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The Heckscher-Ohlin Model with Endogenous Sector-Specific Capital

Jürgen Meckl*
University of Konstanz

Abstract

This paper considers the long-run properties of a dynamic specific-factors model with endogenous capital stocks providing a dynamic foundation for the Heckscher-Ohlin model. The long-run equilibrium is fully determined by a static Heckscher-Ohlin model in primary factors although capital rentals may not be equal between sectors. All theorems of the static model carry over to the present model's steady state as long as countries diversify. Primary-factor endowments determine long-run comparative advantage and long-run capital stocks. Capital endowments are completely irrelevant for the determination of the long-run trade pattern.

I. Introduction

The extreme assumptions on intersectoral capital mobility characteristic of the specific-factors (SF) and the Heckscher-Ohlin (HO) model usually are justified with the different time horizons of these models. Empirical evidence supports the interpretation of the SF model as a short-run model ⟨*cf.* Grossman and Levinson [1989]⟩. As pointed out by Mayer [1974] and Jones [1975], short-run intersectoral differences in rental rates of capital enforce capital real-

* University of Konstanz/SFB178, PO-Box 5560, D-78434 Konstanz, Germany. I am grateful to Max Albert and two referees for valuable comments and suggestions. Financial support from the "Deutsche Forschungs Gemeinschaft" is gratefully acknowledged.

location processes, reducing rental-rate differences over time and generating a long-run equilibrium with the characteristics of the HO model. This adjustment process was first analyzed by Neary [1978, 1982], but with an exogenously given, and hence arbitrary, speed of adjustment. The first microeconomic foundation of such a process of time-consuming adjustment was provided by Mussa [1978]. He introduced investment theory to analyze the capital-reallocation process. Time-consuming adjustment is the consequence of increasing marginal costs of adjustment (strictly convex adjustment costs) arising from decreasing returns in a special sector reallocating capital. This approach, however, is subject to Neary's [1978] critique that in reality investment is the process of building up sector-specific capital stocks rather than transferring physical capital units from one sector to the other. In this view reallocation of capital between sectors is the result of different rates of sectoral net investment and involves no physical mobility of capital. It is impossible to discriminate between reallocation and accumulation of capital: reallocation is only a side effect of different sectoral rates of accumulation. Looking at capital allocation this way, the HO model's requirement of a fixed total stock of capital will never be satisfied; the HO model seems to be inappropriate even for long-run analysis and "the usual long-run Heckscher-Ohlin-Samuelson predictions will not follow in general" ⟨Neary [1978], p. 508⟩.

Taking account of Neary's critique, Murphy [1988] developed a dynamic model of a small open economy with endogenous sector-specific capital stocks. He used the adjustment-cost approach of Hayashi [1982] and Abel and Blanchard [1983] in a two-sector framework with one single primary factor (labor). Due to this specification of the adjustment cost function long-run capital rentals are fixed. As a consequence of the nonsubstitution theorem the steady state is characterized by specialization in general. In order to overcome this implausible result, Murphy endogenizes commodity prices by assuming one good to be non-traded. Unfortunately a simple extension of Murphy's framework to a two-country model with two traded goods yields rather unsatisfying results concerning the validity of the HO model: Assuming identical preferences in both countries, as it is usually done in the HO model, autarky prices will be identical in the long-run and no trade will take place; as long-run capital rentals must be identical between sectors *and* countries in this case, each country will accumulate capital up to the point where per-head capital endow-

ments coincide in the steady state. On the other hand, if preferences differ internationally at least one country must specialize in the long-run free-trade equilibrium ⟨*cf.* Stiglitz [1970]⟩. Hence a simple generalization of Murphy's model cannot be used to prove whether the core propositions of neoclassical trade theory – the Stolper-Samuelson, Rybczynski, factor-price-equalization, and HO theorem – carry over to the long-run equilibrium of a model with endogenous sector-specific capital stocks.

The present paper uses an adjustment-cost approach in a dynamic model with two primary factors, thus preventing specialization due to the non-substitution theorem in a small open economy. Primary factors (say, labor of different qualities) are perfectly mobile between sectors and in perfectly inelastic supply. We will analyze, under what conditions the theorems derived from the static HO model hold in the long-run equilibrium. It will be shown that the long-run equilibrium is determined solely by the endowments of primary factors. Capital as a produced factor is completely irrelevant in that respect; the steady-state analysis of this dynamic three-factor, two-sector model can be reduced to the analysis of an ordinary static two sector, two-primary-factors HO model. In particular capital has no influence on the trade pattern in the long run.

The paper proceeds as follows. The basic dynamic HO model of a small open economy is developed in section II. Section III presents comparative-static results and shows that the Stolper-Samuelson and the Rybczynski theorem hold in the long run. Section IV turns to the two country model and discusses long-run factor-price equalization. Section V shows that the HO theorem holds for balanced trade, whereas the the factor-content version of that theorem, known as the Heckscher-Ohlin-Vanek (HOV) theorem holds only in certain cases. Section VI gives some concluding remarks.

II. The Model

A. Firm Behavior

The core of the present two-sector model is a dynamic model of a competitive firm representing one sector of the economy. The firm is a price taker on all relevant markets and operates under strictly convex costs of adjustment. This model of the firm is well-known from the literature ⟨*cf.* Lucas [1967],

Treadway [1969]). Actually we apply the adjustment-cost specification of Abel and Blanchard [1983].[1] At each point in time t the firm tries to maximize the present value of expected future cash flows over an infinite horizon. Assuming rational expectations a firm solves the following problem:[2]

$$\max_{I(s),L(s)} \int_t^{\infty} \left\{ p \cdot x(s) - \mathbf{w}^T(s) \cdot \mathbf{L}(s) - p_I \cdot I(s) \cdot \left[1 + h\left(\frac{I(s)}{K(s)} \right) \right] \right\} \cdot e^{i \cdot (t-s)}, \quad (1)$$

$$\text{s.t.} \begin{cases} x(s) = F(K(s),\ \mathbf{L}(s)) \\ \dot{K}(s) = I(s) - \delta \cdot K(s) \\ K(t) = K(0) + \int_0^t \dot{K}(s) ds. \end{cases}$$

i is the interest rate at which the firm can borrow or lend financial capital; the firm's production of output x can be described by a linearly homogeneous production function F with capital K and a vector of primary factors $\mathbf{L} = (L_1, L_2)^T$ as arguments, where the superscript T stands for transpose; $\mathbf{w}$ is the vector of primary factor prices. The price of the investment good is denoted by p_I, and I stands for gross investment. $h(I/K)$ gives the amount of investment goods required for installation of one unit of investment; this function has the follow-

