trade of the subsidy-imposing home country are improved.

Proposition 1: *Consider a model with two countries and three goods, with fixed good supplies and the utility functions of (1a) and (1b). Goods are consumed, produced, and imported as in Table 1. Then the home country can improve its terms of trade by instituting a subsidy on imports of good 2.*

This counter-intuitive welfare enhancement for the home country occurs because prices of its two import goods (1 and 2) are fixed with respect to each other (so long as the home country continues to consume both). This fixing occurs because of the perfect substitutability of the two goods in home consumption. The import subsidy serves to lower the domestic price of good 2 and thereby also lower the price of good 1 as well. The world price (=ROW price) of the subsidized good remains constant, but the world price of the other imported good (good 1) is driven down relative to the price of the export good.[3] The home terms of trade improve. This improvement is reflected in consumption levels. By assumption, the home country continues to consume all of good 1 and ROW continues to consume all of good 3. However, some good 2 consumption shifts from ROW to the home country, thereby enhancing home welfare. Note that the desirability of the import subsidy occurs in a world with strong cross price effects on the consumption side.

Proposition 2: *Assuming the conditions of Table 1, the level of good 2 import subsidy which maximizes home welfare in this model is* $s^0 = 1 - (a_2/a_1)/(a_2^*/a_1^*)$. *This subsidy provides a first best optimum for the home country.*

So long as the price of good 1 is not driven down too far, then the foreign country will continue to consume only goods 2 and 3. The home country gains by using the subsidy to drive down P_1^*, to the point where the foreign country is ready to consume good 1 (and reduce or eliminate consumption

3. Of course, re-export of the subsidized import back to the originating country must be precluded. We assume transportation costs prevent such re-export.

of good 2. That optimal subsidy point occurs where:

$$\frac{P_1^*}{P_2^*} = \frac{a_1^*}{a_2^*}$$

Using this condition, along with $P_2 = P_2^*(1-s)$, $P_1/P_2 = a_1/a_2$, and $P_1 = P_1^*$ gives the optimal subsidy above.

The tariff normalization used in this case is to set the tariff on good 1 to zero. When that tariff is zero, and imports of good 2 are subsidized at a level of $s = s^0$, then an import tariff or subsidy on good 3 has no additional effect. Any effect of such a measure at home falls on the home price of good 3, but since there is no consumption or import flow, and supply is completely inelastic, there are no home welfare consequences of an import tariff. The whole benefit of optimal commercial policy can be achieved with the good 2 import subsidy alone.

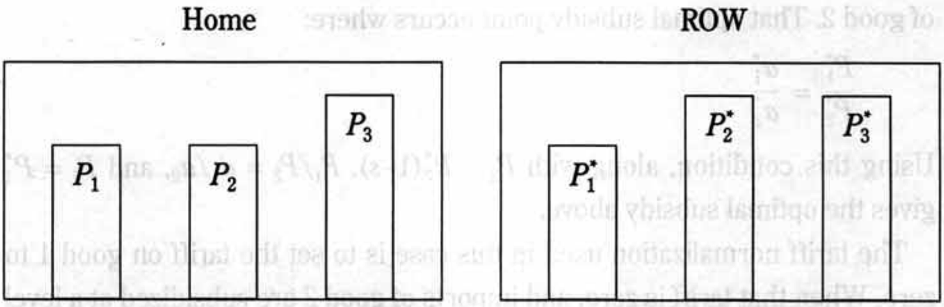
Of course, tariff revenue in this situation is negative. It does not meet the conditions for nonnegative revenue mentioned by Bond. In addition, the least protected good (as measured by the ratio of the home price to the world price) is good 2 and is an import rather than an export. In that respect this model violates Bond's proposition that the least protected good should be an export as long as all goods are gross substitutes abroad.[4]

Proposition 3: ***The home welfare effects of the import subsidy of proposition 1 can be duplicated in this model with an import tariff on good 1. The equivalent tariff would be $t = s/(1-s)$.***

If the home country applies a tariff so that $P_1 = P_1^*(1 + t)$, then relative prices at home and abroad will be:

Home	ROW
$\frac{P_2}{P_1} = \frac{a_2}{a_1}, \quad \frac{P_3}{P_1} = \frac{a_2 a_3^*}{a_1 a_2^*},$	$\frac{P_2^*}{P_1^*} = \frac{a_2}{a_1}(1+t), \quad \frac{P_3^*}{P_1^*} = \frac{a_2 a_3^*}{a_1 a_2^*}(1+t)$

4. It would be equivalent in effect in this model to retain the import subsidy on good 2 and add an export tax on good 3. Such a tax would lower the domestic price of good 3 relative to good 2 but leave that relative price unchanged abroad. If the tax were high enough, then good 3 would become the least protected good and Bond's proposition would hold.



If $t = s/(1-s)$, the effects are almost identical to those of the import subsidy. Relative world prices, which determine the terms of trade, are exactly the same. Welfare effects of trade, therefore, are exactly the same.

Domestic prices are different from the subsidy case, in that the relative price of good 3 is higher. As noted earlier, in this particular model that difference is of no consequence, since there is no home consumption of good 3 in either case and domestic production is unresponsive to prices.

B. General Model

The model used in the preceding section was stylized to simplify solving for equilibrium conditions and to clarify the results. This section contains a more general model. We start with a set of equilibrium conditions employing the commonly used expenditure and revenue functions. The first two are overall budget constraints for the two countries of the model. Trade flows are assumed throughout to be as they were in the previous model; ROW exports goods 1 and 2 and the home country exports good 3. Good 1 is treated as the numeraire good. The resource constraints for the two countries are:

$$E(P_2(1-s), P_3, U) - R(P_2(1-s), P_3, V) + sP_2[E_2(P_2(1-s), P_3, U) - R_2(P_2(1-s), P_3, V)] = 0 \tag{2}$$

$$E^*(P_2, P_3, U^*) - R^*(P_2, P_3, V^*) = 0 \tag{3}$$

Equation (2) is for the home country. P_2 and P_3 are world prices for goods 2 and 3. The domestic price of good 2 reflects an import subsidy. U is utility and

V is a vector of fixed factors of production. Equation (3) reflects conditions in the foreign country, with asterisks representing variables for that country.

