Tariffs and Income Distribution under Domestic Monopoly

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Abstract

This paper investigates the impact of protective tariff on the distribution of income in a Ricardo-Viner model that admits product market monopoly. Monopolistic producer is assumed to own capital employed in the industry and therefore earn all nonwage income in the form of rental and superprofits. In this model, an increase in the relative price of importable good caused by an increase in tariff rate may raise the real wage of labor and the real income of the monopolist in terms of either good, and may thus lead to a possible resolution of the inter-group conflict of interests within the protected industry.

I. Introduction

In the Ricardo-Viner production model, a tariff-induced increase in the relative price of the importable good unambiguously raises the real rental on capital in the protected industry, but it may raise or lower the real wage rate depending on the preferences of the workers. Consequently, the interests of labor and capital-owner within the protected industry may come into conflict in case of a protective policy. This is the so-called neo-classical ambiguity problem in the international trade literature [see e.g., Jones (1971), Musa (1974), and Ruffin and Jones (1977)]. All these studies, however, use the standard assumption of a perfectly competitive market structure, No attempt has yet been made to analyze the distributional implications of tariff in the domestic monopoly version of the Ricardo-Viner model. This paper is a

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modest attempt in this direction.

Melvin and Warne (1973) analyzed the implications of domestic monopoly in a general equilibrium trade model where domestic monopoly perpetuates despite international trade.1 Following Melvin and Warne, Batra (1973) offered the most comprehensive treatment of monopoly in a H-O-S trade model. Specifically, Batra has shown that the basic trade theorems including the Stolper-Samuelson theorem of income distribution would hold in such a model. In this paper, I extend Batra's analysis further by replacing his generic capital with sector specific one, and thereby construct a domestic monopoly version of the Ricardo-Viner model. As in Batra, here also I assume that the monopolist owns capital so that all non-wage income accrues to the monopolist in the form of rental and super profits. In this model, what would be the impact of a tariff induced increase in the price of importable on the distribution of income? The tariff would cut import, and allow the domestic monopoly sector to expand. The monopolist could then raise the price by less than the full amount of tariff so that its profit maximizing price could be below the international price plus tariff. In such a situation, could there be a matching of interests between the monopolistic capital owner and the labor in monopoly sector, and thus a possible resolution of the so-called neoclassical ambiguity? The intent of this paper is to provide a possible affirmative answer to this theoretical question. Specifically, it is demonstrated that under a set of sufficient conditions in terms of elasticity parameters of the model, the tariff-induced increase in the relative price of importable could increase the relative income of both the monopolist and the labor in terms of either good.

The plan of the paper is as follows. In section II, I present the basic argument with the help of a diagram, while in section III, I rigorously derive the main results in terms of a formal model. Finally, some concluding remarks are made in section IV.

II. Monopoly and the Ricardo-Viner Model

For a clear view of the neoclassical ambiguity under perfect competition and its

1. See Melvin and Warne (p. 125) for details of the circumstances under which such a monopolist can exist, Casas (1989) also analyzes the case of domestic monopoly as a basis of trade.
possible resolution under domestic monopoly, I start with a perfectly competitive market structure in each of the product markets. Two commodities, exportable \( X_1 \) and importable \( X_2 \) are produced by using two factors, capital \( (K) \) and labor \( (L) \) under neoclassical technology. Capital is sector-specific, while labor is intersectorally mobile. In Figure 1, \( OO' \) denotes the total supply of labor. \( VMP_{L1} \) and \( VMP_{L2} \) schedules are the initial value marginal product of labor for sector 1 and 2 respectively. These are the inverse of labor demand in each sector with respect to the respective real wage rates. Initially, the wage rate is \( OA \) at which \( OM \) and \( O'M \) amounts of labor are employed in sectors 2 and 1 respectively. Let the price of importable, \( X_2 \) rise due to a tariff such that the \( VMP_{L2} \) shifts proportionately to \( VMP_{L1}' \). I assume no change in the price of exportable, \( X_1 \) so that \( VMP_{L1} \) does not shift. The new wage rate is \( OC \). The nominal wage rate rises less than the price of importable \( (0 < AC/OA < AB/OA) \), leading to a fall in the real wage rate in terms of importable. However, the real wage rate increases in terms of exportable since the price of exportable remains unchanged. This is the so-called neoclassical ambiguity problem under perfectly competitive market structure [see Mussa (1974) for details].