1. An alternative formulation would make installation costs a function of net investment (net-investment approach). Following Sargent [1987] in assuming that the derivative of this installation cost function equals zero if net investment is zero, gives equivalent results to that derived in the paper. However, Kemp and Wan [1974] emphasize that *realistic* adjustment costs involve a positive marginal cost of adjustment even at a zero rate of net investment. With this specification the net-investment approach generates a range of values of sectoral capital stocks surrounding the unique long-run equilibrium of the Sargent specification, for which the economy is in its steady state in the sense that the incentives for capital-stock adjustments are insufficient to overcome costs. The comparative-static results derived in this paper carry over to the Kemp-Wan specification only if the exogenous shocks are of sufficient magnitude, *i.e.* such a model will be characterised by hysteresis. On the other hand, Kemp and Wan assert that their specification of the adjustment-cost function is realistic. However, it is an empirical problem to decide which specification of the adjustment-cost function is more realistic. The formulation used in this paper is quite well supported by empirical studies (*cf.* Sensenbrenner [1991])
2. Indices pertaining to the sector are dropped at this stage.

ing properties:

$$h(0) = 0,\ h'(.) > 0,\ 2 \cdot h'(.) + (I/K) \cdot h''(.) > 0$$

ensuring that total installation costs $I \cdot h(.)$ are nonnegative and strictly convex, with a minimum value of zero if gross investment equals zero. δ is a constant rate of depreciation. The current-value Hamiltonian for the firm's maximization problem is

$$H(K, I, q) = p \cdot F(K, \mathbf{L}) - \mathbf{w}^T \mathbf{L} - p_I \cdot I \cdot \left[1 + h\left(\frac{I}{K}\right)\right] + q \cdot (I - \delta \cdot K), \quad (2)$$

where q is the costable variable associated with the dynamic constraint. The necessary conditions for the choice of L and I maximizing the Hamiltonian are

$$p \cdot \frac{\partial F(K, \mathbf{L})}{\partial L_j} = w_j,\ j = 1,\ 2 \quad (3)$$

and

$$p_I \cdot \left[1 + h\left(\frac{I}{K}\right) + \frac{I}{K} \cdot h'\left(\frac{I}{K}\right)\right] = q. \quad (4)$$

Eq.(4) can be solved for optimal investment I^{opt}

$$I^{opt}(q, p_I, K) = g\left(\frac{q}{P_I}\right) \cdot K, \quad (5)$$

where $g(1) = 0, g'(.) > 0$. With an optimal choice of L we may write $p \cdot x - \mathbf{w}^T \mathbf{L}$ as $r \cdot K$, where r is the marginal value product (rental rate) of capital determined by the prices of primary factors. Inserting the solution of eq.(4) into the Hamiltonian yields the maximized Hamiltonian $H^{max}(K, q)$ as

$$H^{\max}(K, q) = r \cdot K - p_I \cdot g\left(\frac{q}{P_I}\right) \cdot K \cdot \left[1 + h\left(g\left(\frac{q}{p_I}\right)\right)\right] + q \cdot K \cdot \left[g\left(\frac{q}{p_I}\right) - \delta\right]. \quad (6)$$

From H^{max} we get the necessary condition for the behavior of q:

$$\dot{q} = (i + \delta) \cdot q - r - p_I \cdot g\left(\frac{q}{P_I}\right)^2 \cdot h'\left(g\left(\frac{q}{p_I}\right)\right). \quad (7)$$

Solving (7) for $q(t)$ gives

$$q(t) = a \cdot e^{i \cdot t} + \int_t^{\infty} \left[r + p_I \cdot g\left(\frac{q}{P_I}\right)^2 \cdot h'\left(g\left(\frac{q}{p_I}\right)\right)\right] . e^{i \cdot (t-s)} ds, \tag{8}$$

where a is a constant yet to be determined. We now use the following theorem to simplify eq.(8) ⟨*cf.* Feichtinger and Hartl [1986], pp. 42-43⟩. If H^{max} is concave in K, and the solution $q^{opt}(t) \cdot K^{opt}(t)$ satisfies the first-order conditions, and the following transversality condition

$$\lim_{t \to \infty} e^{i \cdot t} \cdot q^{opt}(t) \cdot [K(t) - K^{opt}(t)] \geq 0 \tag{9}$$

holds for all admissible $K(t)$, then $I(t)$ is optimal. Now H^{max} is linear and thus concave in K. The shadow price of capital q is nonnegative since an exogenous increase in the firm's capital stock can never reduce the present value of net revenues. K must be nonnegative for all t, too. Furthermore, we know that on a stable path $e^{-i \cdot t} \cdot q^{opt}(t) \cdot K^{opt}(t)$ goes to zero. Thus the stable solution fulfills eq.(9): stability is sufficient for optimality. Stability requires that in eq. (8) the constant a must be zero, and the integral must converge. Given stability, we may write eq. (8) as

$$q(t) = \int_t^{\infty} \left[r + p_I \cdot g\left(\frac{q}{p_I}\right)^2 \cdot h'\left(g\left(\frac{q}{p_I}\right)\right)\right] . e^{i \cdot (t-s)} ds. \tag{10}$$

B. The Two-Sector Model

In the following, we consider a two-sector economy producing an investment good x_I, and a consumption good x_c. The market-clearing conditions for primary factors are

$$\mathbf{L}_I + \mathbf{L}_C = \mathbf{L}. \tag{11}$$

Sectoral demands for primary factors ($\mathbf{L}_j$) at any given moment depend on the current stock of capital and fulfill the usual optimality condition

$$w_k = p_I \cdot \partial F_j(K_j, \mathbf{L}_j) / \partial L_{jk} \qquad j = I, C;\ k = 1, 2. \tag{12}$$

We set $p_I = 1$ and $p := p_c/p_I$. We get rid of all equations pertaining to reallocation of primary factors by making use of the gross domestic product (GDP) function of the *SF* model, which is defined by

$$y(p, \mathbf{L}, \mathbf{K}) := \max_{x_j, a_{jk}} \{x_I + p \cdot x_c\} \tag{13}$$

$$s.t. \begin{cases} \sum_{j=I,C} a_{jk} \cdot x_j = L_k, & \forall k \\ a_{jk} \cdot x_j = K_j, & \forall j \\ F_j(a_{jk}, a_{j1}, a_{j2}) \geq 1, & \forall j, \end{cases}$$

where **K** is the vector of capital stocks.[3] The GDP function describes the primary-factor allocation at each point in time. As long as the ratio of primary-factor inputs differ between sectors y is strictly concave in (K_I, K_c)[4] and yields the rental rates of capital as $\partial y/\partial K_j = r_j$, if $K_j > 0$. As long as both capital stocks are positive, y is strictly convex in p; if one capital stock is zero, y is convex in p. Strict concavity of the maximand in eq.(13) in L_j ensures existence and uniqueness of the temporary equilibrium.