In addition to these two equations, we can add market clearing equations for goods 2 and 3. In this system of four equations, P_2, P_3, U and U^* are endogenous. V and V^* are treated as fixed. The subsidy s is the source of exogenous shock. This system will indicate conditions under which an

$$E_2(P_2(1-s), P_3, U) - R_2(P_2(1-s), P_3, V) + \\ E_2^*(P_2, P_3, U^*) - R_2^*(P_2, P_3, V^*) = 0 \qquad (4)$$

$$E_3(P_2(1-s), P_3, U) - R_3(P_2(1-s), P_3, V) + \\ E_3^*(P_2, P_3, U^*) - R_3^*(P_2, P_3, V^*) = 0 \qquad (5)$$

import subsidy may be optimal.

Performing comparative statics around $s = 0$ gives the following terms of trade effects:

$$\frac{dP_2/P_2}{ds} = \frac{M_2 M_3^*}{P_2 P_3 \Delta}\left\{\eta_{22}(\tilde{\eta}_{33}^* - \tilde{\varepsilon}_{33}) + \varepsilon_{32}(\tilde{\eta}_{23} - \tilde{\varepsilon}_{23}^*)\right. \\ \left. - P_2 M_2\left[(\tilde{\varepsilon}_{33} - \tilde{\eta}_{33}^*)\frac{E_{2U}}{M_2} + (\tilde{\eta}_{23} - \tilde{\varepsilon}_{23}^*)\frac{E_{3U}}{M_3^*}\right]\right\} \qquad (6)$$

$$\frac{dP_2/P_3}{ds} = \frac{M_2 M_3^*}{P_2 P_3 \Delta}\left\{\tilde{\varepsilon}_{22}^* \tilde{\varepsilon}_{32} - \tilde{\eta}_{32}^* \tilde{\eta}_{22} - P_2 M_2\left[(\tilde{\eta}_{32}^* - \tilde{\varepsilon}_{32})\frac{E_{2U}}{M_2} + (\tilde{\varepsilon}_{22}^* - \tilde{\eta}_{22})\frac{E_{3U}}{M_3^*}\right]\right\} \qquad (7)$$

where around $s = 0$:

$$\Delta = \frac{M_2 M_3^*}{P_2 P_3}\left\{(\tilde{\eta}_{22} - \tilde{\varepsilon}_{22}^*)(\tilde{\eta}_{33}^* - \tilde{\varepsilon}_{33}) - (\tilde{\eta}_{32}^* - \tilde{\varepsilon}_{32})(\tilde{\eta}_{23} - \tilde{\varepsilon}_{23}^*)\right\} > 0$$

Here M_i is imports of good i by the home country, η_{ij} is the compensated (Hicksian) elasticity of excess demand for good i with respect to the price of good j, ε_{ij} is the compensated elasticity of excess supply of i with respect to the price of j, E_{iu} is the elasticity of consumption of good i with respect to real income, $\tilde{\eta}_{ij}$ and $\tilde{\varepsilon}_{ij}$ are the same parameters but in uncompensated (Marshallian) form, and asterisks indicate values for ROW.

Market stability implies that Δ is positive. The effect of s on P_2 then is almost surely positive.[5] Subsidizing imports will tend to raise the world price of the imported good, as expected. P_3, on the other hand, may rise or fall as a result of the subsidy.

Proposition 4: *The world price of the home country's exported good (good 3) may increase in response to a home import subsidy on good 2. Assuming income effects are positive, own and cross price effects have "typical" signs, $M_2 > 0$, $M_3^* > 0$, and cross price effects on each good are smaller than own price effects, then P_3 is more likely to rise:*

the ***larger (in algebraic value)*** *is the Marshallian home cross price elasticity of excess supply of good 3 with respect to P_2 ($\tilde{\varepsilon}_{32}$),*

the ***larger (in absolute value)*** *is the Marshallian home own price elasticity of excess demand for good 2 ($\tilde{\eta}_{22}$),*

the ***smaller*** *is the foreign Marshallian cross price elasticity of excess demand for good 3 with respect to P_2 ($\tilde{\eta}^*_{32}$),*

the ***smaller*** *are home imports of good 2,*

the ***larger*** *are foreign imports of good 3,*

the ***smaller*** *is the home income effect in demand for good 2 (E_{2U}), and*

the ***smaller*** *is the home income effect in demand for good 3 (E_{3U}).*

The role of each parameter or variable is evident in equation (7).

Since good 3 is the home country's only export good, raising its price tends to improve the home country's terms of trade. The overall effect on home welfare of the subsidy depends on how both P_3 and P_2 move in response to the subsidy. If the import subsidy has a large enough positive effect on P_3 and a small enough effect on P_2, then it will have a positive effect on home welfare.

To check welfare effects, solve for the optimum value of s, the value which maximizes home welfare.[6] That optimum occurs where $dU/ds = 0$.

5. In a two-good model, the effect of s on P_2 would be unambiguously positive. In the present three-good context, there is some ambiguity. The effect will be positive if own price effects are large enough relative to cross price effects.

6. This welfare maximum is a second best. A first best optimum would involve a vector of tariff measures, generally with tariffs or subsidies on at least two goods (the third tariff would typically be normalized to zero.)

The result is:

$$s = \frac{\eta_{22}(\varepsilon_{33} - \tilde{\eta}^*_{33}) - \varepsilon_{32}(\eta_{23} - \tilde{\varepsilon}^*_{23}) + \dfrac{P_3 M^*_3}{P_2 M_2}(\tilde{\varepsilon}^*_{22}\varepsilon_{32} - \eta_{22}\tilde{\eta}^*_{32})}{\tilde{\eta}_{22}[\tilde{\eta}^*_{33}\tilde{\varepsilon}^*_{22} - \tilde{\eta}^*_{32}\tilde{\varepsilon}^*_{23}] - \tilde{\varepsilon}^*_{22}[\varepsilon_{33}\eta_{22} - \eta_{23}\varepsilon_{32}]}$$

The denominator is positive, given market stability abroad and at home ($\tilde{\varepsilon}^*_{22}\tilde{\eta}^*_{33} - \tilde{\varepsilon}^*_{23}\tilde{\eta}^*_{32} < 0$ and $\eta_{33}\varepsilon_{22} - \eta_{23}\varepsilon_{32} < 0$) and assuming good 2 is not a Giffen good abroad ($\tilde{\varepsilon}^*_{22} > 0$). The numerator may be negative or positive.

Of the three major terms in the numerator, the first two together are likely to be negative, assuming own price effects are larger in absolute value than cross price effects. The third major term is ambiguous in sign. If η_{22} and $\tilde{\eta}^*_{32}$ are large enough in absolute value, and $\tilde{\varepsilon}^*_{22}$ and ε_{32} are small enough in absolute value, then the third term will be positive and may outweight the first two.