Now I replace the perfectly competitive market structure assumption by introducing monopoly in the importable sector while allowing perfect competition in the exportable sector. The initial wage rate would be \( Ow_0 \) where the marginal revenue product of labor schedule for sector 2 \( MR_{PL2} \) intersects the \( VMP_L \). Now suppose the government imposes a tariff to protect the domestic monopoly from increasing foreign competition. The tariff would raise the price and the marginal revenue for the importable sector. Let the increase in marginal revenue be proportionately more than the price. Then the \( MR_{PL2} \) curve would shift proportionately more than the \( VMP_{L2} \). In Figure 1, \( MR_{PL2} \) shifts to \( MR_{PL2}' \) such that \( wDL/\overline{Ow_0} > AB/OA > 0 \). \( MR_{PL2}' \) intersects \( VMP_L \) to yield a new wage rate \( Ow_1 \). One can observe that the real wage rate has risen in terms of either good since \( wDL/\overline{Ow_0} > AB/OA > 0 \). Thus, if the marginal revenue rises more than the price of the protected good, nominal wage that equals labor's marginal revenue product might rise more than the price leading to a rise in the real wage in terms of importable good. And since the production functions are linearly homogeneous and the monopolist also owns capital invested in the importable sector, real income (rent plus profit) of the monopolist also rises in terms of either good [see Mussa's (1974) excellent argument on this in a perfectly competitive set up]. The exact impact of the price increase, of course,
depends upon the extent of the shift\(^2\) in MRPL\(_k\) and also on the elasticity of demand for labor in each sector. Specifically, given an increase in price of importable and the associated shift in MRPL\(_k\), the higher (lower) the elasticity of labor demand in the importable (exportable) sector, the greater is the chance of a rise in the real wage in terms of either good. Similarly, given the elasticity of demand for labor in each sector, higher the shift in MRPL\(_k\), the greater is the probability of a rise in the real wage rate in terms of either good. The sufficient conditions for the resolution of the neoclassical ambiguity are rigorously derived in the next section where I build up a general model allowing monopoly in both the sectors.

III. Analytical Results

The standard two-good, two-factor model of international trade with monopoly in commodity market a la Batra (1973) is deployed in the present paper. In the

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2. In the context of Stolper-Samuelson theorem in a H-O-S model, Melvin and Warne (p. 132) explained the effect of tariff on elasticity of demand for the good produced in the protected industry.
production side, I specify the following equations.

\[ X_i = X_i(K_s, L_j), \quad j = 1, 2. \]  \hspace{1cm} (1)

\[ \bar{L} = C_{1L}X_1 + C_{12}X_2. \]  \hspace{1cm} (2)

\[ \bar{K}_1 = C_{2L}X_1. \]  \hspace{1cm} (3)

\[ \bar{K}_2 = C_{2L}X_2. \]  \hspace{1cm} (4)

\[ P_1 = C_{11}w + C_{13}r_1 + C_{12}, \]  \hspace{1cm} (5)

\[ P_2 = C_{12}w + C_{13}r_2 + C_{12}. \]  \hspace{1cm} (6)

\[ MR_j = P_j(1 - (1/e_j)), \quad j = 1, 2. \]  \hspace{1cm} (7)

\[ C_{1L} = P_j/e_j, \quad j = 1, 2. \]  \hspace{1cm} (8)

\[ \sigma_j = \frac{d\ln(k_j/L_j)}{d\ln(w/r_j)}, \quad j = 1, 2. \]  \hspace{1cm} (9)

\[ z_j = c_{1j}(P - C_{1L}), \quad j = 1, 2. \]  \hspace{1cm} (10)

Equations (1) denote the neoclassical production functions: equations (2), (3) and (4), the full employment conditions: (5) and (6), the price equations that include \( C_{1L} \), the monopoly profit per unit of output in the \( j \)-th industry: equation (7), the familiar relation among marginal revenue (MR), price (Pj) and elasticity of demand (e_j) in the \( j \)-th sector: equation (8), the relationship among \( P_j, C_{1L} \) and \( e_j \), (for details see Batra, p. 283): identity (9), the definition of elasticity of substitution in production, and finally, \( z_j \) in equation (10) represent income of the monopolist per unit of capital in sector \( j \). Note that the monopolist earns the residual after labor is paid.