C. The Long-Run Equilibrium

Over time capital stocks adjust according to the dynamic system

$$\begin{aligned} \dot{K}_j &= g_j(q_j) \cdot K_j - \delta_j \cdot K_j & j = I,\ C \\ \dot{q}_j &= i \cdot g_j - r_j - g_j(q_j)^2 \cdot h_j'(g_j(q_j)) & j = I,\ C \end{aligned} \tag{14}$$

and generate a sequence of temporary equilibria. The long-run equilibrium of the model is defined by $\dot{K}_j = \dot{q}_j = 0$.[5] Note that in the steady state both q_j are equal to $1 + h_j(\delta_j) + \delta_j \cdot h_i'(\delta_j)$ and capital rentals are given by $r_j = (i + \delta_j) \cdot [1 +$

3. This definition of the SF model's GDP function is slightly different from its standard definition ⟨cf. Dixit and Norman [1980], p. 39⟩ since specialization is possible.
4. As we will see (cf. figure 1) there is only one price vector that generates identical ratios of primary-factor inputs in both sectors. This case hao the probability zero.
5. The proof of stability of eq.(14) is straightforward. A complete analysis of the model's dynamic behavior is available from the author on request.

$h_j(\delta_j)] + i \cdot h_i'(\delta_j)$. Only in the special case of identical installation cost functions and identical depreciation rates in both sectors capital rentals equalize in the long run.

The steady state of the model can be described by the following maximization problem

$$G(p, \mathbf{L}, i) := \max_{K_I, K_c} \left\{ y(p, \mathbf{L}, \mathbf{K}) - \sum_{j=I,C} [(i+\delta_j)\cdot[1+h_j(\delta_j)] + i \cdot h_j'(\delta_j)] \cdot K_j \right\} \quad (15)$$
$$s.t. \quad K_j \geq 0 \quad \forall j.$$

This problem states that in the long run the income derived from primary factors is maximized: The term subtracted from GDP is simply the sum of the replacement costs of the long-run capital stock, $\sum \delta_j \cdot [1 + h(\delta_j)] \cdot K_j$, and the opportunity costs of holding the total capital stock, $i \cdot \sum [1 + h(\delta_j) + h'(\delta_j)] \cdot K_j$, both terms add up to $\sum r_j K_j$. The maximand in eq. (15) is almost always strictly concave in (K_I, K_c). Thus the solution of eq. (15), and hence the steady state, exists and is almost always unique. First-order conditions of eq. (15) are given by:

$$r_j = \frac{\partial y}{\partial K_j} \leq (i + \delta_j) \cdot [1 + h_j(\delta_j)] + i \cdot h_j'(\delta_j), \; K_j \geq 0 \quad j = I, C. \quad (16)$$

with complementary slackness. For positive capital stocks these first-order conditions are identical to the steady-state conditions of the dynamic system eq. (14).

Given the steady-state value of their capital rental, firms will choose their steady-state capital stocks K_j^* such that $\phi_j(i, \delta_j) := (i + \delta_j) \cdot [1 + h_j(\delta_j)] + i \cdot h_j'(\delta_j)$ equals the respective marginal value product of capital $p_j \cdot \partial F_j(K_j^*, \mathbf{L}_j^*) / \partial K_j$. These equations can be solved for K_j^* as a function of $(p_j, \mathbf{L}_j^*, \phi_j(i, \delta_j))$: $K_j^* = f_j(p_j, \mathbf{L}_j^*, \phi_j(i, \delta_j))$, where the function f_j is monotonically increasing and homogeneous of degree one in $\mathbf{L}_j^*$. Steady-state output can then be described by

$$x_j^* = F_j^*(p_j, \mathbf{L}_j^*, (i, \delta_j)) = F_j(\mathbf{L}_j^*, f_j(p_j, \mathbf{L}_j^*, \delta_j(i, \delta_j))), \quad (17)$$

where the composite function $F_j^*(.)$ is also linearly homogeneous in $\mathbf{L}_j^*$. This ensures constant returns to scale in the long run.

Long-run primary-factor prices can be derived from the zero-profit conditions. These conditions require that, given the vector of long-run capital rentals $\mathbf{r}^*$, the unit costs $c_j(r_j^*, \mathbf{w}^*)$ must be equal to p_j. Since the unit cost functions are homogeneous of degree one we can write the zero-profit conditions as

$$\mathbf{A}(\mathbf{w}^*) \cdot \mathbf{w}^* = \mathbf{p}, \tag{18}$$

where $\mathbf{A}(\mathbf{w}^*) := Dc(\mathbf{w}^*)$ is the 2x4 matrix of factor-input coefficients defined by the derivative of the unit-cost function $c_j(r_j^*, \mathbf{w}^*)$ with respect to factor prices. The input-coefficient matrix $\mathbf{A}$ is partitioned as follows:

$$\mathbf{A}(\mathbf{w}^*) \,| = \left(\begin{array}{cc|cc} a_{IK} & 0 & a_{I1} & a_{I2} \\ 0 & a_{CK} & a_{C1} & a_{C2} \end{array}\right) =: (\mathbf{A}_K, \mathbf{A}_p), \tag{19}$$

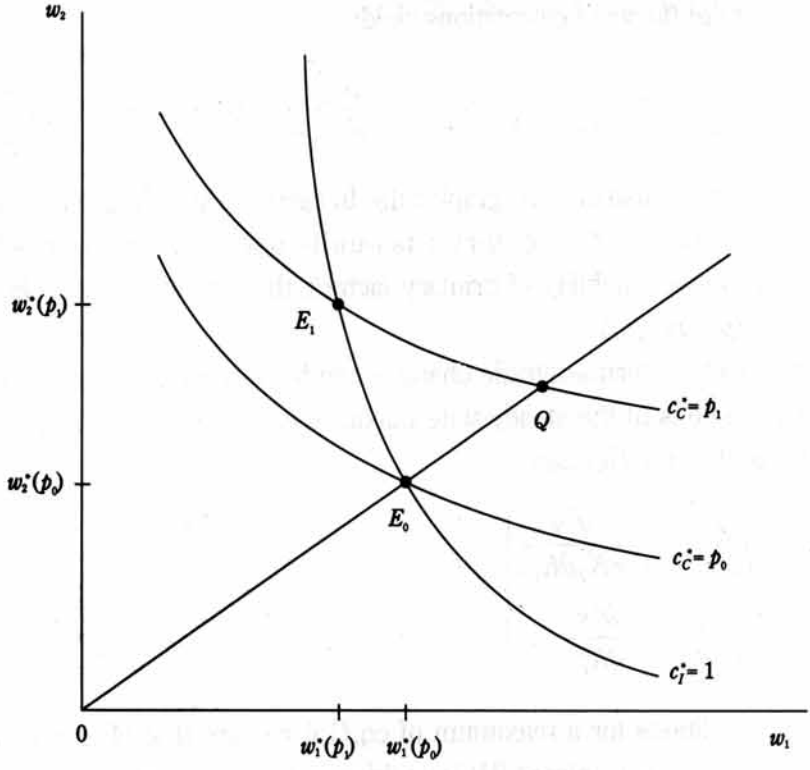
where $\mathbf{A}_K$ is the matrix of capital-input coefficients and $\mathbf{A}_p$ is the matrix of primary-factor-input coefficients. We will make use of this partition in order to simplify the algebra.