A subsidy is most likely to be optimal if home excess demand for good 2 responds strongly to P_2 and home excess supply of good 3 does not. Likewise, likelihood that a subsidy is optimal is also enhanced if the foreign excess demand for good 3 responds strongly to P_2 and foreign excess supply of good 2 does not.

The optimality of an import subsidy in this model is analogous to the optimality of an export subsidy in the Feenstra model.

III. Situation Two: Three Countries and Two Goods

A. Simple Model

As Jones [1977] has pointed out, there is often a symmetry between numbers of countries and numbers of goods in trade models. That is the case here. It is possible to alter the previous model to a *three* country, *two* good model and obtain results similar to those above.

For purposes of demonstration, it is again helpful to use a very simple model with extreme elasticity assumptions which make equilibrium prices easy to determine. Consider a case in which the home country imports good 2 from each of two foreign countries (*A* and *B*). It in turn exports good 1 to those countries. For some reason (*e.g.*, geographic distance), no trade occurs between A and B. Supplies of the two goods in each country are fixed. Goods 1 and 2 are perfect substitutes in consumption in each of the three countries. Because of this perfect substitutability, a country will consume both goods only if the ratio of goods prices equals the ratio of utility weights in that country. Those utility weights are such that:

$$\frac{a_2}{a_1} > \frac{a_2^A}{a_1^A} > \frac{a_2^B}{a_1^B}$$

The home country has a comparative preference for good 2 relative to both other countries and country A has a comparative preference for good 2 relative to country B.[7]

Endowments and relative utility parameters are such that at free trade equilibrium prices only country A consumes both goods. The home country consumes only good 2 and country B consumes only good 1. Because country A consumes both goods, relative prices of the two goods must equal relative utility weights in A. Under these conditions, the net supply of good 2 to the foreign sector is completely inelastic in B and completely elastic in A. Relative prices with no intervention would be:

<u>Home</u>	<u>A</u>	<u>B</u>
$\frac{P_2}{P_1} = \frac{a_2^A}{a_1^A}$	$\frac{P_2^A}{P_1^A} = \frac{a_2^A}{a_1^A}$	$\frac{P_2^B}{P_1^B} = \frac{a_2^A}{a_1^A}$

where a_1^A and a_2^A are utility parameters in country A.

For political reasons the home country is not allowed to use tariffs; however, it is allowed to use import subsidies. (A possible scenario is that the home country and country B form a free trade area, the existence of which prevents imposition of a tariff on imports from B.)

Proposition 5: *Under the conditions described above, the home country would benefit from applying a subsidy to imports from country A.*

Assuming country A continues to consume both goods and the other two countries still consume one good each, relative prices in the three countries

7. The chain of comparative preference here is similar to the chain of comparative advantage in Jones [1961].

with the subsidy would be:

$$\underline{\text{Home}} \qquad\qquad \underline{A} \qquad\qquad \underline{B}$$

$$\frac{P_2}{P_1}=\left[\frac{a_2^A}{a_1^A}\right](1-s) \qquad \frac{P_2^A}{P_1^A}=\frac{a_2^A}{a_1^A} \qquad \frac{P_2^B}{P_1^B}=\left[\frac{a_2^A}{a_1^A}\right](1-s)$$

So long as country A continues to consume both goods, utility parameters determine relative prices of the two goods there. Relative prices in the other two countries are then determined by international trade equilibrium conditions.

The important aspect from the standpoint of the home country is that B's export price is caused to be lowered by the subsidy on imports from A. The home terms of trade with A are unaffected by the subsidy, but the terms of trade with B are improved. This difference in treatment of A and B is a form of monopsonistic price discrimination, designed to exploit the difference in the elasticities of excess supply of countries A and B.

Proposition 6: *The good 1 import subsidy which maximizes home welfare is*

$$s^0 = 1 - (a_2^B/a_1^B)/(a_2^A/a_1^A).$$

So long as the consumption pattern listed above for each country holds, the home country can benefit from further increases in the subsidy. If the subsidy is raised too high, however, then the price of good 2 in country B will drop to the point where B begins shifting consumption from good 1 to good 2 and the home country starts losing its export market. If it is lowered still further, then B will stop consuming good 1 entirely, since the elasticity of demand in B at that point is infinite. The consumption-shifting point occurs where $P_2^B/P_1^B = a_2^B/a_1^B$. It would also still be true that $P_2^B/P_1^B = [a_2^A/a_1^A](1-s)$. Putting these two equations together yields the optimal subsidy.[8]

Proposition 7: *The home welfare effects of the import subsidy of proposition 5 can be duplicated in this model with a tariff on imports of good 1 from coun-*

8. As the terms of trade turn against B and its income declines, its demand for good 1 decreases. The amounts of good 1 freed up are consumed by A. If they are large enough, A may stop consuming good 2 entirely, in which case the relative price of good 1 can be driven down in A. This effect will further decrease its relative price in country B and at home as well.

try B. The equivalent tariff would be $t = s/(1-s)$.

Relative prices with such a tariff would be:

Home	A	B
$\frac{P_2}{P_1} = \frac{a_2^A}{a_1^A}$	$\frac{P_2^A}{P_1^A} = \frac{a_2^A}{a_1^A}$	$\frac{P_2^B}{P_1^B} = \left[\frac{a_2^A}{a_1^A}\right] / (1+t)$

If $t = s/(1-s)$, then these relative prices are almost exactly the same as with the subsidy. They are not the same in the home country, but that fact has no influence on consumption decisions, so long as the home country continues to consume only good 1. The tariff has the same effect as the subsidy in improving home terms of trade in a monopsonistic price discriminating manner.

B. General Model

Comparative statics with a more general model reveal the roles of key elasticities in determining whether a positive subsidy is called for. Expenditure and revenue functions for each of the three countries give the three equilibrium conditions:

$$E(P, U) - R(P, V) + [sP/(1-s)][R_2^A(P/(1-s),\ V^A) - E_2^A(P/(1-s),\ U^A] = 0 \quad (5)$$

$$E^A(P/(1-s),\ U^A) - R^A(P/(1-s),\ V^A) = 0 \quad (6)$$

$$E^B(P, U^B) - R^B(P, V^B) = 0 \quad (7)$$

where E and R denote expenditure and revenue functions, P is the relative price of good 2 at home and in country B (good 1 is the numeraire), $P(1-s)$ is the relative price of good 2 in country A, U denotes utility level, V denotes a vector of fixed factor inputs, and subscripts indicate partial derivatives. The home country is assumed to export good 1 and import good 2 from both A and B.