On the consumption side of the model, following Batra I assume a CES utility function, \( U = U(D_1, D_2) = (ax_1^\beta + bx_2^\beta)^{-1/\beta} \) (11) with \( x_i = D_i \) under autarky, and \( a, b, \beta \) positive constants. The demand functions that originate from (11) yield the following relationships between elasticity of substitution in consumption, \( \sigma \), the price ratio, \( P = P_2/P_1 \), and the elasticity of demand, \( e_j \):

\[ e_1 = \frac{(1 + \sigma P_2^{\beta})}{(1 + P^{\beta})}, \]  \hspace{1cm} and

\[ e_2 = \frac{(1 + \sigma P_1^{\beta})}{(1 + P_1^{\beta})}. \]  \hspace{1cm} (12)

As Batra has shown, in a general equilibrium model of monopoly, there exists a unique wage-rental ratio at which the producer price ratio becomes equal to the consumer price ratio, \( P \). Notice that in the utility function there are two arguments, (1) the quantity demanded of the exportable good (\( D_1 \)) and (2) the quantity demanded of the importable good (\( D_2 \)). Here I assume that the importable good produced by the domestic monopoly is a perfect substitute of the foreign import which
is subjected to a prohibitive tariff. It is the latter which protects the domestic monopoly from foreign competition. One could introduce the assumption of imperfect substitutability between the import and the importable, and thereby introduce three consumption goods in the utility function. Since the major focus of the paper is on the income distribution aspects of the tariff policy, I do not intend to complicate the analysis by introducing the imperfect substitutability assumption. Besides, imperfect substitutability assumption would inevitably bring us to the world of monopolistic competition which by itself is a broad new area of theoretical research, and is beyond the scope of this paper.

Differentiating equations (2) through (8) and (11), and using identity (9) and the cost minimizing conditions, I derive the following equations of change (see details in Appendix 1).

\[-(\sigma_{1}\Delta_{1} + \sigma_{2}\Delta_{2})w^{*} + \sigma_{1}\Delta_{1} r_{1}^{*} + \sigma_{2}\Delta_{2} r_{2}^{*} = 0\]

\[\theta_{1}w^{*} + \theta_{1}r_{1}^{*} = [(1-\theta_{1}) + A_{1}\theta_{1}\pi_{1}^{*} - A_{1}\theta_{1}\pi_{1}^{*}\]

\[\theta_{2}w^{*} + \theta_{2}r_{2}^{*} = -\theta_{2}A_{2}\pi_{2}^{*} + [(1-\theta_{1}) + A_{2}\theta_{2}\pi_{2}^{*}\]

(13)

Here an asterisk over a variable or parameter denotes its proportionate change i.e., \(r_{i}^{*} = \frac{dr_{i}}{r_{i}}\), \(\pi_{j}^{*} = \frac{d\pi_{j}}{\pi_{j}}\) etc. \(\Delta_{i}\) is the fraction of labor employed in sector \(j\); \(\theta_{i}\) is the share of \(i\)-th factor in the total value of output in \(j\)-th sector, \(i = K, L\) and \(j = 1, 2\); \(\theta_{3}\) is the fraction of \(j\)-th sector output received by the monopolist as superprofits. Note that \(\lambda_{1} + \lambda_{2} = 1\) and \(\theta_{1} + \theta_{2} + \theta_{3} = 1\), \(j = 1, 2\). Finally,

\[A_{1} = (\beta\sigma P^{b})(1+\sigma P^{b})(1+\sigma P^{b})\]

\[A_{2} = (\beta\sigma P^{c})(1+\sigma P^{c})(1+\sigma P^{c})\]

