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The Depressing Effect of Randomly Migrating Population on Wage Share

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Abstract

In this paper, a stochastic framework is developed to study the effects of randomly migrating population. The analysis borrows a number of intrinsic characteristics from the classic Goodwin model. While Goodwin formalizes his model as a Lotka Volterra system, the present study converges into a stochastic version of the Lotka-Volterra system. In particular, it is shown that randomly migrating population has a depressing effect on wage share. The employment ratio, on the other hand, exhibits the natural rate property.

I. Introduction

With expanded trade zones and removal of trade restrictions, an increase in population migration is seen in many parts of the globe. Under accelerating political and economic integration, migration among member nations of the European Community rises significantly. In the fast-growing Pacific zone, international migration also increases substantially. Studies of migration are prevalent. Galor and Stark [1991] study the impact of technological differences on population migration. Hatzipanaystou [1991] analyses international migration and remittance in a two country equilibrium model. Zhang [1990] demonstrates the presence of economic cycles created by brain drain and population migration.

An issue at hand is the random nature of international population migration. In

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particular, the sizes of migration can be very sporadic. Though stochastic elements are usually regarded as intrinsic characteristics of economic variables, random fluctuations in migration may be attributed to rational economic behaviours like job searching and trials activities. Moreover, Wiener [1987] suggests that 'there may be as many refugees in the world as there are people who migrated in response to employment opportunities'. Haphazard refugee movements also contribute to the randomness of migration. Recent studies on trends of migration show that most international migration during recent decades has been made without the intention of permanent settlement (see for example, Salt [1989]). Actual statistics (Mitchell [1992]) on migrations to and from nations display a large degree of fluctuation. The purpose of this paper is to develop a framework for studying the effects of randomly migrating population on an economy. The analysis borrows a number of intrinsic characteristics from the deterministic models of Goodwin [1967] and Mehrling [1986]. Stochastic elements in the migration process are introduced to formulate a stochastic model. While it is well known that Goodwin formalizes his analysis as a Lotka-Volterra model [Volterra 1931], the present study converges into Yeung's [1986, 1988] stochastic version of the Lotka-Volterra system. In particular, it is shown that random population migration has a depressing effect on wage share while the employment ratio exhibits the natural rate property.

The paper is organized as follows. Section II presents the conceptual framework of the model. Section III examines the properties of the model and the implications of randomly migrating population on an economy. Section IV concludes.

II. The Conceptual Framework

Following Goodwin [1967] and Mehrling [1986], we consider an economy with two classes of participants – the entrepreneurs (capitalists) and workers. Technology is characterized by a fixed coefficient production function and the output produced can be either consumed or used as an input to create capital. Total output at time t is $Q(t) = ak(t)$, where $k(t)$ is the capital stock and a reflects capital productivity. Labor input requirement is $L(t) = \frac{1}{\lambda} Q(t)$ where λ represents the average product of labour. Labour service is supplied at wage $w(t)$. The size of the population is $N(t)$ and its growth rate is affected by two factors – the natural growth rate c and the migration rate B. In mathematical notations we have $\dot{N}(t) / N(t) = c + B$.

Define:

Labour share of output $u(t) = w(t)\, L(t) / Q(t) = w(t) / \lambda$, and employment ratio

$$\upsilon(t) = L(t) / N(t) \tag{1}$$

Capital share than becomes $1 - u(t)$ and all wages are consumed. Instead of following Goodwin to assume that all profits are invested, we hypothesize that the proportion of capital share invested is

$$1 - u(t) - b\frac{Q(t)}{N(t)} \tag{2}$$

where b is a positive constant and $b\frac{Q(t)}{N(t)}$ is the portion of capital share that is consumed by the capitalists. Such behaviours can be interpreted as the case where capitalists consumption is related to the state of the economy. In time of prosperity, a higher percentage out of the capital share is consumed and *vice versa*. Hence the change in capital stock becomes the product of the proportion of capital share invested (2) and output:

$$\dot{k}(t) = \left[1 - u(t) - b\frac{Q(t)}{N(t)}\right] Q(t) \tag{3}$$

With $Q(t) = \lambda L(t)$, we obtain $\dot{Q}(t) / Q(t) = \dot{L}(t) / L(t)$ and hence derive $\dot{L}(t) / L(t) = \dot{k}(t) / k(t)$. Making use of the fact that $Q(t) = ak(t) = \lambda L(t)$, we have

$$\frac{\dot{k}(t)}{k(t)} = a - au(t) - a\lambda b\frac{L(t)}{N(t)} = \frac{\dot{Q}(t)}{Q(t)} = \frac{\dot{L}(t)}{L(t)} \tag{4}$$

From $\upsilon(t)$ in (1), we can express the dynamics of the employment ratio as:

$$\frac{\dot{\upsilon}(t)}{\upsilon(t)} = \frac{\dot{L}(t)}{L(t)} - \frac{\dot{N}(t)}{N(t)} = [a - au(t) - a\lambda b\ \upsilon(t)] - (c + B) \tag{5}$$

Condition (5) shows that the percentage change in the employment ratio is negatively related to the current size of the labor share, the employment ratio and the population growth rate.

