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Factor Prices and the Shape of Average Cost Curves with Special References To International Trade**

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Abstract

This paper shows that the shapes of average cost and marginal cost curves and the possibility of scale economies can be quite sensitive to factor price configuration. We illustrate our analysis in terms of "international competitiveness."

Since the days of Frank Knight (1921) and Jacob Viner (1931), microeconomic analysis in terms of the usual U-shaped average cost curve and the marginal cost curve (which passes through the minimum point of the AC curve) has become a common intellectual tool of economists, and the analysis in terms of these devices has produced a bulk of interesting and fruitful results. Today, students in economics learn this apparatus early and virtually all intermediate price theory textbooks contain a discussion of such cost curves and their applications.

Needless to say, it is well-known that the shape of the AC curve is not necessarily U-shaped. For example, any homogeneous production function does not yield U-shaped AC curve (under any factor price configurations).[1] It is also well-known that the shape of the average and the marginal cost curves depend on factor prices as well as output, and hence any change in factor prices would shift these curves. On the other hand, it is not well-recognized that a slight change in factor prices *can* result in a drastic change in the AC rather than a simple shift. In this paper, we intend to show, by way of simple

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1. Function $y=f(x)$ is said to be homogeneous of degree m if $f(\lambda x)=\lambda^m f(x)$ for all $x>0$ and $\lambda>0$ (e. g., Takayama [1972], p. 31). It can be shown easily that the AC curve associated with production function $f(x)$ is monotonically decreasing or increasing depending on whether $m>1$ or $m<1$. When $m=1$, $AC=MC=k$, where k depends only on factor prices and it is indepenednt of the level of output (the so-called Shephard-Samuelson theorem).

examples, that such is indeed possible.[2] These examples will then be useful in en hancing our understanding of this important analytical apparatus. It will also serve to caution students and others on its use.

Needless to say, there can be many applications of the sensitivity of AC with respect to factor prices to various fields of economics such as industrial organization (e. g., the determination of a particular industrial structure). One interesting application may be found in the field of international trade. Consider two countries which are capable of producing the same commodity under the identical technology. Since the factor price configuration is in general different between the two countries, the shape of AC would also be different. For example, it is possible that in one country the AC curve is strictly decreasing (i. e., "economies of scale" prevail)[3] over the relevant range of output, whereas in the other country the AC curve is strictly increasing. This then may affect the "competitiveness" of producing the commodity (say, "steel") between the two countries (say, "Japan"and "Korea"), even if both countries have identical production technology. On the other hand, the existence of scale economies may have little to do with "competitiveness". Namely, Japan may not be able to sell steel in the world market at a lower price compared to Korea, even if scale economies prevail in the Japanese steel industry and scale diseconomies prevail in the Korean steel industry. The reader may obtain some other applications of our observations to international trade theory.[4] Note also that the present paper makes a clear departure from the traditional Heckscher-Ohlin world in which production functions are assumed to be homogeneous of degree one. This in turn rules out variable returns to scale and nonhomotheticity.[5]

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2. This should not be confused with the fact that the AC function is continuous with respect to factor prices. The point in issue will become clear as our discussion develops. We consider those examples in which the input price vector is exogeneously given to the firm. Note that this then enables us to avoid further complications associated with monopsony.
3. The concept of economies of scale or returns to scale is, contrary to the usual perception, not well understood in the literature. See Hanoch (1975) and Ide-Takayama ([1987], [1989]). Here we simply define scale economies and scale diseconomies in terms of a falling AC and a rising AC, respectively.
4. For a recent excellent survey of the role of scale economies on trade theory, see Helpman (1984). Our result on the sensitivity of AC by a change in factor prices would become more serious in a general equilibrium context in which the number of inputs and outputs respectively exceeds one, and constant returns to scale are absent in any industry. An example is given by Chao-Takayama ([1987], [1989]). There it is shown that the well-known Chamberlin "tangency solution" needs not be stable, if we use such a general equilibrium framework.
5. Under such postulate, factor price equilization will not in general occur even if both countries produce the same mix of goods