Determination of long-run primary-factor prices can be illustrated in the (w_1, w_2)-space, as shown in figure 1. Given $\mathbf{r}^*$, long-run unit cost functions are strictly concave in (w_1, w_2). Steady-state iso-unit-cost curves are strictly convex toward the origin in the (w_1, w_2)-space. The only difference to the unit-cost curve analysis of the static HO model is that the level of the iso-unit cost curves depends on capital rentals, and that unit cost functions are no longer homogeneous in (w_1, w_2) in general.[6] Long-run iso-unit cost curves will intersect exactly once, if there are no reversals in the ratio of primary-factor inputs L_{j2}/L_{j1} for all primary-factor-prices. According to Shepard's lemma the factor intensity L_{j2}/L_{j1} is given by the respective derivatives of the unit-cost function. This requires that the primary-factor-input-vector is perpendicular to the support line of the unit-cost curve. In figure 1 it is assumed that sector I is factor-1 intensive relative to sector C. With $p = p_0$, steady-state prices of primary factors are given by $(w_1^*(p_0), w_2^*(p_0))$.

The steady-state factor-market clearing conditions can be written as

6. Only in the case of Cobb-Douglas production functions unit-cost functions arc homogeneous of the degree m < 1 in (w_1, w_2).

Figure 1
Determination of Steady-State Primary-Factor Pries



$$\mathbf{A}^T(\mathbf{w}^*) \cdot \mathbf{x} = \mathbf{V}, \tag{20}$$

where $\mathbf{V} = (K_I^*, K_C^*, L_1, L_2)^T$. These four equations determine the unknowns $\mathbf{K}^*$ and $\mathbf{x}^*$ as linear functions of the two primary-factor endowments. Steady-state outputs are fully determined by the third and fourth equation of (20);[7] the remaining two equations in eq.(20) serve to determine steady-state capital stocks $\mathbf{K}^*$ of the economy.

III. Comparative Statics of the Long-Run Equilibrium

We consider the effects of a change in the terms of trade p on the steady state. The first-order condition eq.(16) of the steady-state maximization prob-

7. Note that $\mathbf{A}_p(.)$ gives the *direct* primary-factor-input coefficients. In contrast to the time phased models (see below) no aggregation is involved.

lem show that long-run capital rentals do not react. The validity of the Stolper-Samuelson theorem and the magnification effect can be proved by differentiating eq.(18); straightforward calculations yield:

$$\frac{\partial w_1}{\partial p} = -\frac{a_{I2}}{|\mathbf{A}_p|} < 0; \frac{\partial w_2}{\partial p} = -\frac{a_{I2}}{|\mathbf{A}_p|} > 0; \frac{\partial w_1}{\partial p} \cdot \frac{p}{w_2} > \frac{a_{I1}}{a_{I1} - a_{I1} \cdot / a_{C2}} > 1. \quad (21)$$

This result can be also derived graphically. In figure 1 the increase in terms-of-trade shifts sector C's long-run iso-unit-cost curve outwards[8] to $c_C^* = p_1$. With perfect mobility of primary factors, the new steady-state factor prices are $(w_1^*(p_1), w_2^*(p_1))$.

Quantity effects of terms-of-trade changes can be derived by using the second-order conditions of the steady-state maximization problem eq.(15); these are summarized by the Hessian

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 y}{\partial K_I^2} & \frac{\partial^2 y}{\partial K_I \partial K_C} \\ \frac{\partial^2 y}{\partial K_C \partial K_I} & \frac{\partial^2 y}{\partial K_C^2} \end{pmatrix} \quad (22)$$

Sufficient conditions for a maximum of eq.(15) require that $\mathbf{H}$ is negative definite, *i.e.* that the determinant $|\mathbf{H}|$ is positive and the elements of its principal diagonal are negative. The latter is straightforward. The determinant of $\mathbf{H}$ is almost always positive since $G(p, \mathbf{L}, i)$ is strictly concave in capital stocks. Differentiation of the first-order conditions eq.(16) and exploiting sign patterns yields

$$\frac{\partial K_I^*}{\partial p} < 0, \quad \frac{\partial K_C^*}{\partial p} > 0. \quad (23)$$

Given these steady-state capital-stocks reactions, primary factors are reallocated as follows. Since K_I^* is a function of $\mathbf{L}_I$, the employment of at least one primary factor must fall in sector I. Furthermore, we derive from the analysis of the unit-cost curves that the factor-price ratio w_2/w_1 determines the input ratio of primary factors in both industries. Figure 1 illustrates this point. Comparing

8. Note that the iso unit-cost curve is not shifted by a common factor v along every ray through the origin since the unit-cost function is not homogeneous in (w_1, w_2).

the old steady state at E_0 to the new one at E_1, it is obvious that the increase in w_2/w_1 generates a fall in factor intensities L_{j2}/L_{j1}, in both sectors. Full employment of primary factors can be ensured only if sector *I* employs less of both factors. An increase in the price of the consumption good increases (decreases) steady-state employment of all factors in sector C (sector *I*).

The proof of the Rybczynski theorem, and the determination of quantity adjustments caused by an exogenous change in primary-factor endowments proceed along the same line. Instead delving into this, one parallel to the literature merits comment at this point, the validity of the Stolper-Samuelson (and Rybczynski) theorem for steady states has been also shown in the neo-Ricardian time-phased models allowing for heterogeneous capital goods. For example Kemp [1973] proved that the Stolper-Samuelson theorem holds in the steady-state of a discrete-time model with two primary factors and heterogeneous capital. Kemp treats capital as a bundle of intermediate goods attracting interest payments. Although the sign of factor-price adjustments are determined by the matrix of direct primary-factor input coefficients, the actual change in factor prices depend on the matrix of *true* primary-factor input coefficients made up by the sum of direct primary-factor input coefficients aggregated in the capital-input coefficient. Kemp has to account for indirect coefficients since changes in product prices alter the cost of intermediate inputs and thus change the composition of capital. Treating capital as a stock (as it is done in the present approach) rather than a flow of intermediate goods renders changes in relative prices unnecessary for the composition of capital and the direct input coefficients are sufficient to determine Stolper-Samuelson and Rybczynski results. The same holds in Kemp's model if one assumes that capital is made up by only one intermediate good.

IV. Long-Run Factor-Price Equalization

In order to complete the proof of our proposition that the steady state of the present model is fully equivalent to the static HO model with two primary factors, we must proof the validity of the factor-price-equalization and the HO theorem. That, however, requires endogenization of goods prices and the interest rate and thus analysis of the demand side. In the following, consumer behavior is not explicitly derived from a consumer's optimization problem; only long-run

equilibrium conditions will be used. We consider both the case of optimal saving and of non-optimal saving. The latter, however, cannot be the optimal result of a fully intertemporal optimization behavior of the consumer; it is either an optimal result of an intertemporal consumer optimization or a non-optimal result of an intertemporal optimization behavior. As in the static HO model we assume preferences to be homothetic and identical.