The workers, on the other hand, bargain for wage settlement. Their ability to

bargain for a higher wage share depends on the current rate of unemployment. In particular, at high levels of employment workers can raise their wage faster because the threat of unemployment is reduced. This notion of labor market behaviour is shown in Lipsey [1960] and Phelps [1970]. Using Goodwin and Mehrling's linear approximation, we have

$$\frac{\dot{w}(t)}{w(t)} = -\gamma + \rho v(t) \tag{6}$$

where γ and ρ are positive constants. The wage dynamics (6) also reflect the fact that in steady-state (*i.e.* $\dot{w} / w = 0$), the employment ratio would approach a long-run natural rate γ / ρ.

To characterize the random nature of population migration, we let the migration rate be:

$$B + \Omega(t) \tag{7}$$

where $\Omega(t)$ is a Gaussian white noise with zero mean and covariance $\sigma^2 \delta(t-s)$; $\delta(.)$ is the Dirac delta function, σ^2 measures the degree of uncertainty in population migration. A stochastic version of (5) can be obtained as:

$$\frac{\dot{v}(t)}{v(t)} = (a - c - B) - au(t) - a\lambda b v(t) + \xi(t) \tag{5'}$$

where $\xi(t)$ is the negative of $\Omega(t)$.

Equations (5′) and (6) becomes a system random differential equations which can be expressed as a pair of Ito differential equations (see Soong [1973]):

$$dv(t) = [(a - c - B) - au(t) - a\lambda b v(t)]\ v(t)dt + \sigma v(t)\ dz(t) \tag{8}$$

$$du(t) = [-\gamma + \rho v(t)]\ u(t)dt \tag{9}$$

where $z(t)$ is a Brownian motion process with $E(dz(t)) = 0$ and var $(dz(t)) = dt$. The degree of uncertainty in the change in the employment ratio, $v(t)$, is $\sigma\, v(t)$.

System (8) – (9) characterizes the outcome of the economy in terms of the employment ratio and wage share. Moreover, (8) – (9) is a 2-level model of Yeung's [1988] n-level stochastic version of the Lotka Voltena food chain.

III. Properties of the Model and Implications of Randomly Migrating Population

In this section, we examine properties of the model developed in the previous section and the economic consequences of randomly migrating population. The first task is to solve system (8) – (9). Contrary to deterministic models in which the solution of a system is a set of particular time paths of the variables, the solution to a stochastic model is a set of stochastic processes. The joint transition density function of the solution processes, denoted by $\phi(u, \upsilon, t)$, must satisfy the Fokker-Planck *forward* equation (see Karlin and Taylor [1981]):

$$\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial \upsilon}\{[(a-c-B)-au-a\lambda b\upsilon]\upsilon\phi\} - \frac{\partial}{\partial u}\{(-\gamma+\rho\upsilon)u\phi\} + \frac{1}{2}\frac{\partial^2}{\partial \upsilon^2}(\sigma^2\upsilon^2\phi) \tag{10}$$

Analogous to the long-run equilibrium in deterministic models, there is a *stationary* solution for the stochastic system (also known as a stochastic equilibrium or a stochastic steady state). The stationary equilibrium is characterized by a time-invariant density function, that is where $\partial\phi / \partial t = 0$. With this, the solution to (10) will give the long-run (stationary) equilibrium joint density function of u and υ which indicates the possible realizations of these variables. Following Yeung's [1986] analysis, we first perform the following variable transformation:

$$\begin{aligned} x(t) &= ln\ \upsilon(t) \\ y(t) &= ln\ u(t) \end{aligned} \tag{11}$$

The resultant dynamics of x and y can be derived, using Ito's lemma, as:

$$\begin{aligned} dx(t) &= [(a-c-B-\tfrac{1}{2}\sigma^2) - a\ exp(y(t)) - a\lambda b\ exp\ (x(t))]\ dt + \sigma\ dz(t) \\ dy(t) &= [-\gamma + \rho\ exp\ (x(t))]\ dt \end{aligned} \tag{12}$$

The long-run (stationary) equilibrium joint density function of x and y denoted by $\varphi(x, y)$ is obtained as follows.

