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# Optimal Storage for a Small Open Economy under Price and Production Uncertainty

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## Abstract

*In this paper we examine welfare-maximizing storage for a small open economy subject to price and production uncertainty. We show that production decisions, and the pattern of trade in some periods, may be affected by the storability of a good.*

## I. Introduction

Although Taub (1987) discusses a case of storage in the case of certainty, where storage maximizes utility directly because production and consumption are not coincident, nearly all storage literature concerns storage as a price stabiliser. Turnovsky (1978) gives an excellent survey of this literature, starting with the classic papers by Oi and Massell, who assume additive supply and demand uncertainty, and implicitly, risk-neutrality on the part of both producers and consumers. J. M. Keynes (1938, 1974) proposed an international storage oragnization to stabilize both prices and market supply, particularly those of the volatile primary commodities heavily produced by the LDCs. He posited this organisation as a natural companion to the IMF and the IBRD. Yet while price stabilisation makes production and therefore input decisions much easier as the variance

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of price decreses, it may well not be optimal in the sense of maximizing a country's welfare (see Oi, Massell, and Wright(1979)). Other segments of the literature explore the effects of output stabilisation, which is subject to the same critique. Brian Wright and Jeffrey C. Williams (1979, 1984), however, consider storage as a stabiliser of income which might potentially stabilise welfare.

Very little work has been done on storage in the context of international trade, despite the potential role of storage as an income-stabilising welfare-stabiliser, a la Keynes. Both Newbery and Stiglitz (1981) and Batra (1975) include brief discussions which principally indicate that storage by a large country affects the vector of relative prices. Although a small country cannot, by definition, change the price vector through storage, a small country may store in order to maximize a group utility function over a period of time. In this paper, we consider the determinants of optimal storage for a small country with a von Neumann-Morgenstern type utility or welfare function, with discount factor $0<r<1$. Helpman and Razin (1978) have shown that the pattern of trade may be reversed under uncertainty if securities, like storage a method of risk-sharing, are not internationally traded. We show that storage itself may change the pattern of trade.

Like Wright and Williams (1984), we assume known multiplicative risk in production, so that the output of good X at time t, using amounts of capital and labour $K_t$and $L_t$, is

$$X_t=\theta(\alpha_t)f\ (K_t,\ L_t)$$

where $\alpha_t$ is the state of the world obtaining at time t. $E(\theta\ (\alpha_t))=\bar{\theta}=1$, $var(\theta(\alpha_t))=\sigma^2$, $\sigma^2>0$ if the distribution is non-degenerate. Since risk is multiplicative, the amount of production lost or gained depends on the quantity of production planned.

As is customary, we will write the actualized price of good one in terms of good two as $p(\alpha_t)$, $E(p(\alpha_t))=\bar{p}$, without attempting to specify the form of risk. Clearly, there will in general be a strong relation between production and price uncertainty in the world in toto or within a closed economy. In this paper, only a small open economy is discussed, so that the sign of $cov(\theta(\alpha_t),\ p(\alpha_t))\equiv cov(\theta,p)$ may be positive, negative or zero for any good. If $cov\ (\theta,\ p)=0$,

$$E(\theta(\alpha_t)\ P(\alpha_t))=\bar{\theta}\bar{p}+cov(\theta,\ p)=\bar{\theta}\bar{p}.$$

If $\mathrm{cov}(\theta, p)<0$, this means that domestic production and world prices tend to move in opposite directions. In the case of stochastic prices determined principally by world quantity supplied, this would tend to indicate that world fluctuations in productions agree in sign with domestic ones. If $\mathrm{cov}(\theta, p)>0$, the opposite is the case.

While the expectations for price and production risk are known and there is no method of minimizing income risk which would alter the production plan, a country producing two goods will plan to produce on the expected PPF (given by expected production functions) at the point of tangency with the expected price vector $p(\alpha_t)=p_1(\alpha_t)/p_2(\alpha_t)$. As we shall see, the possibility of storing one commodity can change the planned production.

We assume that only good 1 is storable, and denote storage at time t by $s_{1t}$. Storage is of course negative when net withdrawal occurs. Withdrawal is constrained by the amount of good 1 in store at time t. The amount of good 1 in storage at t+1 from $s_{1t}$ stored at time t is denoted $\delta(s_{1t})$. We assume that $\delta(s_{1t})$ is continuous and differentiable, with $\delta(s_{1t})>0$, and that $s_{1t}>\delta(s_{1t})>0$ where net storage occurs, so that storage cost $s_{1t}-\delta(s_{1t})$ are positive but not prohibitive.