Note further that \(MR_{1}^{*} = e_{1}^{*} + P_{1}^{*}\), and \(A_{1} = -e_{1}^{*}/P^{*}\), \(A_{2} = e_{2}^{*}/P^{*}\) [see Batra (1973), p. 288 for details]. Assuming commodity 1 to be the numeraire (so that \(P_{1}^{*} = 0\) and \(P^{*} = P_{1}^{*}\)), \(MR_{1}^{*}/P^{*} = -A_{1}\) and \(MR_{2}^{*} = (A_{2}+1)\). It should be noted here that \(A_{1}\) and \(A_{2}\) which denote the relation between price and marginal revenue in sector 1 and 2 respectively, are neither constant, nor are they each equal to unity despite the fact that the model has a CES utility function. As a matter of fact, they individually depend on commodity prices and other demand parameters of the model. Consequently, the percentage rate of change in price could very well be different from that in marginal revenue, and this could lead to the possibility of the resolution of the neoclassical ambiguity. This point has also been emphasized in the previous section where I have presented the result diagrammatically.
From the solution of equations (13) [given in Appendix 2], I derive the following propositions (proofs are in Appendix 3).

Proposition 1: Given \( \theta_{11} > 0 \) and \( \theta_{12} > 0 \), \( z_i^* > w^* > P_i^* > 0 > z_i \) if
\[
\left[ \frac{A_2}{(\lambda_1 \lambda_2 \theta_1 (A_1 \theta_{11} + (1 - \theta_{11})))} \right] - \left[ \frac{1}{(\lambda_1 \lambda_2 \lambda_1 (1 - \theta_{11})(1 - \theta_{11}^{-1}))} \right] > \left[ \frac{1}{(\lambda_1 \lambda_2 \lambda_1 (1 - \theta_{11})(1 - \theta_{11}^{-1}))} \right].
\]

Proposition 2: Given \( \theta_{11} = 0 \) and \( \theta_{12} > 0 \), \( z_i^* > w^* > P_i^* > 0 > z_i \) if
\[
\left[ \frac{A_2}{(\lambda_1 \lambda_2 \lambda_2 \theta_2 \theta_{12})} \right] - \left[ \frac{1}{(\lambda_1 \lambda_2 \lambda_2 \theta_2 \theta_{12} (1 - \theta_{12}))} \right] > \left[ \frac{1}{(\lambda_1 \lambda_2 \lambda_2 \theta_2 \theta_{12} (1 - \theta_{12}))} \right].
\]

Proposition 3: Given \( \theta_{11} = \theta_{12} = 0 \), \( z_i^* (= r_i^*) > P_i^* > w^* > 0 > z_i \) if
\[
1 \left( \frac{1}{(\lambda_1 \lambda_2 \lambda_2 \theta_2 \theta_{12} (1 - \theta_{12}))} \right) > \left[ \frac{1}{(\lambda_1 \lambda_2 \lambda_2 \theta_2 \theta_{12} (1 - \theta_{12}))} \right].
\]

As noted earlier, \( \text{MR}_1^* / P_i^* = (1 + A_1) \) and \( \text{MR}_1^* / P_i^* = -A_1 \). Thus, \( A_2(\theta_2) \) denotes the rate at which marginal revenue in sector 2 (1) rises (falls) as a result of an increase in relative price of the second good. Furthermore, in this model the partial elasticity of demand for labor in sector j is given by (see Appendix 4)
\[
\eta_i = \left( \frac{P_i^*}{w^*} \right) \left( \frac{A_i \theta_{11} + (1 - \theta_{11})}{A_i + A_1 \theta_{11}} \right) < 0.
\]

Therefore,
\[
\frac{A_i \theta_{11} + (1 - \theta_{11})}{A_i + A_1 \theta_{11}} = -\eta_i (A_i + A_1 \theta_{11}). \tag{14}
\]

Substitution of (14) in the sufficient condition stated in proposition 1 will reveal that higher the values of \( \eta_i \) and \( A_2 \) and/or lower the values of \( \eta_i \) and \( A_1 \), the greater is the likelihood of \( (w^* - P_i^*) / P_i^* > 0 \) and \( (w^* - P_i^*) / P_i^* > 0 \). Thus, the real wage rate may rise in terms of either good if the partial elasticity of labor demand in the protected (unprotected) industry are relatively higher (lower), and/or if the tariff-induced rise in price leads to a relatively larger (smaller) increase (decrease) in the marginal revenue in the protected (unprotected) industry.