Proposition 1: *The long-run (stationary) equilibrium joint density function φ can be solved explicitly as:*

$$\varphi(x,y) = m\ exp\left[\frac{2}{\sigma^2}\left(\frac{a\lambda b\gamma}{\rho}x - a\lambda b\ exp\,(x) + a\lambda b\right)\right]$$

$$exp\left\{\frac{2a\lambda b}{\rho\sigma^2}\left[\left(a - c - B - \tfrac{1}{2}\sigma^2 - \frac{a\lambda b\gamma}{\rho}\right)y - a\ exp\,(y) + a\right]\right\}$$

where m is the normalization factor such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varphi(x,y)dxdy = 1.$$

Proof: See Appendix Q.E.D.

The function $\varphi(x, y)$ characterizes the long-run stationary distribution of x and y. Using straight-forward differentiation, one can readily show that the marginal density function $\varphi(x \mid y = y_0)$ reaches a maximum at $x = ln(\gamma/\rho)$ for any y_0. Similarly, the marginal density function $\varphi(y \mid x = x_0)$ reaches a maximum at $y = ln\left[\frac{a - c - B - \frac{1}{2}\sigma^2}{a} - \frac{\lambda b\gamma}{\rho}\right]$ for any x_0. Thus, the probability density concentrates around a spike point

$$(x^*, y^*) = \left\{ln\,(\gamma/\rho),\ ln\left(\frac{a - c - B - \frac{1}{2}\sigma^2}{a} - \frac{\lambda b\gamma}{\rho}\right)\right\} \tag{13}$$

As (x, y) digresses from (x^*, y^*), the probability density declines.

The transformations $x = ln\,\upsilon$ and $y = ln\,u$ are one-to-one mappings and the corresponding point at which the probability density reaches a maximum in the υ-u space is

$$(\upsilon^*, u^*) = \left\{\gamma/\rho,\ \frac{a - c - B - \frac{1}{2}\sigma^2}{a} - \frac{\lambda b\gamma}{\rho}\right\} \tag{14}$$

Two interesting implication follows immediately.

Proposition 2: *The greater the degree of uncertainty in population migration (measured by σ^2), the lower the level of the most likely realizable wage share u^*.*

Proof: Given that

$$u^* = \frac{a - c - B - \frac{1}{2}\sigma^2}{a} - \frac{\lambda b\gamma}{\rho}$$

we have

$$\frac{\partial u^*}{\partial \sigma^2} = \frac{-1}{2a} < 0$$

Thus an increase in σ^2 would lower u^*. Q.E.D.

Proposition 2 demonstrates the depressing effect of randomly fluctuating population on wage share.

The economic intuition behind Proposition 2 is as follows. As the population growth rate randomly fluctuates, the capitalists adjust the capital stock with these random fluctuations. The capitalists' adjustments to offset random shrinkages and those to capture the gain in a random expansion favour their cause, and hence give rise to the depressing effect on wage share.

Proposition 3: *The most likely realizable level of employment ratio is not affected by the degree of uncertainty in population migration.*

Proof: Since $v^* = \gamma / \rho$, a change in σ^2 would not affect the most likely realizable level of employment ratio. Q.E.D.

Finally, Proposition 3 indicates that the model exhibits the natural rate (of unemployment) property which follows from wage dynamics (6).

IV. Concluding Remarks

As Zolberg [1989] noted, "the dynamics that have propelled international population movements to the forefront of humanistic and political concerns during the past quarter century are likely to be amplified in the next." Issues concerning migration would occupy a high priority in most nations' policy agenda. Randomly migrating population, which is likely to be augmented by the current trend of temporary migration and haphazard refugee movement, is the focus of this paper. A stochastic model is developed to analyze the effect of random population migration on an economy. The economic structure in the analysis is characterized by a modified version of the classic Goodwin model. Stochastic elements are added to the population migration process and a system of stochastic differential equations (instead of the Goodwin Lotka-Volterra type of deterministic equations) is obtained to describe the dynamics of the economy. The joint stationary density function of wage share and employment ratio is derived to delineate the long-run equilibrium of the economy. It is shown that randomness in migration *per se* would cause a de-

pressing effect on wage share. At the same time, the behaviour of the employment ratio reflects the natural rate property.

Appendix

Proof of Proposition 1 (from Yeung [1986, 1988]).