Following modern analysis of factor-price equalization within the static two-by-two HO model, we start with the long-run equilibrium of a fully integrated world economy <cf. Dixit and Norman [1980], pp. 108-114>. The integrated equilibrium is a hypothetical construct where all goods as well as all factors are assumed to be perfectly mobile internationally. Thus we in fact consider a closed economy. Having solved for the integrated equilibrium we then ask under what conditions that equilibrium is replicated by a two-country equilibrium where at least primary-factors are completely immobile internationally. In the case of non-optimal saving, long-run equilibrium conditions of the production sector eqs. (18) and (20) are completed by the conditions of commodity-market clearing eqs.(24),(25), and financial-market clearing eq.(26), and the determination of long-run capital rentals eq.(27):

$$x_c^* = [1 - s(p^*, i^*)] \cdot \frac{x_I^* + p \cdot x_C^*}{p^*} \tag{24}$$

$$x_I^* = \sum_{j=I,C} \delta_j \cdot K_j^* \tag{25}$$

$$\sum_{j=I,C} \delta_j \cdot K_j^* = s(p^*, i^*) \cdot (x_I^* + p^* \cdot x_C^*) \tag{26}$$

$$r_j^* = i^* + \delta_j, \tag{27}$$

where $s(p, i)$ is the rate of saving. With homothetic preferences, s does not depend on income. It is well known from the optimal growth literature that in the case of optimal saving the steady-state interest rate must equal the rate of time preference ρ of the representative consumer:

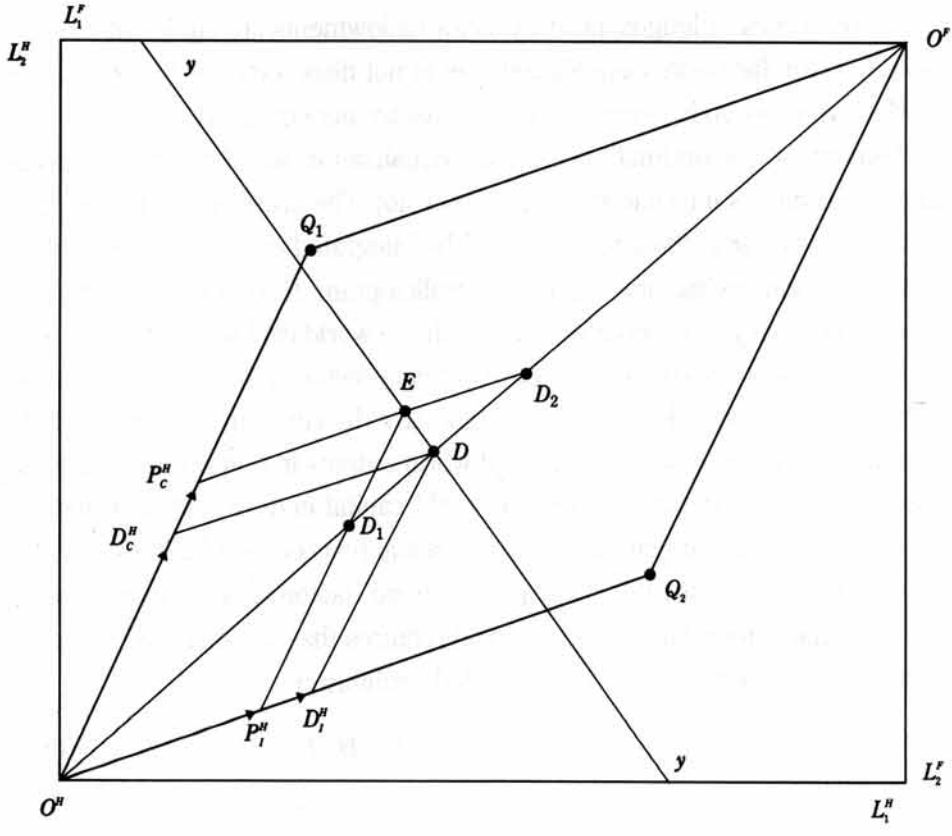
$$i^* = \rho. \tag{28}$$

With optimal saving the above system is not overdetermined since the rate

of saving s will cease to be a function of p and i and adjust to ensure equilibrium. Thus we have 12 equations in 11 unknowns ($\mathbf{w}, \mathbf{r}, \mathbf{x}, p, \mathbf{K}, i, s$) in the case of optimal saving, and 11 equations in 10 unknowns otherwise; since Walras' Law makes one equation redundant we have a determinate system. Let ($\tilde{\mathbf{w}}, \tilde{\mathbf{r}}, \tilde{\mathbf{x}}, \tilde{p}, \tilde{\mathbf{K}}, \tilde{i}, \tilde{s}$) be the steady-state solution of the integrated equilibrium with optimal saving. The corresponding primary-factor input-coefficient matrix then is $\mathbf{A}_p(\tilde{\mathbf{w}})$. Each row of that matrix gives the unit primary-factor requirements for the respective sector in the integrated equilibrium. As in the static two-by-two HO model, the vector sum of total equilibrium factor requirements defines a parallelogram $O^H Q_1 O^F Q_2$, illustrated in figure 2, in the space representing given world endowments of primary factors (endowment box).

Figure 2

Equalization of Primary-Factor Prices and Primary-Factor Content of Trade



Having found the solution of the integrated equilibrium, the factor-price-equalization problem can be reformulated as follows. Does any split of world output $\tilde{\mathbf{x}}$ into non-negative national outputs clear national factor markets at identical interest rates on international or national financial markets? In the case of optimal saving the answer is straightforward. Identical preferences in both countries imply identical rates of time preference and hence equalization of long-run interest rates, irrespective of whether financial capital is internationally mobile or immobile. Given factor prices of the integrated equilibrium, each distribution of primary-factor endowments between the two countries lying within the parallelogram $O^H Q_1 O^F Q_2$ is compatible with an appropriate split of world outputs into non-negative national outputs. Figure 2 illustrates this point. The two countries are denoted home (H) and foreign (F). Measuring the home country's primary-factor inputs from O^H and the foreign country's from O^F, all distributions of primary factors lying within the parallelogram $O^H Q_1 O^F Q_2$ replicate the integrated equilibrium and thus yield long-run factor-price equalization. With optimal saving unrestricted international trade is sufficient for efficient use of resources as long as primary-factor endowments are sufficiently similar. Note that the factor-price-equalization set is not degenerate as in the static SF model, even though in general capital rentals are not equalized.