For the sake of simplicity, our examples are concerned with the case of two inputs (denoted by x_1 and x_2 and a single output(y)[6]. We denote the production function by f (x), where $x=(x_1, x_2)$. Suppose (in the wusual way) that our firm minimizes its cost, $w \cdot x(\equiv w_1x_1+w_2x_2)$ subject to $f(x) \geqq y$ and $x \geqq 0$ where $w=(w_1, w_2)$ is the input price vector which is exogeneously given to the firm. We denote the solution of this cost minimization problem as $x_i(w, y)$, $i = 1, 2$, and let $C(w, y) = w \cdot x(w, y)$ (the total minimum cost). Also define α and μ, respectively, by $\alpha(w, y) \equiv C(w,y)/y$ (average cost) and $\mu(w, y) \equiv \partial C(w, y)/\partial y$ (marginal cost). We assume $w_1>0$, $w_2>0$ and $y>0$.

Example 1:

(1) $f(x) \equiv x_1 + \log(x_2+0.5)$, where $x \in \{x \geqq 0: f(x) \geqq 0\}$.

Usual cost minimization yields:

(2-a) $x_1(w, y)= y - \log q$, $x_2(w, y)=q - 0.5$, $C(w, y)=w_1(y - A)$,

(2-b) $\alpha(w, y)=w_1(1 - A/y)$, $\mu(w, y)=w_1$,

where $q \equiv w_1/w_2$ and $A \equiv \log q - 1+1/(2q) \equiv A(q)$. Assume an interior solution, i.e.,$x_1(w, y)>0$ and $x_2(w, y)>0$, which in turn requires $y>\log q$ and $q>0.5$, respectively. Also $C(w, y)>0$ requires $y>A$, which is satisfied for and $q>0.5$, (as $y>\log q$ and $\log q>A$ for $q>0.5$). Note that A is determined solely by factor prices.[7] Next, we define q^* by $A(q^*)=0$, i. e.,

$\log q^*=1-1/(2q^*)$.

Clearly such a q^* exists uniquely, and $q^*>1$. By a simple iteration, q^* can be computed as, $q^*=2.1555\cdots$. Also, we have:

(3) $A(q)<0$ for $0.5<q<q^*$, and $A(q)>0$ for $q>q^*$.

Note that, the AC curve is defined only for $y>a(\equiv \log q)$ if $q>1$, whereas the AC

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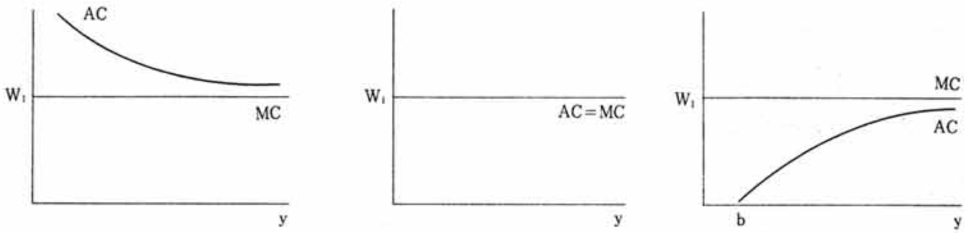
6. Also, for the sake of simplicity, we assume away the complication due to such factors as "fixed costs" and "indivisibilities". The emphasis of these factors, which is seen in some literature to obtain falling average costs, has an important element of truth. However such should not be overemphasized or at least should be understood correctly. To this end, it may suffice to recall Frank Knight's seminal paper (1921), in which he, for example, states, "There is a Fallacy in overlooking the fact that any amount of commodity *could* be made by any one of the methods available" (p. 333). Also, the following remark by Kaldor ([1935], p. 34. fn. 1) may be of some interest: " in the long-run the supply of all factor--even the resources supplied by entrepreneur himself--can be assumed variable, and consequently there are no 'fixed' costs." In any case, the presence of such factors as "fixed costs" and "indivisibilities" will not affect the argument of the present paper in any essential way.