Since the economy discussed is not monetary, the interest cost of storage is irrelevant. The rate of time preference, however, performs a similar function, in that at any moment in time, future consumptions is valued less highly than present consumption. Further, since goods "lent" to the future by storage result in a smaller volume being retrieved later, the real rate of return on storage is negative.

In this respect, storage functions not only as a means of production, but as an imperfect capital market, since the rates of interest to "borrowers" and "lenders" differ. Therefore, the Fisher Separation Theorem does not hold, and consumptior decisions cannot be made separate from production decisions. Producers will not in general aim to profit maximize. Because their incomes are uncertain and their utility functions (if storage is to occur) display risk-aversion, they will not find profit maximization desirable unless their utility depends directly on profit or the profit maximizing plan also minimizes risk. Production choices are no longer independent of tastes. When we assume preferences are risk-averse, the riskiness of a type of production determines its desirability.

We have assumed that storage is a costly form of production riskless with respect to its planned output (though, like other forms of production, risky with respect to prices actualized after the planning period). Since production by storage is certain in the quantity sense, it can be shown that storage will occur if the cost of storage is

low enough or risk aversion(in the Arrow-Pratt sense) is great enough. Without loss of generality, we assume that storage does occur in period zero, the first period in the objective function sum. Only three factors are affected by the number of periods over which the objective function is summed. If one cares only about consumption over a fixed number of periods, all available resources are consumed in the final period. In addtion, the relation between storage in a given period and marginal utility or welfare of consumption in previous periods becomes more complex as more previous periods exist. Since neither of these factors seems fundamental, we give the model in two periods only and thereby avoid the difficulties of the stock-out problem (which, as Wight [1982] has indicated, occurs with probability 1 in finite time).

In assuming only two periods, we minimize both the benefits and costs of storage. Storage occurs in period zero only if $p(\alpha_t)<\bar{p}$. While the probability in a symmetric distribution is greater than 1/2 that $p(\alpha_{t+1})>p(\alpha_t)$, the probability that a higher price will occur at some later period over ten, one hundred, or an infinite number of periods approaches 1, and correspondingly the expected benefits of storage rise as more periods are considered. At the same time, however, as more periods follow initial storage, more of the stock is eroded away.

We assume that there is perfect information on the distribution of price and production uncertainty and realized price and production on the part of both public and the central authority. Each of the two goods is produced with CRS technology. With an open economy, good 1 produced at home or abroad may be consumed or stored, with no preference between the two. Production is by means of two endowed factors, capital (K) and labour (L). The stored good is not capital for future production. Endowment and technology are assumed to be constant over time, and the endowment fully employed, so that at time t

$$\bar{K}_t=\bar{K},\ \bar{L}_t=\bar{L}.$$
$$f_{1t}(K_{1t},\ L_{1t})=f_1(K_{1t},\ L_{1t}).$$
$$f_{2t}(K_{2t},\ L_{2t})=f_2(\bar{K}-K_{1t},\ \bar{L}-L_{1t}) \text{ for all t.}$$

We assume that production, trade, and storage are planned by either the population of the small country or its government, shares of the value of production being distributed among the populace in a welfare maximizing fashion.

The planning of inputs into each type of production is done in period $t-1$, when the distributions of price and production in period t are known, but the realizations

are not known. In period t, these random factors are realized and known. Trade, storage, consumption, and planning for t+1 are undertaken on the basis of realized prices and production and the known, stable distributions which will continue to obtain in future.

The prices of both goods are in terms of good 2, whose relative price is always 1.

## Ⅱ. The Closed Economy

In a closed economy, price and production uncertainty will be highly correlated, since prices must adjust so that all goods produced are consumed or stored. Nonetheless, in terms of the budget constraint, prices in a given period are irrelevant, since good 1 cannot be instantaneously transformed into good 2 or vice versa. Production must eventually be consumed. Below, in stating the closed economy problem, we assume that the government stores optimally with respect to some social welfare function or, equivalently, that the social welfare function is a community untility function which is the sum of n identical homothetic utility functions, where n is the population. In the latter case, government storage is equivalent to storage by all individuals so long as public and private costs of storage do not differ. The two-period economy problem is :

$$\max E[W(c_{10}, c_{20}\Gamma W(c_{11}, c_{21})] \text{ such that } \underline{c}_{1t}, \underline{c}_{2t}, \underline{K}, \underline{L}, \underline{W}_{10}$$
$$c_{10}=\theta_1(\alpha_0)f_1(K_{10}, L_{10})-s_{10}$$
$$c_{20}=\theta_2(\alpha_0)f_2(\overline{K}-K_{10}, \overline{L}-L_{10})$$
$$c_{11}=\theta_1(\alpha_1)f_1(K_{11}, L_{11})+\delta(s_{10})$$
$$c_{21}=\theta_2(\alpha_2)f_2(\overline{K}-K_{11}, \overline{L}-L_{11})$$
$$\theta_1(\alpha_0)f_1(K_{10}, L_{10})\geq s_{10}\geq 0.$$