Note that the subsidy on imports from country A is included as an additional expenditure for the home country. A fourth equation can be derived from equilibrium in the market for good 2.

$$E_2(P,U)-R_2(P,V)+E_2^A(P/(1-s),U^A)-R_2^A(P/(1-s),V^A) \\ +E_2^B(P,U^B)-R_2^B(P,V^B)=0 \tag{8}$$

(The good 1 version of this equation is redundant, by Walras' Law.) These equations form a system in four unknowns, P, U, U^A, and U^B. The vectors of inputs are considered fixed. The ad valorem subsidy s is the exogenous variable.

Complete differentiation of the system and some work yields equations for effects of the subsidy on P, the price paid to country B (and also the domestic price) and for P^A, the price paid to country A, where $P^A = P/(1-s)$. (See Appendix 2 for the derivations.)

$$\frac{dP}{ds} = -\frac{M^A}{\Delta}\left\{\varepsilon_{22}^A + P\ E_{2U}\right\}$$

$$\frac{dP^A}{ds} = P\left\{\frac{M^B\tilde{\varepsilon}_{22}^B - M\,\tilde{\eta}_{22} - M^A PE_{2U}}{M^A\tilde{\varepsilon}_{22}^A + M^B\tilde{\varepsilon}_{22}^B - M\,\tilde{\eta}_{22}}\right\}$$

where M^A and M^B are home imports from A and B, ε_{22}^i is the elasticity of excess supply of good 2 in country i, η_{22} is the elasticity of excess demand at home, E_{2u} is the income effect for good 2 at home, and a "~" indicates an uncompensated (Marshallian) effect. Also:

$$\Delta = \left(M^A\tilde{\varepsilon}_{22}^A + M^B\tilde{\varepsilon}_{22}^B - \tilde{\eta}_{22}\right)/P + M^A\frac{s}{1-s}E_{2U}\left(1+\tilde{\varepsilon}_{22}^A\right)$$

This value exceeds zero with the usual price effects and $s = 0$.

The price paid to country A almost surely rises, although less than one for one with the subsidy. The price paid to country B almost definitely drops. The latter effect improves the home country's terms of trade. If strong enough it will dominate the effect of a higher P^A on those terms of trade.

Proposition 8: *The optimal import subsidy of the home country on goods from country A may be positive. A positive optimal subsidy is more likely:*

the ***larger*** *is the elasticity of excess supply of good 2 from country A* ($\tilde{\varepsilon}_{22}^A$),

the ***smaller*** *is the elasticity of excess supply of good 2 from country B* ($\tilde{\varepsilon}_{22}^B$),

*the **smaller in absolute value** is the excess demand elasticity for good 2 at home ($\tilde{\eta}_{22}$),*
*the **larger** is the level of imports from country B (M^B), and*
*the **smaller** is the total level of imports (M).*

Comparative statics yields a value for the optimum subsidy, *i.e.,* the subsidy which maximizes the utility level of the home country:

$$s = \frac{\tilde{\varepsilon}_{22}^A - \tilde{\varepsilon}_{22}^B + \tilde{\eta}_{22} M / M^B}{\tilde{\varepsilon}_{22}^A (1 + \tilde{\varepsilon}_{22}^B - \eta_{22} M / M^B)}$$

That subsidy may be positive or negative. It is straightforward to confirm that the partial derivatives of s with respect to $\tilde{\varepsilon}_{22}^A$ and $\tilde{\varepsilon}_{22}^B$ are positive and negative respectively, and that the partials of s with respect to M^B and M are also positive and negative, confirming the effects of Proposition 8. The partial of s with respect to $\tilde{\eta}_{22}$ is positive, indicating that a subsidy is more likely if the home excess demand elasticity for good 2 is small in absolute value.

The likelihood that an import subsidy is optimal is not insignificant. For example, if the excess supply elasticities of good 2 from A and B are 2 and .5, the excess demand elasticity at home is –.5, and eighty percent of good 2 imports come from country B, then the optimal subsidy is 20.6 percent. This scenario is not unreasonable.

The results of this section are fully consistent with the simpler model presented earlier. There $\tilde{\varepsilon}_{22}^A$ was assumed to be infinite and $\tilde{\varepsilon}_{22}^B$ and $\tilde{\eta}_{22}$ were assumed to be zero. Such a case clearly calls for a subsidy.

IV. Summary and Conclusions

This paper develops the concept of comparative preference and uses it to model situations in which a subsidy on an import may enhance welfare for the importing country. The model is a three-good two-country one analogous to a model of Itoh and Kiyono used to illustrate use of welfare-enhancing export subsidies. An import subsidy on a marginal good may not only enhance home welfare but may be a first-best optimal use of monopoly power by the home country. In addition, such a subsidy may imply negative tariff revenue for the home country from a set of optimum tariffs. The import subsidy is optimal under a certain pattern of comparative preference.

The paper also indicates a three-country two-good model somewhat isomorphic to the first case. A country-specific import subsidy may be optimal as a way of exploiting monopsony power when the same good is imported from two countries and monopsonistic price discrimination is possible. As with the first case, optimal policy in this model depends on the patterns of comparative preference of the three countries. It relies on different comparative preferences of the two foreign countries for the home import.

In addition to the use of the two simple comparative preference models, the paper develops more general models which illustrate the welfare-enhancing effects of import subsidies. Expressions are provided for the optimal subsidy levels as functions of model parameters.

The practical significance of import subsidies may be limited by political forces. However, the considerations calling for such subsidies may also imply that certain tariffs are all the worse in their effects on welfare. That is, if a positive subsidy is called for but a tariff is applied, it is probably more harmful than if a zero or positive tariff is optimal.

Appendix 1
Derivation of 3-Good 2-Country Model

Differentiated versions of equations (2) through (5) can be written:

$$\begin{bmatrix} M_2+s(1-s)P_2D_{22} & M_3+sP_2D_{23} & E_U+sP_2E_{2U} & 0 \\ M_2^* & M_3^* & 0 & E_U^* \\ (1-s)D_{22}-S_{22}^* & D_{23}-S_{23}^* & E_{2U} & E_{2U}^* \\ -(1-s)S_{32}+D_{32}^* & -S_{33}+D_{33}^* & E_{3U} & E_{3U}^* \end{bmatrix} \begin{bmatrix} dP_2 \\ dP_3 \\ dU \\ dU^* \end{bmatrix} = \begin{bmatrix} sP_2D_{22} \\ 0 \\ D_{22} \\ -S_{32} \end{bmatrix} P_2ds$$

where M_i and M_i^* are net imports of the home and foreign countries of good i. D_{ij} is the derivative of good i net demand for good i with respect to price j, and S_{ij} is a similar derivative of net supply, with derivatives for the foreign country indicated with an asterisk. E_U and E_{iU} are the derivative of expenditure with respect to real income and the derivative of good i demand with respect to real income. Similar foreign effects are denoted with an asterisk. Without loss of generality, we can let $E_U = E_U^* = 1$. Note that: $D_{ij} = E_{ij} - R_{ij}$ and $S_{ij} = R_{ij} - E_{ij}$.