7. Let $a \equiv \log q$. The $x_1>0$(at optimum) requires $y>a$. This condition is always satisfied for all $y>0$ if $0.5<q\leqq 1$. For $q>1$, we have $a>0$. Hence $y>a$ implies that y must be bounded away from $a>0$. The value of a increases as q increases.

curve is defined for all y if $0.5<q<1$. With (2-b) and (3), we can now illustrate the average cost (AC) and the marginal cost curves in Figure 1, where $b \equiv A(q)$ (which is less than $a>0$ for case c).

Figure 1: The Shapes of AC and MC for Example 1



Case a: $0.5<q<q^*$ Case b: $q=q^*$ Case c: $q^*<q$

Thus, for Example 1, we may conclude:

(i) The marginal cost (MC) is constant regardless of y: the MC line is parallel to the output axis, where $MC = w_1$.

(ii) There exists a "critical value" q^* such that (a) if $q(\equiv w_1/w_2)<q^*$, the AC curve is always above the MC line and decreases as y increases, asymptotically approaching the MC line, (b) if $q=q^*$, AC=MC for all y, and (c) if $q>q^*$, the AC is always below the MC line and increases as y increases, asymptotically approaching the MC line. Thus, AC is not U-shaped, and the shape of AC can change drastically by a slight change in the input price configuration.

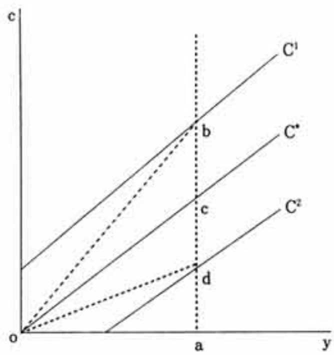
(iii) Constant returns to scale (i. e., AC=MC) prevail if and only if $q=q^*$, in which case $AC=w_1$.

Note that the AC function obtained in (2-b) is a continuous function of factor prices over the relevant range of q. Namely, statement (ii) is different from the discontinuity of the AC function.

It is not easy to obtain intuitive explanation of the phenomena such as the one described in statement (ii). However the following consideration may help us to understand the question at issue. Recalling $C(w, y)=w_1(y-A)$ from (2-a), we may illustrate the total minimum cost line (TMC) in Figure 2. The lines C^2, C^* and C^1 depict TMC's for q^2, q^*, and q^1, respectively, where $A(q^2)<0$, $A(q^*)=0$, and $A(q^1)>0$. From (3), we have, $0.5<q^2<q^*<q^1$. In Figure 2, the slopes of the lines C^2, C^*, and C^1 represent MC's, whereas the slopes of dotted lines ob and od and solid line ac represent AC's for q^2, q^1, and q^*, respectively, at the output level a. It can be seen that as the level of output (y)

increases for fixed factor prices along the C^2-line (resp. the C^1-line), the value of AC decreases (resp. increases). Along the C^*-line, the value of AC stays constant for an

Figure 2: AC, MC, and TMC for Example 1



increase in y. From this we may conclude that scale economies prevail if $0.5<q<q^*$, scale diseconomies prevail if $q^*<q$, and constant returns to scale prevail if $q^*=q$. It can also be easily seen that AC$\gtreqless$MC according to whether $q^*\gtreqless q$. From this we may also conclude that scale economies prevail if AC>MC, and that scale diseconomies prevail if AC<MC. Now consider a change in q from q^2 to q^1. This change in q causes a shift in the TMC line from the C^2-line to the C^1-line, so that the value of AC becomes less than the value of MC at each output level. Consequently, scale economies turn into scale diseconomies as q moves from q^2 to q^1.

Let us return to the example in international trade. Consider two countries, H and F, which have the same technology in the production of y, where x_1 and w_2 may be called "capital" and "labor," respectively. Suppose that capital is completely mobile between the two countries (so that $w_1^H=w_1^F$), and that country F is relatively more labor abundant than H in the sense that $q^H<q^F$. If $q^H<q^*<q^F$, then for each level of output, country F can produce the commodity less expensively than country H, whereas scale economies prevail in H and scale *dis*economies prevail in F. So country H cannot compete with F in the world market in the production of y, and scale economies and diseconomies matter very little in determining the international competitiveness.

Example 2:

(4) $f(x) \equiv [x_1+\log(x_2+0.5)]^{1/2}$, where $x \in \{x\geqq 0: f(x)\geqq 0\}$.