If saving is non-optimal, factor-price equalization will depend on whether financial capital is internationally mobile or not. The problem is whether financial markets clear at the interest rate of the integrated equilibrium for all distributions of primary factors within the parallelogram. Due to linearity of capital stocks in primary-factor endowments, both the world capital stock and national capital stocks are linear in respective primary-factor endowments and so are national incomes and national savings. Any distribution of primary-factor endowments compatible with a split of world outputs in non-negative parts neither changes world savings nor the world capital endowment. International mobility of capital then ensures that the capital market clears at the integrated equilibrium's interest rate. On the other hand, factor-price equalization with internationally immobile financial capital requires that national capital markets clear at the interest rate $\tilde{i}$ of the integrated equilibrium:

$$\sum_{j=I,C} \delta_j \cdot K_j^k = s(\tilde{i}, \tilde{p}) \cdot (x_I^k + \tilde{p} \cdot x_C^k); \qquad k = H, F. \tag{29}$$

With national capital demands derived by the the first two equations of(20), using national outputs instead of world outputs, it follows that the clearing condition of a national financial market can be written as

$$\frac{x_1}{x_C} = \frac{\delta_C \cdot a_{CK} - s(\tilde{p}, \tilde{i}) \cdot \tilde{p}}{s(\tilde{p}, \tilde{i}) - \delta_I \cdot a_{IK}}. \tag{30}$$

This condition must also hold for the integrated equilibrium. For given prices the right side of eq.(30) is fixed. Hence output ratios must be identical in both countries to replicate the integrated equilibrium allocation by goods' trade; identical output ratios, however, can be only established by identical relative primary-factor endowments. Countries can differ in size, but must be otherwise identical to ensure factor-price equalization if saving is non-optimal and no international financial markets exist. The factor-price-equalization set shrinks to the diagonal of the endowment box.

It is interesting to compare these results to the literature on the interest-rate-equalization problem. Samuelson [1965] argued that factor-price equalization implies equalization of the interest rates in the long-run equilibrium. He interprets the interest rate as an endogenous discount factor and assumes capital stocks to be given exogenously ⟨Samuelson [1975], p. 326⟩. In this context, however, it is not clear how the interest rate is influenced by the households' savings decisions. With optimal saving[9] the steady-state interest rate is given exogenously; long-run factor-price equalization then presupposes identical long-run interest rates and therefore identical rates of time preference. Under the assumption of identical preferences interest-rate equalization implies equalization of primary-factor prices in the long run, as long as endowments are sufficiently similar. On the other hand, due to our assumption of (at least) two primary factors, countries do not necessarily specialize in our model if long-run interest rates do not coincide. Stiglitz's [1970] argument that specialization is inevitable if interest rates differ only holds if labor is the only primary factor; it is the consequence of the nonsubstitution theorem. In the present model primary-factor prices also differ in the long-run if steady-state interest rates differ. Nevertheless, countries must not necessarily specialize in this case: Differ-

9. The same will hold for the case of *classical* saving, *i.e.* if a fixed fraction of capital income and nothing of labor income is saved, if long-run capital rentals are identical in both sectors; cf. Stiglitz [1970].

ences in the interest rate imply different long-run cost functions in both countries and generate the same consequences as different technologies. Hence equalization of steady-state interest rates *and* sufficiently similar endowments of primary factors are necessary conditions for equalization of primary-factor prices the long-run.

V. Trade Pattern and Factor-Content of Trade

In order to prove the validity HO theorem we look at the case of balanced trade. Furthermore, we concentrate on the case of optimal savings, since with nonoptimal savings, the model in general behaves like a model with differences in technologies, and the trade pattern cannot be explained exclusively by differences in factor endowments in that case. The literature distinguishes between two versions of the HO theorem ⟨*cf.* Ethier [1984], p. 141⟩. The quantity version of the theorem states that in a world with two countries, two sectors, and two factors a country will export that good that intensively uses its relatively abundant factor under balanced trade. Referring to primary factors, this theorem carries over to the steady state of the present model. The proof is as follows. Factor abundance is defined by a country's endowments of primary factors relative to world endowments: A country is factor-2 abundant if $L_2/L_2^{\mathrm{w}} > L_1/L_1^{\mathrm{w}}$, where the index **W** stands for world endowments. The vector of net exports **t** of a country is the difference between the vectors of national production **x** and national demand **d**. With identical preferences (*i.e.* identical rates of time preference) the latter is a fraction of world production $\mathbf{x}^W$ depending on the country's share in world income $\eta := \mathbf{p}^T \cdot \mathbf{x}/\mathbf{p}^T \cdot \mathbf{x}^W$. With $\mathbf{x}^W$ being determined by the third and fourth equation in eq.(20), **d** can be written as: $\mathbf{d} = \alpha \cdot [\mathbf{A}_p^T]^{-1} \cdot \mathbf{L}^W$. The trade vector *t* then is

$$\mathbf{t} = [\mathbf{A}_p^T]^{-1} \cdot (\mathbf{L} - \eta \cdot \mathbf{L}^W). \tag{31}$$

However, in the present three-factor model the country's share in world income η must not lie between its relative primary-factor endowments. Specifically:

$$\eta = \frac{\mathbf{p}^T \cdot \mathbf{x}}{\mathbf{p}^T \cdot \mathbf{x}^W}$$

$$= \frac{m_1 \cdot L_1^W \cdot (L_1 / L_1^W)^T + m_2 \cdot L_2^W \cdot (L_2 / L_2^W)}{m_1 \cdot L_1^W + m_2 \cdot L_2^W}, \tag{32}$$

where

$$m_1 = \frac{a_{C2} - p \cdot a_{I2}}{|\mathbf{A}_p|}, \; m_2 = \frac{p \cdot a_{I2} - a_{C1}}{|\mathbf{A}_p|}.$$

Since factor prices must be positive both m_j, must be also positive in the static two-factor model, thus ensuring that η is a weighted average of relative factor endowments. In the dynamic HO model with three factors, however, one m_j may be negative and consequently the sign pattern of the excess supply vector of primary factors $\mathbf{L} - \eta \cdot \mathbf{L}^W$ may be $(-,-)^T$ or $(+,+)^T$. It will be shown that this has no consequences for the validity of the HO theorem, but it has consequences for the HOV theorem.

Suppose first that $\mathbf{L} - \eta \cdot \mathbf{L}^W$ has the sign pattern $(-,+)^T$. If sector I is relatively factor-l intensive, the sign pattern of the vector of net exports of a country that is relatively abundant in factor 2 is then

$$\mathbf{t} = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \cdot \begin{pmatrix} - \\ + \end{pmatrix} = \begin{pmatrix} - \\ + \end{pmatrix}. \tag{33}$$

This implies that the relatively factor-2 abundant country exports x_c, the good that uses the primary factor 2 relatively intensively. If $\mathbf{L} - \eta \cdot \mathbf{L}^W$ has the sign pattern $(-,-)^T$ or $(+,+)^T$, the trade vector will have the same sign pattern, because otherwise clearing of factor markets is impossible. This can be proven by the following counterargument: Suppose $\mathbf{t}$ has the sign pattern $(-,-)^T$ and $\eta > L_2/L_2^W > L_1/L_1^W$. According to eq.(31) our trade pattern then implies $a_{I2}/a_{I1} < a_{C2}/a_{C1} < L_2^W/L_1^W \cdot (L_2/L_2^W - \eta)/(L_1/L_1^W - \eta)$. Since $(L_2/L_2^W - \eta)/(L_1/L_1^W - \eta)$ is less than unity, that relation is incompatible with clearing of primary-factor markets. Hence $\mathbf{t}$ cannot have the stated signs pattern. An equivalent argument holds if $\mathbf{L} - \eta\mathbf{L}^W$ consists of positive arguments. Thus the quantity version of the HO theorem interpreted for primary factors holds although capital is sector-specific and endogenous.