The determinant of the matrix is:

$$\begin{vmatrix} M_2+s(1-s)P_2D_{22} & M_3+sP_2D_{23} & 1+sP_2E_{2U} & 0 \\ M_2^* & M_3^* & 0 & 1 \\ (1-s)D_{22}-S_{22}^* & D_{23}-S_{23}^* & E_{2U} & E_{2U}^* \\ -(1-s)S_{32}+D_{32}^* & -S_{33}+D_{33}^* & E_{3U} & E_{3U}^* \end{vmatrix}$$

Subtract M_2 times column 3 from column 1 and M_3 times column 3 from column 2. Subtract M_2^* times column 4 from column 1 and M_3^* times column 4 from column 2. Let a tilde over a derivative indicate an ***uncompensated*** excess demand or supply effect (those without tildes are compensated).

$$\begin{vmatrix} s(1-s)P_2\tilde{D}_{22} & sP_2\tilde{D}_{23} & 1+sP_2E_{2U} & 0 \\ 0 & 0 & 0 & 1 \\ (1-s)\tilde{D}_{22}-\tilde{S}_{22}^* & \tilde{D}_{23}-\tilde{S}_{23}^* & E_{2U} & E_{2U}^* \\ -(1-s)\tilde{S}_{32}+\tilde{D}_{32}^* & -\tilde{S}_{33}+\tilde{D}_{33}^* & E_{3U} & E_{3U}^* \end{vmatrix}$$

Subtract $1/sP_2$ times row 1 from row 3 and simplify.

$$\begin{vmatrix} s(1-s)P_2\tilde{D}_{22} & sP_2\tilde{D}_{23} & 1+sP_2E_{2U} \\ -\tilde{S}_{22}^* & -\tilde{S}_{23}^* & -1/sP_2 \\ -(1-s)\tilde{S}_{32}+\tilde{D}_{32}^* & -\tilde{S}_{33}+\tilde{D}_{33}^* & E_{3U} \end{vmatrix}$$

Simplify and convert to elasticities.

$$\begin{vmatrix} sM_2\tilde{\eta}_{22} & sM_2P_2\tilde{\eta}_{23}/P_3 & 1+sP_2E_{2U} \\ -M_2\tilde{\varepsilon}_{22}^*/P_2 & -M_2\tilde{\varepsilon}_{23}^*/P_3 & -1/sP_2 \\ M_3\tilde{\varepsilon}_{32}/P_2-M_2\tilde{\eta}_{32}^*/P_2 & M_3\tilde{\varepsilon}_{33}/P_3-M_3\tilde{\eta}_{33}^*/P_3 & E_{3U} \end{vmatrix}$$

Here η_{ij} and ε_{ij} are demand and supply elasticities of good i with respect to price j at home. A " ~ " over an elasticity indicates it is an uncompensated or Marshallian elasticity.

$$\Delta=\frac{M_2M_3^*}{P_2P_3}\left\{(\tilde{\eta}_{22}-\tilde{\varepsilon}_{22}^*)(\tilde{\eta}_{33}^*-\tilde{\varepsilon}_{33})-(\tilde{\eta}_{32}^*-\tilde{\varepsilon}_{32})(\tilde{\eta}_{23}-\tilde{\varepsilon}_{23}^*)\right\}+$$

$$s\left\{P_2E_{2U}\frac{M_2M_3^*}{P_2P_3}\left[\tilde{\varepsilon}_{23}^*(\tilde{\eta}_{32}^*-\tilde{\varepsilon}_{32})-\tilde{\varepsilon}_{22}^*(\tilde{\eta}_{33}^*-\tilde{\varepsilon}_{33})\right]+P_3E_{3U}\frac{(M_2)^2}{(P_3)^2}(\tilde{\eta}_{23}\tilde{\varepsilon}_{22}^*-\tilde{\eta}_{22}\tilde{\varepsilon}_{23}^*)\right\}$$

The effects of s on P_2 and P_3 around $s = 0$ can be determined using

Cramer's rule. First, solve for the following determinant for dP_2/ds.

$$\begin{vmatrix} sP_2D_{22} & M_3 + sP_2E_{2U} & 1+sP_2E_{2U} & 0 \\ 0 & M_3^* & 0 & 1 \\ D_{22} & D_{23} - S_{23}^* & E_{2U} & E_{2u}^* \\ -S_{32} & -S_{33} + D_{33}^* & E_{3U} & E_{3u}^* \end{vmatrix}$$

Subtract M_2 times column 3 from column 1, subtract M_3 times column 3 from column 2, and subtract M_3^* times column 4 from column 2.

$$\begin{vmatrix} sP_2\tilde{D}_{22} - M_2 & sP_2\tilde{D}_{23} & 1+sP_2E_{2U} & 0 \\ 0 & 0 & 0 & 1 \\ \tilde{D}_{22} & \tilde{D}_{23} - \tilde{S}_{23}^* & E_{2U} & E_{2U}^* \\ -\tilde{S}_{32} & -\tilde{S}_{33} + \tilde{D}_{33}^* & E_{3U} & E_{3U}^* \end{vmatrix}$$

Setting $s = 0$ and simplifying:

$$\begin{vmatrix} -M_2 & 0 & 1 \\ \tilde{D}_{22} & \tilde{D}_{23} - \tilde{S}_{23}^* & E_{2U} \\ -\tilde{S}_{32} & -\tilde{S}_{33} + \tilde{D}_{33}^* & E_{3U} \end{vmatrix}$$

Convert to elasticities.

$$\begin{vmatrix} -M_2 & 0 & 1 \\ M_2\tilde{\eta}_{22} / P_2 & M_2\tilde{\eta}_{23} / P_3 - M_2\tilde{\varepsilon}_{23}^* / P_2 & E_{2U} \\ M_3\tilde{\varepsilon}_{32} / P_2 & M_3\tilde{\varepsilon}_{33} / P_3 - M_3\tilde{\eta}_{33}^* / P_3 & E_{3U} \end{vmatrix}$$