Proposition 2 gives the sufficient conditions for resolution of the conflict when the importable sector is monopolistic, but the exportable sector is competitive. This is the case I discussed diagrammatically in section II. Proposition 3 is the traditional result. If superprofits equal zero (i.e., perfect competition) in both sectors, we get the neoclassical ambiguity. Whereas real wage rate in terms of good 1 rises, it falls in terms of good 2, thus leading to a conflict interests as between labor and capital-owners in the protected industry.

IV. Concluding Remarks

In this paper I have shown that in the Ricardo-Viner model of production, the
perpetuation of domestic monopoly protected by tariffs may lead to a possible resolution of the neoclassical ambiguity in the distributional implications of protection. The basic argument centers on the price searching ability of the domestic monopolist who is assumed to produce a good that is a perfect substitute of the foreign import. The monopolist is protected by a prohibitive tariff. As is well known, because of the price searching phenomenon, there arises a divergence between price and marginal revenue under imperfectly competitive markets. If, as a result of the tariff, marginal revenue rises proportionately more than the price, real wage of labor and real income of capital-owner may rise in terms of either good. Thus, in the light of the preceding analysis, it is not surprising that both labor and capital-owner in an imperfectly competitive industry might be unanimous in their demand for protection from foreign competition. Sufficient conditions for the existence of such an outcome are rigorously derived in terms of various parameters such as partial elasticities of demand for labor in each sector, elasticities of substitution in production and consumption, and the distributive as well as the allocative parameters of the model.

Appendix 1

From the unit-cost minimization conditions and also the definition of elasticity of substitution in production, I derive the following:

\[ C_i^* = -\theta_i \sigma_i (w^s - r_i^s) / (1 - \theta_i) \]  
\[ C^*_k = \theta_k \sigma_k (w^s - r_k^s) / (1 - \theta_k) \]  
\[ A1.1 \]
\[ A1.2 \]

Since capital is sector-specific,

\[ X_i = -C_i^* = -\sigma_i \theta_i (w^s - r_i^s) / (1 - \theta_i). \]  
\[ A1.3 \]

From equation (2) in the text I have

\[ \lambda_{1i} X_i^* + \lambda_{2i} X_2^* = -\lambda_{1i} C_i^* - \lambda_{2i} C_2^*. \]  
\[ A1.4 \]

Substituting equations (A1.1) through (A1.3) in equations (A1.4) and simplifying, I obtain the first equation of (13) in the text. The last two equations in (13) are derived by differentiating and simplifying equations (5) and (6) in the text.

Appendix 2

The coefficient determinant of the system (13) is
\[ J^* = -[\sigma_{12} \theta_{12}(1-\theta_{11}) + \sigma_{12} \theta_{12}(1-\theta_{11})] < 0. \]

The solution of (13) yields the following comparative static derivatives.