From eq.(31) we can also calculate the net content of primary factors in net exports. Primary factors embodied in trade form the vector $\mathbf{A}_p^T \cdot \mathbf{t}$ and eq.(31) can be written as

$$\mathbf{A}_P^T \cdot \mathbf{t} = \mathbf{L} - \eta \cdot \mathbf{L}^W. \tag{34}$$

This states that primary factors embodied in net exports equal excess primary-factor supplies. This is identical to the HOV equation known from the static HO model. If $\eta \in (L_1/L_1^W, L_2/L_2^W)$, we can interpret eq.(34) as the HOV theorem: A country will export the services of its relatively abundant primary factors and import services of its relatively scarce primary factors.[10] This theorem stresses that it is factor services embodied in commodities that are traded. However, if η does not lie between relative primary-factor endowments, then services of *both* primary factors are either imported or exported in exchange for capital services.

Trade pattern and primary-factor content of trade can be analyzed graphically in the endowment box as shown in figure 2. Suppose that E describes the distribution of primary-factor endowments. Obviously the home country is relatively factor-2 abundant. The sum of its primary-factor input vectors $O^H P_I^H$ and $O^H P_C^H$ ensures full employment and determines output levels. The vector $O^H E$ then gives the primary-factor content of production. The budget line *yy* gives all possible redistributions of primary factors between countries that do not change national income at given factor prices. Making use of eqs.(18) and (20) we can calculate the slope of the budget line as

$$\left.\frac{dL_2}{dL_1}\right|_{yy} = \frac{p \cdot a_{I2} - a_{C2}}{p \cdot a_{I1} - a_{C1}}, \tag{35}$$

which may be positive. With identical preferences, the world endowment ratio of primary factors is identical to both countries' primary-factor services embodied in demand. Suppose that *yy* is negatively sloped as drawn in figure 2. The home country's total primary-factor content of demand is then given by the vector $O^H D$. The difference between primary-factor-content of production and of demand then gives the primary-factor content of trade as the vector ED. At E the home country is a net exporter of factor-2 services and an importer of factor-l services. By splitting up the total primary-factor content of demand into

10. The advantage of the HOV-theorem compared to the HO theorem is that it also holds for the case of imbalanced trade, stating that the trade vector is linear in the vector of excess primary-factor supplies $\mathbf{L} - \eta \cdot \mathbf{L}^w$. The value of the demand share η indicates trade imbalances.

its sectoral components we can interpret the HO theorem geometrically. Sectoral demands in the home country are then given by $O^H D_I^H$, and $O^H D_C^H$. Home demand for the investment good exceeds its supply, and production of the consumption good exceeds its demand. Hence the home country exports x_C that intensively uses its relatively abundant primary factor 2 and imports x_I. This proves the validity of the HO theorem. If yy is positively sloped, it must be either flatter than the ray $O^H Q_1$ or steeper than the ray $O^H Q_2$, since otherwise the home country would either import or export both goods. In general, the primary-factor content of demand must lie between $O^H D_1$ and $O^H D_2$. In the case of a positive sloped budget line, however, the home country either exports services of both primary factors and imports capital services, or exports capital services and imports services of both primary factors. Capital, however, is completely irrelevant in determining comparative advantage in the long run.

The price version of the HO theorem states that in a $2 \times 2 \times 2$ world each country has a lower relative autarky price of that good that intensively uses the relative cheap factor. With respect to primary factors this version also carries over to the present model. This is a direct consequence of the validity of the Stolper-Samuelson theorem. With identical technologies and both goods produced in autarky, w_2^*/w_1^* can be only higher in one country if the relative price of the factor-2 intensive good exceeds the other country's. Figure 1 illustrates that point. Suppose that p_0 is the autarky price in the foreign country. Furthermore suppose that factor 2 is relatively expensive in the home country H. With the factor-l intensive good x_I as numeraire, the home price of x_C must ensure that sector C's iso-unit-cost curve cuts sector I's at a point left of E_0, say, at E_1; otherwise the home country would specialize. Thus the requirement that both goods must be produced in autarky yields a higher price of the factor-2 intensive good at home, if factor 2 is relatively expensive compared to the foreign country. The endogeneity of capital does not imply a breakdown of the HO theorem as a theorem holding for a long-run equilibrium.

One characteristic of the price version of the HO theorem is that it holds independent from international differences in preferences. From our graphical analysis (figure 1) we know, that interest-rate differentials generate different levels of the iso-unit-cost curves for both countries. Hence it is possible, that even with identical endowments of primary factors and identical prices the

ratio of primary-factor prices may differ in autarky if the rates of time preference do not coincide. It cannot be excluded then that a country has a lower relative price of the good in autarky that intensively uses its expensive primary factor. International differences in the rate of time preference may cause a breakdown of the price version of the HO theorem. However, international differences in preferences are not necessarily differences in time-preference rates, but may be differences in the instantaneous utility functions. With this kind of preference dissimiliarities the price version of the HO theorem still holds for long-run equilibria.

These results demonstrate that it is not the existence of a positive interest rate or the incorporation of time into trade models that prevent the HO theorem to hold in the steady state. The breakdown of the quantity version derived by Metcalfe and Steedman [1977] rests on the heterogeneity of capital goods that generates changes in the composition of capital stocks as variables change. Since in their model long-run capital rentals depend on the relative price, which is an endogenous variable in a two-country framework, capital as well as the rental rate of capital are endogenous in the long run. Changes in the relative price then change the steady-state capital rentals and hence generate the breakdown of the HO-theorem's quantity version in the same way as differences in the rates of time preference in the dynamic HO model. The pure incorporation of time into trade models or the existence of a positive interest rate, however, do no harm to the results derived from the static models as far as steady states are considered.