Evaluating, and adding the factor P_2 (see the initial matrix equation above), gives:

$$\frac{dP_2 / P_2}{ds} = \frac{M_2M_3^*}{P_2P_3\Delta}\left\{\eta_{22}(\tilde{\eta}_{23}^* - \tilde{\varepsilon}_{33}) + \varepsilon_{32}(\tilde{\eta}_{23} - \tilde{\varepsilon}_{23}^*)\right\}$$

$$- \frac{M_2}{P_3\Delta}\left\{M_3^*E_{2U}(\tilde{\varepsilon}_{33} - \tilde{\eta}_{33}^*) + M_2E_{3U}(\tilde{\eta}_{23} - \tilde{\varepsilon}_{23}^*)\right\}$$

To compute the effect of s on P_3, follow a similar procedure.

$$\begin{vmatrix} M_2 + s(1-s)P_2D_{22} & sP_2D_{22} & 1+sP_2E_{2U} & 0 \\ M_2^* & 0 & 0 & 1 \\ (1-s)D_{22} - S_{22}^* & D_{22} & E_{2U} & E_{2U}^* \\ -(1-s)S_{32} + D_{32}^* & -S_{32} & E_{3U} & E_{3U}^* \end{vmatrix}$$

Subtract $(1 - s)$ times column 2 from column 1, subtract M_2 times column 3 from column 2, and add M_2 times column 4 to column 1.

$$\begin{vmatrix} M_2 & sP_2\tilde{D}_{22} - M_2 & 1 + sP_2E_{2U} & 0 \\ 0 & 0 & 0 & 1 \\ -\tilde{S}^*_{22} & \tilde{D}_{22} & E_{2U} & E^*_{2U} \\ \tilde{D}^*_{32} & -\tilde{S}_{32} & E_{3U} & E^*_{3U} \end{vmatrix}$$

Simplify, set $s = 0$, and convert to elasticities.

$$\begin{vmatrix} M_2 & -M_2 & 1 \\ -M_2\tilde{\varepsilon}^*_{22} / P_2 & M_2\tilde{\eta}_{22} / P_2 & E_{2U} \\ -M_3\tilde{\eta}^*_{32} / P_2 & M_3\tilde{\varepsilon}_{32} / P_2 & E_{3U} \end{vmatrix}$$

$$\frac{dP_3 / P_3}{ds} = \frac{M_2M_3^*}{P_2P_3\Delta}\left\{\tilde{\varepsilon}^*_{22}\tilde{\varepsilon}_{32} - \tilde{\eta}^*_{32}\tilde{\eta}_{22} + P_2M_2\left[\left(\tilde{\varepsilon}_{32} - \tilde{\eta}^*_{32}\right)E_{2U} / M_2 + \left(\tilde{\eta}_{22} - \tilde{\varepsilon}^*_{22}\right)E_{3U} / M_3^*\right]\right\}$$

The optimum value of s is found by setting $dU/ds = 0$. This condition implies that the following determinant must equal zero.

$$\begin{vmatrix} M_2 + s(1-s)P_2D_{22} & M_3 + sP_2D_{23} & sP_2D_{22} & 0 \\ M_2^* & M_3^* & 0 & 1 \\ (1-s)D_{22} - S^*_{22} & D_{23} - S^*_{23} & D_{22} & E^*_{2U} \\ -(1-s)S_{32} + \tilde{D}^*_{32} & -S_{33} + D^*_{33} & -S_{32} & E^*_{3U} \end{vmatrix}$$

To simplify, subtract (1 – s) times column 3 from column 1.

$$\begin{vmatrix} M_2 & M_3 + sP_2D_{23} & sP_2D_{22} & 0 \\ M_2^* & M_3^* & 0 & 1 \\ -\hat{S}^*_{22} & D_{23} - S^*_{23} & D_{22} & E^*_{2U} \\ D^*_{32} & -S_{33} + \tilde{D}^*_{33} & -S_{32} & E^*_{3U} \end{vmatrix}$$

Subtract M_2^* times column 4 from column 1 and M_3^* times column 4 from column 2. Note that using the Slutsky equations, the results in rows three and four are uncompensated price effects for the foreign country.

$$\begin{vmatrix} M_2 & M_3 + sP_2D_{23} & sP_2D_{22} & 0 \\ 0 & 0 & 0 & 1 \\ -\tilde{S}^*_{22} & D_{23} - \tilde{S}^*_{23} & D_{22} & E^*_{2U} \\ \tilde{D}^*_{32} & -S_{33} + \tilde{D}^*_{33} & -S_{32} & E^*_{3U} \end{vmatrix}$$

This can be simplified to:

$$\begin{vmatrix} M_2 & M_3+sP_2D_{23} & sP_2D_{22} \\ -\tilde{S}_{22}^{*} & D_{23}-\tilde{S}_{23}^{*} & D_{22} \\ \tilde{D}_{32}^{*} & -S_{33}+\tilde{D}_{33}^{*} & -S_{32} \end{vmatrix}$$

Convert to elasticities:

$$\begin{vmatrix} M_2 & M_3+sP_2M_2\eta_{23}/P_3 & sM_2\eta_{22}/(1-s) \\ -M_2\tilde{\varepsilon}_{22}^{*}/P_2 & (\eta_{23}-\tilde{\varepsilon}_{23}^{*})M_2/P_3 & M_2\eta_{22}/P_2(1-s) \\ -M_2\eta_{32}^{*}/P_2 & (\varepsilon_{33}-\tilde{\eta}_{33}^{*})M_3/P_3 & M_2\varepsilon_{32}/P_2(1-s) \end{vmatrix}$$

Divide through by factors where appropriate.

$$\begin{vmatrix} 1 & P_3M_3/P_2M_2+s\,\eta_{23} & s\,\eta_{22} \\ -\tilde{\varepsilon}_{32}^{*} & \eta_{23}-\tilde{\varepsilon}_{23}^{*} & \eta_{22} \\ -\tilde{\eta}_{32}^{*} & \varepsilon_{33}-\tilde{\eta}_{33}^{*} & \varepsilon_{32} \end{vmatrix}$$