\[
\begin{align*}
(w^* - P_2^*) / P^* &= J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) - \sigma_{12} \theta_{12}(A_2 \theta_{11})] \geq 0. \quad (A2.1) \\
(w^*) / P^* &= J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} - \sigma_{12} \theta_{12}(A_2 \theta_{11} + (1-\theta_{11})))] \geq 0. \quad (A2.2) \\
(r_1^* - P_2^*) / P^* &= J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11}))(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] \\
&\quad + A_1 \theta_{11}(\sigma_{12} \theta_{12} + \delta_{12} \theta_{12}) < 0. \quad (A2.3) \\
(r_1^*) / P^* &= J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) + A_1 \theta_{11}(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] \geq 0. \quad (A2.4) \\
(r_2^* - P_2^*) / P^* &= -J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) + A_1 \theta_{11}(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] > 0. \quad (A2.5) \\
(r_2^*) / P^* &= -J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) + A_1 \theta_{11}(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] > 0. \quad (A2.6) \\
(w^* - r_1^*) / P^* &= -J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) + A_1 \theta_{11}(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] > 0. \quad (A2.7) \\
(w^* - r_2^*) / P^* &= J^{-1} [\sigma_{12} \theta_{12}(A_1 \theta_{11} + (1-\theta_{11})) + A_1 \theta_{11}(\theta_{12}(\sigma_{12} + \sigma_{12}) + \sigma_{12} \theta_{12})] > 0. \quad (A2.8) \\
(z_1^* - P_2^*) / P^* &= J^{-1} [\theta_{12}(1-\theta_{12})^{-1} [A_2 (\sigma_{12} \theta_{12} A_1 \theta_{11}(1-\theta_{11})) + A_1 \theta_{11}(1-\theta_{11}) - \sigma_{12} \theta_{12}(\theta_{12} + \sigma_{12} \theta_{12})] > 0. \quad (A2.9) \\
(z_1^* - P_2^*) / P^* &= 1 + [(z_1^* - P_2^*) / P^*] > 0. \quad (A2.10) \\
(z_1^* - P_2^*) / P^* &= -[1 - (1-\theta_{12})^{-1} - [\theta_{12}(1-\theta_{12})]^{-1}] J^{-1} [1 + \sigma_{12} \theta_{11}(1-\theta_{11})^{-1}] \\
&\quad \geq 0. \quad (A2.11) \\
(z_1^* - P_2^*) / P^* &= -[1 - (1-\theta_{12})^{-1} - [\theta_{12}(1-\theta_{12})]^{-1}] J^{-1} [1 + \sigma_{12} \theta_{11}(1-\theta_{11})^{-1}] \\
&\quad \geq 0. \quad (A2.12) \\
(z_1^* - w^*) / P^* &= -J^{-1} [\theta_{12}(1-\theta_{12})^{-1} - (\sigma_{12} \theta_{12} A_1 \theta_{11}(1-\theta_{11})^{-1} + A_1 \theta_{11}(1-\theta_{11})^{-1} + \sigma_{12} \theta_{12} A_1 \theta_{11}(1-\theta_{11})^{-1}] \\
&\quad \geq 0. \quad (A2.13) \\
(z_1^* - w^*) / P^* &= -J^{-1} [\sigma_{12} \theta_{12} A_1 \theta_{11}(1-\theta_{11})^{-1} + A_1 \theta_{11}(1-\theta_{11})^{-1} + \sigma_{12} \theta_{12} A_1 \theta_{11}(1-\theta_{11})^{-1}] \\
&\quad \geq 0. \quad (A2.14)
\end{align*}
\]
Appendix 3

Proof of proposition 1:

Let \( P^* = P^*_r > 0 \). Then from equation (A2.1), \((w^* - P^*_r) > 0\), if \( \sigma_{\lambda_4, \theta_{HH}} A_\theta H > \sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})) \), or if

\[
[A_\theta \left/ (\sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})) \right)] > \left[1/\sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \right].
\] (A3.1)

From equation (A2.2) \( w^* > 0 \), if

\[
\sigma_{\lambda_4, \theta_{HH}} A_\theta H > \sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})) \), or if,
\[
\sigma_{\lambda_4, \theta_{HH}} A_\theta H > \sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})) - \{ \sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) + \sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \}.
\] (A3.2)

Note that (A3.2) holds, if (A3.1) is satisfied since \( \theta_{HH} < 1 \).