The first four constraints must hold and so are represented as equalities. Since storage has been assumd to occur in period 0, $s_{10}>0$, and the right hand side of the fifth constraint has been presumed to occur and need not appear in the Lagrangian. The left hand side of the fifth constraint will not in general be binding, since it states only that no more of good 1 can be srored in period 0 than has been produced, so that in general, $\mu=0$.

Solving the maximization problem, we find the follwing relations as first order conditions :

$$E[W_1(c_{10}, c_{20})-\lambda_1]=0,$$
$$E[W_2(c_{10}, c_{20})-\lambda_2]=0,$$
$$E[\Gamma W_1(c_{11}, c_{21})-\lambda_3]=0,$$
$$E[\Gamma W_2(c_{11}, c_{21})-\lambda_4]=0,$$
$$E[(\lambda_1+\mu)\theta_1(\alpha_0)f_{1k}(K_{10}, L_{10})-\lambda_2\theta_2(\alpha_0)f_{2k}(\overline{K}-K_{10}, \overline{L}-L_{10})]=0,$$
$$E[\lambda_1+\mu)\theta_1(\alpha_0)f_{11}(K_{10}, L_{10})-\lambda_2\theta_2(\alpha_0)f_{2L}(\overline{K}-K_{10}, \overline{L}-L_{10})]=0,$$
$$E[\lambda_3\theta_1(\alpha_1)f_{1k}(K_{11}, L_{11})-\lambda_4\theta_2(\alpha_1)f_{2k}(\overline{K}-K_{11}, \overline{L}-L_{11})]=0,$$
$$E[\lambda_3\theta_1(\alpha_1)f_{11}(K_{11}. L_{11})-\lambda_4\theta_2(\alpha_1)f_{2L}(\overline{K}-K_{11}, \overline{L}-L_{11})]=0,$$
$$E[\lambda_1-\lambda_3\delta(s_{10})+\mu]=0.$$

where $\lambda_1$ is the marginal return to consumption of good 1 in period zero, $\lambda_2$ to consumption of good 2 in period zero, $\lambda_3$ to consumption of good 1 in period one, $\lambda_4$ to consumption of good 2 in period one and $\mu$ to storage in period zero.

The consumption first order conditions are satisfied ex ante. after the price and production have been realized, since people can consume until they are and

$$W_1(c_{10}, c_{20})/ W_2(c_{10}, C_{20})=\lambda_1/\lambda_2,$$
$$\Gamma W_1(c_{11}, c_{21})/\Gamma W_2(C_{11}, C_{21})=W_1(C_{11}, C_{21})/ W(C_{11}, C_{21})=\lambda_3/\lambda_4,$$
$$W_1(c_{10}, c_{20})/\Gamma W_1(C_{11}, C_{21})=\lambda_1/\lambda_3,$$
$$W_2(c_{10}, c_{20})/\Gamma W_2(C_{11}, C_{21})=\lambda_2/\lambda_4.$$

None of these relations are astounding : they are basic to the analysis of time preference and recur in the following models though they will not be restated below.

Production is planned in period $-1$, so the amount of capital and labour applied to each form of production depends on their expected marginal products :

$$\frac{E(\lambda_1+\mu)\theta_1(\alpha_0)}{E\lambda_2\theta_2(\alpha_0)}=\frac{f_{2k}(\overline{K}-K_{10}, \overline{L}-L_{10})}{f_{1k}(K_{10}, L_{10})}=\frac{(\lambda_1+\mu)}{\lambda_2},$$

$$\frac{E(\lambda_1+\mu)\theta_1(\alpha_0)}{E\lambda_2\theta_2(\alpha_0)}=\frac{f_{2L}(\overline{K}-K_{10}, \overline{L}-L_{10})}{f_{1L}(K_{10}, L_{10})}=\frac{(\lambda_1+\mu)}{\lambda_2},$$