Evaluating the determinant and setting it to zero yields:

$$s\left\{\eta_{22}\left[\tilde{\eta}_{32}^{*}(\eta_{23}-\tilde{\varepsilon}_{23}^{*})-\tilde{\varepsilon}_{22}^{*}(\varepsilon_{33}-\tilde{\eta}_{33}^{*})\right]-\eta_{23}\left[\eta_{22}\tilde{\eta}_{32}^{*}-\varepsilon_{32}\tilde{\varepsilon}_{22}^{*}\right]\right\}$$
$$+\varepsilon_{32}\left(\eta_{23}-\tilde{\varepsilon}_{23}^{*}\right)-\eta_{22}\left(\varepsilon_{33}-\tilde{\eta}_{33}^{*}\right)-\frac{P_3M_3}{P_2M_2}\left(\eta_{22}\tilde{\eta}_{32}^{*}-\tilde{\varepsilon}_{22}^{*}\varepsilon_{32}\right)$$

$$s=\frac{\eta_{22}\left(\varepsilon_{33}-\tilde{\eta}_{33}^{*}\right)-\varepsilon_{32}\left(\eta_{23}-\tilde{\varepsilon}_{23}^{*}\right)+\dfrac{P_3M_3^{*}}{P_2M_2}\left(\tilde{\varepsilon}_{22}^{*}\varepsilon_{32}-\eta_{22}\tilde{\eta}_{32}^{*}\right)}{\tilde{\eta}_{22}\left[\tilde{\eta}_{33}^{*}\tilde{\varepsilon}_{22}^{*}-\tilde{\eta}_{32}^{*}\tilde{\varepsilon}_{23}^{*}\right]-\tilde{\varepsilon}_{22}^{*}\left[\varepsilon_{33}\eta_{22}-\eta_{23}\varepsilon_{32}\right]}$$

Appendix 2
Derivation of 2-Good 3-Country Model

The system of equations (8) through (11) can be written:

$$\begin{bmatrix} M^B+\dfrac{1}{1-s}M^A+\dfrac{sP}{(1-s)^2}S_{22}^A & E_u & \dfrac{-sP}{1-s}E_{2U}^A & 0 \\ -M^A/(1-s) & 0 & E_U^A & 0 \\ -M^B & 0 & 0 & E_U^B \\ D_{22}-S_{22}^A/(1-s)-S_{22}^B & E_{2U} & E_{2U}^A & E_{2U}^B \end{bmatrix}\begin{bmatrix} dP \\ dU \\ dU^A \\ dU^B \end{bmatrix}$$

$$= \begin{bmatrix} -M^A - \frac{sP}{1-s} S_{22}^A \\ M^A \\ 0 \\ S_{22}^A \end{bmatrix} \frac{P}{(1-s)^2} ds$$

where M is total imports of the home country, M^A is its imports from A and M^B those from B, $D_{22} = E_{22} - R_{22}$, $S_{22}^A = R_{22}^A - E_{22}^A$, $S_{22}^B = R_{22}^B - E_{22}^B$, and subscripts indicate partial derivatives with "2" denoting the derivative with respect to the relative price of good 2. Without loss of generality, we can let $E_U = E_U^A = E_U^B = 1$. The determinant of the 4×4 matrix is:

$$\begin{bmatrix} M + \frac{s}{1-s} M^A + \frac{sP}{(1-s)^2} S_{22}^A & 1 & -\frac{sP}{1-s} E_{2U}^A & 0 \\ -M^A/(1-s) & 0 & 1 & 0 \\ -M^B & 0 & 0 & 1 \\ D_{22} - \frac{S_{22}^A}{1-s} - S_{22}^B & E_{2U} & E_{2U}^A & E_{2U}^B \end{bmatrix}$$

Subtract M times column 2 from column 1. Add $M^A/(1-s)$ times column 3 to column 1 and M^B times column 4 to column 1.

$$\begin{vmatrix} \frac{s}{1-s} M^A + \frac{sP}{(1-s)^2} \tilde{S}_{22}^A & 1 & -\frac{sP}{1-s} E_{2U}^A & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \tilde{D}_{22} - \frac{\tilde{S}_{22}^A}{1-s} - \tilde{S}_{22}^B & E_{2U} & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

Here the " ~ " over a derivative denotes an uncompensated effect, so that, for example: $\tilde{D}_{22} = E_{22} - R_{22} - ME_{2U}$, and $\tilde{S}_{22}^A = R_{22}^A - E_{22}^A - M^A E_{2U}^A$. For the next step, convert to elasticities. Let $\tilde{\eta}_{22} = P\tilde{D}_{22}/M$ be the uncompensated (Marshallian) elasticity of excess home demand, and $\tilde{\varepsilon}_{22}^B = P\,\tilde{S}_{22}^B/M^B$ and $\tilde{\varepsilon}_{22}^A = P\,\tilde{S}_{22}^A/(1-s)M^A$ be the uncompensated elasticities of excess supply for countries B and A.

$$\begin{vmatrix} \frac{s}{1-s} M^A (1 + \tilde{\varepsilon}_{22}^A) & 1 \\ (M\tilde{\eta}_{22} - M^A \tilde{\varepsilon}_{22}^A - M^B \tilde{\varepsilon}_{22}^B)/P & E_{2U} \end{vmatrix}$$

From this we can find the value of the determinant.

$$\Delta = \frac{1}{P}\left[M^A\tilde{\varepsilon}_{22}^A + M^B\tilde{\varepsilon}_{22}^B - M\,\tilde{\eta}_{22}\right] + M^A\frac{s}{1-s}E_{2U}(1+\tilde{\varepsilon}_{22}^A)$$

Next solve for effects of s on P around $s = 0$. To use Cramer's rule, we must evaluate the following determinant:

$$\begin{vmatrix} -M^A - \dfrac{sP}{(1-s)}S_{22}^A & 1 & -\dfrac{sP}{1-s}E_{2U}^A & 0 \\ M^A & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ S_{22}^A & E_{2U} & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

Subtract M^A times column 3 from column 1 and convert to elasticities.

$$\begin{vmatrix} -M^A\left(1+s\,\tilde{\varepsilon}_{22}^A\right) & 1 & -\dfrac{sP}{1-s}E_{2U}^A & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \tilde{\varepsilon}_{22}^A\dfrac{M^A}{P/(1-s)} & E_{2U} & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

From this, and noting the $P/(1-s)^2$ term from the initial equation of Appendix 2:

$$\frac{dP}{ds} = -\left[\frac{M^A}{\Delta}\right]\left\{\tilde{\varepsilon}_{22}^A + PE_{2U}\right\}$$

Here Δ, E_{2U}, and ε_{22}^A are positive, assuming the usual price effects. The overall effect is therefore negative. An increase in s from zero lowers the relative price of good 2 at home. It also lowers the price paid to country B. Note that $P^A = P/(1-s)$. Thus, around $s = 0$:

$$\frac{dP^A}{ds} = \left[\frac{dP}{ds} + P\right]$$

or

$$\frac{dP^A}{ds} = P\left\{\frac{M^B\tilde{\varepsilon}_{22}^B - M\,\tilde{\eta}_{22} - M^A PE_{2U}}{M^A\tilde{\varepsilon}_{22}^A + M^B\tilde{\varepsilon}_{22}^B - M\,\tilde{\eta}_{22}}\right\}$$

A careful look shows that $(dP_A/P_A)/ds$ is positive but in general less than 1.