From (A2.9) \((z^*_r - P^*_r) > 0\), if

\[
\sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) (1-\theta_{HH}) > \sigma_{\lambda_4, \theta_{HH}} \theta_{HH}, \text{ or if}
\]

\[
\left[1/\sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \right] > \left[1/\sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) (1-\theta_{HH}) \right].
\] (A3.3)

In equations (A2.11) and (A2.12) \((z^*_r - P^*_r) < 0\), and \( z^*_r < 0 \), if

\[
\sigma_{\lambda_4, \theta_{HH}} A_\theta H > \sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})), \text{ i.e., if the inequality (A3.1) holds.}
\]

In equation (A2.13) a sufficient condition for \((z^*_r - w^*) > 0\) is

\[
[(\sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) (1-\theta_{HH})) \{ \sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) + \sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \}] >
\]

\[
[(1-\theta_{HH}) (1-\theta_{HH}) (1-\theta_{HH}) + (1-\theta_{HH}) (1-\theta_{HH}) (1-\theta_{HH}) (1-\theta_{HH})] \sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \theta_{HH} \theta_{HH}].
\]

Further simplification of this inequality yields the following condition for a positive \((z^*_r - w^*)\) [see Appendix 5]:

\[
\left[1/\sigma_{\lambda_4, \theta_{HH}} \theta_{HH} \right] > \left[1/\sigma_{\lambda_4, \theta_{HH}} (1-\theta_{HH}) (1-\theta_{HH}) \right].
\] (A3.4)

Finally, from equation (A2.14) \((z^*_r - w^*) < 0\), if

\[
\sigma_{\lambda_4, \theta_{HH}} A_\theta H > \sigma_{\lambda_4, \theta_{HH}} (A_\theta H + (1-\theta_{HH})), \text{ i.e., if the inequality (A3.1) holds.}
\]

The inequalities (A3.1), (A3.3) and (A3.4) together with the fact that \( \theta_{HH} < 1 \) prove the proposition.

Proofs of propositions 2 and 3 trivially follow from proposition 1 under the assumption that \( \theta_{HH} = 0 \) in the former case, and \( \theta_{HH} = \theta_{HH} = 0 \) in the latter case.
Appendix 4

Define partial elasticity of demand for labor in sector j as

$$\eta_j \equiv L_j^* / (w^* - P_j^*).$$  \hfill (A4.1)

Since all variables other than $L_j, w$ and $P_j$ are treated as constant,

$$\sigma_j = -L_j^* / w^*.$$  \hfill (A4.2)

From the price equations

$$\theta_w w^* = P_j^* (1 - \theta_{ll}) + c_j^* \theta_{ll}.$$  \hfill (A4.3)

Substituting the value of $c_j^*$ in equation (A4.3), I have

$$P_j^* = \theta_w w^*/[\theta_{ll} + (1 - \theta_{ll})].$$  \hfill (A4.4)

Now I substitute equations (A4.2) and (A4.4) in (A4.1) to obtain

$$\eta_j = -\sigma_j [\theta_w + (1 - \theta_{ll})]/[\theta_{ll} + \theta_{ll}].$$  \hfill (A4.5)

Appendix 5

Let $N = \sigma_2 \lambda_{ll} (1 - \theta_{ll})^{-1} (1 - \theta_{ll})^{-1}$, and $\alpha = (1 - \theta_{ll})^{-1}$. Then from equation (A2.13)

$$(z^*_2 - w^*) / P^* > 0,$$  \hfill if

$N(\sigma_2 \lambda_{ll} (1 - \theta_{ll}) + \sigma_2 \lambda_{ll} \theta_{ll}) > (N + \alpha) \sigma_2 \lambda_{ll} \theta_{ll} \theta_{ll}$, or if

$[N \theta_{ll} / (N + \alpha)] \sigma_2 \lambda_{ll} (1 - \theta_{ll}) > \sigma_2 \lambda_{ll} \theta_{ll} [\alpha / (N + \alpha)],$ or if

$N \theta_{ll} \sigma_2 \lambda_{ll} (1 - \theta_{ll}) > \sigma_2 \lambda_{ll} \theta_{ll} \theta_{ll}$, or if

$\sigma_2 \lambda_{ll} \theta_{ll} \theta_{ll} (1 - \theta_{ll}) (1 - \theta_{ll})^{-1} > \sigma_2 \lambda_{ll} \theta_{ll} \theta_{ll}$, or if the inequality (A3.4) holds.
References