$$\frac{E\lambda_3\theta_1(\alpha_1)}{E\lambda_4\theta_2(\alpha_1)}=\frac{f_{2k}(\overline{K}-K_{11}, \overline{L}-L_{11}}{f_{1k}(K_{11}, L_{11})}=\frac{\lambda_3}{\lambda_4},$$

$$\frac{E\lambda_3\theta_1(\alpha_1)}{E\lambda_4\theta_2(\alpha_1)} = \frac{f_{2L}(\overline{K}-K_{11}, \overline{L}-L_{11})}{f_{1L}(K_{11}, L_{11})} = \frac{\lambda_3}{\lambda_4}.$$

These conditions state that capital and labour will be used to produce goods 1 and 2 untill production is at the point of tangency between the expected PPF and the expected relative price vector(which is determined in this case by the amount of realized production and the public's prefernces). Planned production is unaffected by the possibility of storage. The last first order condition gives the optimal amount of storage in period zero, which has been assumed strictly positive.

$E[\lambda_1-\lambda_3\delta'(s_{10})+\mu]=\lambda_1-\lambda_3\delta'(s_{10})+\mu=0$ implies
$s_{10}=\delta'^{-1}[(\lambda_1+\mu)/\lambda_3]$.

## Ⅲ. The Open Economy

When the small economy is open, fluctuations in its production no longer necessarily have any correlation with price fluctuations, and its budget constraint now depends on the value of production rather than the absolute amount of each good produced, since a small ecconomy is assumed able to purchase of sell any amount of a good at the prevailing world price without altering it (Hence a small country cannot alter world price through storage, nor can any individual within it.) Further, the country can store good 1 which is purchased from abroad, and may wish to do so in periods in which its production and the price of good 1 are lower than expected.

In this case. the maximization problem is :

$$\max_{c_{1t},\ c_{2t}K_{1t},\ W_{10}} \quad E[W(c_{10}, c_{20})+\Gamma W(c_{11}, c_{21})] \text{ such that}$$

$$P(\alpha_0)c_{10}+c_{20}=p(\alpha_0)\theta_1(\alpha_0)f_1(K_{10}, L_{10})+\theta_2(\alpha_0)f_2(\overline{K}-K_{10}, \overline{L}-L_{10})-p(\alpha_0)s_{10}$$

$$p(\alpha_1)c_{11}+c_{21}=p(\alpha_1)\theta_1(\alpha_1)f_1(K_{11}, L_1+\theta_2(\alpha_1)f_2(\overline{K}-K_{11}, \overline{L}-L_{10})+p(\alpha_1)\delta'(s_{10})$$

$$p(\alpha_0)\theta_1(\alpha_0)f_1(K_{10}, L_{10})+\theta_2(\alpha_0)f_2(K-K_{10}, \overline{L}-L_{10})$$
$$\geq p(\alpha_0)s_{10} \geq 0.$$

As above, the first two constraints must hold, the left hand side of the third inequality will not in general bind, and we have assumed that the right hand side of the third

inequallity is satisfied. The first order conditions are:

$$E[W_1(c_{10}\ ,\ c_{20})-p(\alpha_0)\lambda_1]=0,$$
$$E[W_2(c_{10}\ ,\ c_{20})-\lambda_1]=0,$$
$$E[\Gamma W_1(c_{11})-p(\alpha_1)\lambda_2]=0,$$
$$E[\Gamma W_2(c_{11},\ c_{21})-\lambda_2]=0,$$
$$E(\mu+\lambda_1)\ [p(\alpha_0)\ \theta_1(\alpha_0)\ f_{1k}(K_{10}\ ,\ L_{10})\ -\theta_2(\alpha_0)f_{2k}(\overline{K}-K_{10}\ ,\ \overline{L}-L_{10})]=0,$$

$$E(\mu+\lambda_1)[p(\alpha_0)\ \theta_1(\alpha_0)f_{1L}(K_{10}\ ,\ L_{10})-\theta_2(\alpha_0)f_{2l}(\overline{K}-K_{10}\ ,\ \overline{L}-L_{10})]=0,$$

$$E\lambda_2[p(\alpha_1)\ \theta_1(\alpha_1)f_{1k}(K_{11},\ L_{11})-\theta_1(\alpha_1)f_{2k}(\overline{K}-K_{11},\ \overline{L}-L_{11})]=0,$$

$$E\lambda_2[p(\alpha_1)\ \theta_1(\alpha_1)f_{1L}(K_{11},\ L_{11})-\theta_2(\alpha_1)f_{2L}(\overline{K}-K_{11},\ \overline{L}-L_{11})]=0,$$

$$E[\lambda_1+\mu)\,p(\alpha_0)-\lambda_2 p(\alpha_1)\delta'\,(s_{10})]=0=(\lambda_1+\mu)\,p(\alpha_0)-\lambda_2 p\delta'\,s_{10}).$$

since the realized prices of period zero are known when storage in performed.