Again using the matrix equation above, we can find the optimum subsidy by solving for dU/ds and setting it to zero. $dU/ds = 0$ implies that the following determinant is zero:

$$\begin{vmatrix} M^B + \frac{1}{1-s}M^A + \frac{sP}{(1-s)^2}S_{22}^A & -M^A - \frac{sP}{1-s}S_{22}^A & -\frac{sP}{1-s}E_{2U}^A & 0 \\ -M^A/(1-s) & M^A & 1 & 0 \\ -M^B & 0 & 0 & 1 \\ D_{22} - \frac{S_{22}^A}{1-s} - S_{22}^B & S_{22}^A & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

Add $1/(1-s)$ times column 2 to column 1.

$$\begin{vmatrix} M^B & -M^A - \frac{sP}{1-s}S_{22}^A & -\frac{sP}{1-s}E_{2U}^A & 0 \\ 0 & M^A & 1 & 0 \\ -M^B & 0 & 0 & 1 \\ D_{22} - S_{22}^B & S_{22}^A & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

Subtract M^A times column 3 from column 2 and add M^B times column 4 to column 1. The result is:

$$\begin{vmatrix} M^B & -M^A - \frac{sP}{1-s}\tilde{S}_{22}^A & -\frac{sP}{1-s}E_{2U}^A & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ D_{22} - \tilde{S}_{22}^B & \tilde{S}_{22}^A & E_{2U}^A & E_{2U}^B \end{vmatrix}$$

Simplify:

$$\begin{vmatrix} M^B & -M^A - \frac{sP}{1-s}\tilde{S}_{22}^A \\ D_{22} - \tilde{S}_{22}^B & \tilde{S}_{22}^A \end{vmatrix}$$

Divide column 1 by M^B and column 2 by M^A. Multiply row two by P. Divide and multiply D_{22} by M. Divide and multiply the bottom right element by $(1-s)$. The result is:

$$\begin{vmatrix} 1 & -1 - \frac{sP}{1-s}\frac{\tilde{S}_{22}^A}{M^A} \\ \frac{PD_{22}}{M}\frac{M}{M^B} - \frac{P\tilde{S}_{22}^B}{M^B} & (1-s)\frac{P\tilde{S}_{22}^A}{(1-s)M^A} \end{vmatrix}$$

Convert to elasticity form.

$$\begin{vmatrix} 1 & -1-s\,\tilde{\varepsilon}_{22}^{A} \\ \eta_{22}\dfrac{M}{M^B}-\tilde{\varepsilon}_{22}^{B} & (1-s)\tilde{\varepsilon}_{22}^{A} \end{vmatrix}$$

This yields the expression:

$$s=\frac{\tilde{\varepsilon}_{22}^{A}-\tilde{\varepsilon}_{22}^{B}+\eta_{22}M/M^{B}}{\tilde{\varepsilon}_{22}^{A}(1+\tilde{\varepsilon}_{22}^{B}-\eta_{22}M/M^{B})}$$

References

Abbott, Philip C., Philip L. Paarlberg, and Jerry A. Sharples [1987], "Targeted Agricultural Export Subsidies and Social Welfare," *American Journal of Agricultural Economics* 69; pp. 723-732.

Brander, James, and Barbara Spencer [1984], "Trade Warfare: Tariffs and Cartels," *Journal of International Economics* 16; pp. 227-242.

Brander, James, and Barbara Spencer [1985], "Export Subsidies and International Market Share Rivalry," *Journal of International Economics* 18; pp. 83-100.

Bond, Eric [1990], "The Optimal Tariff Structure in Higher Dimensions," *International Economic Review* 31; pp. 103-116.

Corden, Max [1967], "Monopoly, Tariffs and Subsidies," *Economica* 34; pp. 50-58.

Dutton, John C. [1990], "Targeted Export Subsidies as an Exercise of Monopoly Power," *Canadian Journal of Economics* 23; pp. 705-710.

Feenstra, Robert C. [1986], "Trade Policy with Several Goods and 'Market Linkages," *Journal of International Economics* 20; pp. 249-267.

Gardner, Grant W., and Kent P. Kimbrough [1990], "The Economics of Country-Specific Tariffs," *International Economic Review* 31; pp. 575-588.

Graaf, J. [1949-50], "On Optimal Tariff Structures," *Review of Economic Studies* 17; pp. 47-59.

Gruen, F. H., and W. M. Corden [1970], "A Tariff that Worsens the Terms of Trade," in I. A. McDougall and R. H. Snape, eds., *Studies in Interna-*

tional Economics (Amsterdam: North-Holland); pp. 55-58.

Horwell, David J., and Ivor F. Pearce [1970], "A Look at the Structure of Optimal Tariff Rates," *International Economic Review* 11; pp. 147-161.

Itoh, Motoshige, and Kazuharu Kiyono [1987], "Welfare-Enhancing Export Subsidies," *Journal of Political Economy* 95; pp. 115-137.

Jones, Ronald W. [1987], "Trade Taxes and Subsidies with Imperfect Competition," *Economic Letters* 23; pp. 375-379.

Jones, Ronald W. [1977], "'Two-ness' in Trade Theory: Costs and Benefits," *Special Papers in International Economics,* No. 12, Princeton University.

Jones, Ronald W. [1961], "Comparative Advantage and the Theory of Tariffs: A Multi-Country, Multi-Commodity Model," *Review of Economic Studies,* June; pp. 161-175.

Katrak, Homi [1977], "Multi-national Monopolies and Commercial Policy," *Oxford Economic Papers* 29; pp. 283-291.

Kemp, Murray C. [1967], "Notes on the Theory of Optimal Tariffs," *Economic Record* 43; pp. 395-403.

Tower, Edward [1983], "On the Best Use of Trade Controls in the Presence of Foreign Market Power," *Journal of International Economics* 15; pp. 349-365.

Young, Leslie [1991], "Optimal Tariffs: A Generalization," *International Economic Review* 32; pp. 341-370.