VI. Conclusions

This paper has analyzed the implications of endogenous sector-specific capital stocks for the long-run equilibrium in a dynamic version of the HO model. Instead of viewing capital as physically mobile over time, we assume that capital is sector-specific even in the long run. Reallocation of capital is only a side effect of accumulation and proceeds solely by investment and depreciation. The long-run capital endowment of an economy then is no longer given exogenously, but determined endogenously by endowments of primary factors, which are assumed to be inelastically supplied and perfectly mobile intersectorally. The steady state of a two-primary-factors, two-sector dynamic HO model is com-

pletely determined as in the static two-by-two HO model. All theorems of the HO model, namely the Stolper-Samuelson theorem, the Rybczynski theorem, the factor-price-equalization theorem, the HO theorem, and – with minor qualifications – the HOV theorem, carry over to the steady state of the present model even though capital stocks are completely sector-specific and endogenous and capital rentals do not equalize in general. The reason for this is that steady-state rental rates of capital are given by the rate of interest and the depreciation rate, and hence both capital endowments and outputs are linear functions of primary-factor endowments alone. The long-run equilibrium is fully determined by a static HO model in the two primary factors; thus, the present model provides a dynamic foundation of the long-run validity of the static HO model. Just as intermediate goods, capital endowments (as produced factors of production) are endogenous and have no direct influence on the long-run equilibrium. It is in *this* sense that capital is irrelevant for long-run analysis.

These results have consequences for the (re-)interpretation of the HO model. First, the static HO model in primary factors is appropriate for long-run analysis even if capital is endogenous and sector-specific. This refutes Neary's suspicion that the properties of the HO model will not hold in the long run. Second, long-run comparative advantage is solely determined by primary-factor endowments. International differences in capital endowments reflect differences in endowments of primary factors; capital endowments and trade patterns are determined simultaneously by primary-factor endowments. Third, nonoptimal saving is incompatible with efficient resource allocation if trade must be balanced, unless countries differ only in size. Differences in interest rates generate differences in primary-factor prices. Fourth, empirical tests of the HO theory may concentrate on primary factors only and exclude capital, if one assumes the world to be near the steady state. Including capital in this case must necessarily end up in multi-collinearity ⟨*cf.* Leamer [1984], especially. ch. 5, for a discussion of the multicollinearity problem in empirical studies⟩. Thus already existing empirical tests of the HO model should be replicated in order to ascertain that negative results are not due to the inclusion of capital.

Apart from looking at the empirical literature, future research should concentrate on the analysis of the model's adjustment dynamics. The model could be used to explore the response of capital stocks, income, and the current account to shifts in the terms of trade or to changes in primary-factor endow-

ments. In particular, differences in the intertemporal behavior of the economy arising from alternative specifications of the adjustment-cost function, especially hysteresis effects arising from the specification proposed by Kemp and Wan [1974], could be explored. Furthermore, a deeper analysis of dynamics will also throw light on the stability of the two-country model.

References

Abel, A.B. and O.J. Blanchard [1983], "An Intertemporal Model of Saving and Investment," *Econometrica* 51; pp 675-692.

Benhabib, J. and K. Nishimura [1979], The Hopf Bifurcation and the Existence and Stability of Closed Orbits in Multisector Models of Optimal Economic Growth," *Journal of Economic Theory,* 21; pp 421-444.

Dixit, A.K. and V. Norman [1980], *Theory of International Trade*, Cambridge University Press: Cambridge.

Ethier, W.J. [1984], "Higher Dimensional Issue in Trade Theory," in: Jones, R.W. and P.B. Kenen (eds.), *Handbook of International Economics*, Vol. I, North Holland: Amsterdam; pp 131-184.

Feichtinger, G. and R.F. Hartl [1986], *Optimale Kontrolle Oekonomischer Prozesse*, de Gruyter: Berlin.

Grossman, G.M. and J.A. Levinson [1989], "Import Competition and the Stock Market Return to Capital," *American Economic Review,* 79; pp 1065-1087.

Hayashi, F. (1982), "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica,* 50; pp 213-224.

Jones, R.W. [1975], "Income Distribution and Effective Protection in a Multicommodity Trade Model," *Journal of Economic Theory,* 11; pp 1-15.

Kemp, M.C. [1973], "Heterogeneous Capital Goods and Long-Run Stolper-Samuelson Theorems," *Australian Economic Papers,* 12; pp 253-260.

Kemp, M.C. and H.Y. Wan [1974], "Hysteresis of the Long-Run Equilibrium from Realistic Adjustment Costs," in: Horwich, G. and P.A. Samuelson (eds.), *Trade, Stability,* and *Macroeconomics*, Academic Press: New York.

Leamer, E.E. [1984], *Sources of International Comparative Advantage*, The MIT Press: Cambridge/Mass.

Lucas, R.E. [1967], "Adjustment Costs and the Theory of Supply," *Journal of Political Economy* 75; pp 321-334.

Mayer, W. [1974], "Short-Run and Long-Run Equilibrium for a Small Open Economy, *Journal of Political Economy,* 82; pp 955-967.

Metcalfe, J.S. and I. Steedman [1977], "Reswitching, Primary Inputs and the Heckscher-Ohlin-Samuelson Theory of Trade," *Journal of International Economics,* 7; pp 291-298.

Murphy, R.G. [1988], "Sector-Specific Capital ad Real Exchange Rate Dynamics," *Journal of Economic Dynamics ant Control,* 12; pp 7-12.

Mussa, M. [1978], "Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model," *Journal of Political Economy,* 86; pp 775-791.

Neary, J.P. [1978], "Short-Run Capital Specificity and the Pure Theory of International Trade," *The Economic Journal,* 88; pp 488-510.

Neary, J.P. [1982], "Intersectoral Capital Mobility, Wage Stickiness, and the Case for Adjustment Assistance," in: Bhagwati, J.N. (ed.), *Import Competition and Response*, NBER Conference Report, The University of Chicago Press: Chicago; pp 39-67.

Samuelson, P.A. [1965], "Equalization of the Interest Rate along with the Real Wage Rate," in: Baldwin, R.E. *et al.* (eds.), *Trade, Growth and the Balance of Payments*, Rand McNally: Chicago; pp 35-52.

Samuelson, P.A. [1975], "Trade Pattern Reversals in Time-Phased Ricardian Systems and Intertemporal Efficiency," *Journal of International Economics,* 5; 309-363.

Sargent, T.J. [1987], *Macroeconomic Theory*, 2nd ed., Academic Press: Orlando/Florida.

Sensenbrenner, G. [1991], "Aggregate Investment, the Stock Market, and the Q-Model," Robust Results for Six OECD Countries, *European Economic Journal,* 35; pp 769-827.

Smith, A. [1984], "Capital Theory and Trade Theory," in: Jones, R.W. and P.B. Kenen (eds.), *Handbook of International Economics*, Vol. I, North Holland: Amsterdam; pp 289-324.

Stiglitz, J.E. [1970], "Factor Price Equalization in a Dynamic Economy," *Journal of Political Economy* 78; pp 456-488.

Treadway, A.B. [1969], "On Rational Entrepreneurial Behavior and the Demand for Investment," *Review of Economic Studies,* 36; pp 227-239.

Woodland, A.D. [1982], *International Trade and Resource Allocation*, North Holland: Amsterdam.