The relations between consumption of the two goods in the same period and same good across two periods are similar to those in the closed economy case save that here the prices are important. These relations give expected values for the marginal utility of welfare of consumption of good i in period j, and, although these values may not be realized, people will consume each good in each period until the realized equation holds.

Planned production, on the other hand, may differ radically from that in the closed economy model. Initially, assume that $cov(\theta_1,p)=0$. Then $E(\theta_1(\alpha_t)p(\alpha_t))=\bar{\theta}_1\ \bar{p}$. In this case,

$$\frac{E[p(\alpha_t)\theta_1(\alpha_t)]}{E[\theta_2(\alpha_t)]}=\frac{\bar{p}\bar{\theta}_1}{\bar{\theta}_2}=\frac{f_{2k}(\overline{K}-K_{1t},\ \overline{L}-L_{1t})}{f_{1k}(K_{1t},\ L_{1t})}$$

and production will be as in the close case. It is irrelevant whether only price, only production, or both are risky in determining choice of inputs to production so long as price and production uncertainties are independent. When at least good two has a non-zero covariance between price and output risk, the choice of inputs to production

is made according to the relations

$$\frac{E[p(\alpha_t)\theta_1(\alpha_t)]}{E[\theta_2(\alpha_t)]} = \frac{\bar{p}\bar{\theta}_1 + cov(\theta_1,\ p)}{1} = \frac{f_{2k}(\bar{K}-K_{1t},\ \bar{L}-L_{1t})}{f_{1k}(K_{1t},\ L_{1t})},$$

$$\frac{E[p(\alpha_t)\theta_1(\alpha_1)]}{E[\theta_2(\alpha_t)]} = \frac{\bar{p}\bar{\theta}_1 + cov(\theta_1,\ p)}{1} = \frac{f_{2L}(\bar{K}-K_{1t},\ \bar{L}-L_{1t})}{f_{1l}(K_{1t},\ L_{1t})}.$$

Suppose that good 1 has a positive covariance between price and output uncertainty. Then the left hand side of each relation is greater than in the case of zero covariance, the certainty marginal products of labour and capital in production of good 2 will be set relatively higher with respect to good 1 than in the case of zero covariance in production of each good, and more of each factor will be devoted to production of the first good. If the covariance of price and production of good 1 is negative, the conuntry will plan to produce more of good 2.

This implies that a good which produces either a very low or a very high income, depending on the state of the world, is more desirable to produce than one which produces a more completely random income. This seems reasonable, since the covariance supplies more information on the income resulting from production of the non-zero-covariance good than is available on the income produced by the other. Conversely, a good which produces a relatively stable income is *not* preferred to one producing a more completely random income, which seems paradoxical. We suggest, however, that just as the possibility of income stabilisation through storage nullifies the predictions of simple comparative advantage, it makes risky gambles like producing more of a positive covariance good desirable. This would seem to indicate that methods of income stabilisation may be used as "security" for gambling.

The amount of storage in period zero, $s_{10}$ , is derermined through the last first order condition :

$$s_{10} = E\delta'^{-1}[\lambda_1+\mu)p(\alpha_0)/\lambda_2\bar{p}].$$

This in nearly identical to optimal storage in the closed economy case, except for the inclusion of the ratio of first period realized price to expected price, which we anticipated would influence storage decisions in precisely this way. The lower the realized price in period one, the higher $s_{10}$ is. The larger the argument of $\delta'^{-1}(s)$, the larger the result of applying the function.

## Ⅳ. Conclusion

Price and production risk have been considered in this paper, which considers storage as a method of risk-sharing across time and across states of nature. In the empirical world, storage is a method of smoothing (or raising) incomes long used in the agricultural sector, and there has long been concern about the need for some method of reducing the income-risk of LDCs, for which storage has been posed as a possible solution. We have shown that such a country can enhance its welfare through storage. We have also shown that an optimal storage influences inputs into production, and that storage is sufficient to change the pattern of trade in some periods.

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