Let $\psi(x, y)$ be the stationary density function of x and y. It must satisfy the Fokker-Planck *forward* equation (see Soong [1973]):

$$0 = -\frac{\partial}{\partial x}\left\{[(a - c - B - \tfrac{1}{2}\sigma^2) - a\ exp\,(y) - a\lambda b\ exp\,(x)]\psi\right\}$$
$$-\frac{\partial}{\partial y}[(-\gamma + \rho\ exp\,(x)]\psi + \frac{1}{2}\frac{\partial^2}{\partial x^2}(\sigma^2\psi) \qquad \text{(A1)}$$

According to Liu's analysis [1969], if the Fooker Planck equation of a two dimensional problem can be written in form

$$0 = L_1(x,y)\left[h_1(x)\psi + k_1(x)\frac{\partial\psi}{\partial x}\right] + L_2(x,y)\left[h_2(y)\psi + k_2(y)\frac{\partial\psi}{\partial y}\right] \qquad \text{(A2)}$$

where $L_1(x,y)$ and $L_2(x,y)$ are partial differential operators, then a solution of ψ can be obtained by requiring that $\psi(x,y)$ satisfies.

$$h_1(x)\psi + k_1(x)\frac{\partial\psi}{\partial x} = 0$$
$$h_2(y)\psi + k_2(y)\frac{\partial\psi}{\partial y} = 0 \qquad \text{(A3)}$$

These first order homogeneous differential equations will immediately lead to the result (see Soong [1973] pp. 197-198):

$$\psi(x,y) = m\ exp\left[-\int_0^x \frac{h_1(s)}{k_1(s)}ds\right] exp\left[-\int_0^y \frac{h_2(s)}{k_2(s)}ds\right] \qquad \text{(A4)}$$

where m is the normalization factor such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi(x,y)dxdy = 1 \tag{A5}$$

Upon remangement of terms, equation (5) can be expressed as

$$\left[\frac{\partial}{\partial x} - \frac{\rho}{a\lambda b}\frac{\partial}{\partial y}\right]\left\{\left[\frac{a\lambda b}{\rho}\gamma - a\lambda b\ exp\ (x)\right]\psi - \frac{1}{2}\sigma^2\frac{\partial\psi}{\partial x}\right\}$$

$$+\frac{\partial}{\partial x}\left\{\left[(a-c-B-\sigma^2-\frac{a\lambda b}{\rho}\gamma) - a\ exp\ (y)\right]\psi - \frac{1}{2}\sigma^2\frac{\rho}{a\lambda b}\frac{\partial\psi}{\partial y}\right\} = 0 \tag{A6}$$

Inspection of (A6) shows that it has the form of (A4) with

$$L_1(x,y) = \frac{\partial}{\partial x} - \frac{\rho}{a\lambda b}\frac{\partial}{\partial y}, \qquad L_2(x,y) = \frac{\partial}{\partial x}$$

$$h_1(x) = \frac{a\lambda b}{\rho} - \gamma - a\lambda b\ exp\ (x), \qquad k_1(x) = -\frac{1}{2}\sigma^2$$

$$h_2(y) = (a-c-B-\frac{1}{2}\sigma^2-\frac{a\lambda b}{\rho}\gamma)\ - a\ exp\ (y)$$

$$k_2(y) = -\frac{1}{2}\sigma^2\frac{\rho}{a\lambda b}\frac{\partial\psi}{\partial y} \tag{A7}$$

Therefore the solution of $\psi(x,y)$ can be obtained as:

$$\psi(x,y) = m\ exp\left\{\int_0^x \frac{2}{\sigma^2}\left[\frac{a\lambda b}{\rho}\gamma - a\lambda b\ exp\ (s)\right]ds\right\} \tag{A8}$$

$$exp\left\{\int_0^y \frac{2a\lambda b}{\rho\sigma^2}\left[a-c-B-\frac{a\lambda b}{\rho}\gamma - a\ exp\ (s)\right]ds\right\}$$

Upon integration,

$$\psi(x,y) = m\ exp\left[\frac{2}{\sigma^2}\left(\frac{a\lambda b}{\rho}\gamma x - a\lambda b\ exp\,(x) + a\lambda b\right)\right]$$

$$exp\left\{\frac{2a\lambda b}{\rho\lambda^2}\left[\left(a - c - B - \frac{1}{2}\sigma^2 - \frac{a\lambda b}{\rho}\gamma\right)y - a\ exp\ (y) + a\right]\right\} \quad \text{(A9)}$$

Hence Proposition 1.

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