In Example 1, $\partial^2 f/\partial x_1^2=0$ although $\partial x_2^2<0$. In Example 2, we can show, by a straightforward computation, $\partial^2 f/\partial x_1^2<0$ and $\partial^2 f/\partial x_2^2<0$.Namely, Example 2 more completely incorporates the law of diminishing marginal productivity.

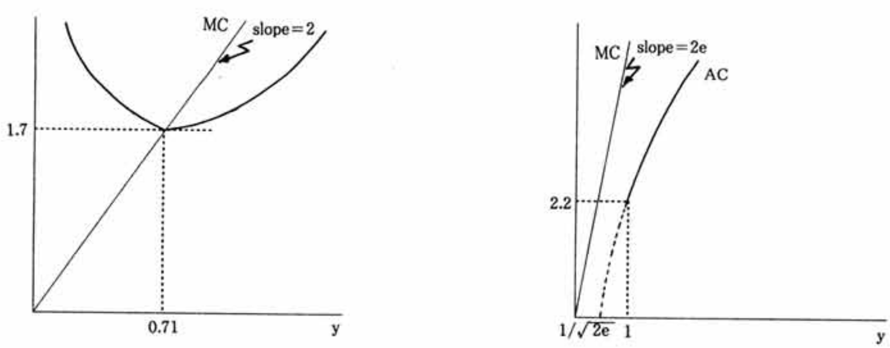
The usual cost minimization problem under (4) yields:

(5-a) $x_1(w, y)=y^2-\log q$, $x_2(w, y)=q-0.5$, $C(w, y)=w_1(y^2-A)$,

(5-b) $\alpha(w, y)=w_1(y-A/y)$, $\mu(w, y)=2w_1 y$,

where $q \equiv w_1/w_2$ and $A \equiv \log q-1+1/(2q)$ as before. Also, $x_1(w, y)>0$, $x_2(w, y)>0$, $y>0$ and $C(w, y)>0$ require $q>0.5$ and $y^2>A$. We henceforth assume $q>0.5$ and $y^2>A$. The slope of the AC curve is obtained as, $\partial\alpha/\partial y=w_1(1+A/y^2)$. Suppose that $w_1=w_2=1$. Then $A(q)=-0.5$, so that $C>0$ for all $y>0$. Let $B \equiv \sqrt{0.5}\simeq 0.71$. Also, $\partial\alpha/\partial y<0$ for $0<y<B$, $\partial\alpha/\partial y=0$ for $y=B$, and $\partial\alpha/\partial y>0$ for $y>B$. Namely, the AC curve is U-shaped in this case, where we may note that $\partial^2\alpha/\partial y^2=y^{-3}>0$. Next suppose that $w_1=e$ and $w_2=1$ then $A(q)=1/(2e)$, so that $\partial\alpha/\partial y>0$. Namely, the AC curve is strictly increasing, where $C>0$(or $y^2>A$) requiresy$>1/\sqrt{2e}$. Thus, again the shape of average costs curve can change drastically due to a change in factor prices. These two cases are illustrated in Figure3.

Figure 3: The Shapes of AC and MC for Example 2



Case a: $w_1=1$, $w_2=1$

Case b: $w_1=e$, $w_2=1$

In the literature it is often argued that scale economies prevail up to a certain level of output (say, y^*) and they "are exhausted" beyond y^*, where scale economies are defined in terms of falling average cost curves. Then we often see empirical studies to determine such a statement for particular industries. Our example indicates that empirical studies with time series data often make little sense. In Example 2, for $w_1=w_2=1$, it is perfectly valid to state that scale economies prevail up to $y^* \equiv B\simeq 0.71$, and they are exhausted beyond y^*. Yet, if factor price changes so that $w_1=e$ and $w_2=1$, scale economies completely cease to exist. Thus, the statement such as "scale economies are exhausted beyond certain points" should be taken with great caution as it can crucially depend on a particular configuration of factor prices.

The present note raises the interesting question of whether or not anything can be said, in general, about the production function yielding the phenomena described herein. This is left to a future study.

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