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The Desirability of a Currency Given a Contrationary Appreciation, Monetary Policy and Concave Supply Relationships

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The paper offers an argument why, given a monetary contraction, a currency appreciation is desirable in that it allows a more favorable tradeoff between aggregate output and inflation. Assume in each of two sectors, traded and nontraded, a concave supply relationship. It follows that to maximize aggregate output for any given inflation rate, contraction or expansion should be shared equally by the two sectors. If a country contracts without currency appreciation, the burden in the domestic country will be borne disproportionately by the nontraded sector, and in the foreign country by the traded sector. Some appreciation is desirable for a balanced economy.

Floating exchange rates have lost a good deal of their appeal in recent years. But a large change in the exchange rate can often be attributed to macroeconomic policy. Much of the criticism of our current international monetary system points out the negative effects of such a change, without properly distinguishing whether the alternative under consideration is a different macroeconomic policy regime to stabilize the exchange rate, or is some method of stabilizing the exchange rate[1] in spite of macroeconmic policy. But some authors do recognize the distinction, and explicitly deplore exchange rate effects independently of macroeconomic policy. Typical is a quote from Dornbusch (1982, pp.595-6):

> There is no sensible argument that tightening of money should involve as a desirable side effect a loss of exports, an increase in imports, and international redistribution of real income and borrowing abroad. Because these side effects are undesirable, both here and abroad, we should attempt to the maximum possible extent to immunize the world economy against these spillovers.

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1.For example, capital controls or sterilized1 foreign exchange intervention, if they are thought effective.

This paper offers an argument of the sort that the Dornbusch quote was unable to envision : an argument why the appreciation of the domestic currency may by the natural concomitant of a monetary contraction, not just in the sense that appreciation is what we would expect from the contraction, but in the sense that it is actually desirable, in that macroeconomic welfare in both the domestic and foreign countries is greater than it would be under fixed exchange rates, given the contraction. It is taken as given that the currency of the domestic country appreciates as the result of a monetary contraction undertaken to fight inflation.

As regards welfare in the domestic country the argument is that it is better off if the currency appreciates because then, given the decision to contract to fight inflation, the loss in demand is felt by the export or tradable sector as well as by the domestic or nontradable sector. While one might make an argument for equal sharing of the pain on equity grounds, the argument made here is on the grounds of obtaining the best possible terms for the tradeoff between aggregate output and aggregate inflation. As regards welfare in the foreign country, the paper will argue that, given the domestic contraction, it too is better off with a domestic/foreign exchange rate that is at least somewhat lower. Under a fixed exchange rate, the foreign country would experience a loss in export demand. If it does change its own demand policy, any gain in competitiveness will mitigate the involuntary movement down the Philips curve (to lower output) that it would otherwise experience. If it does adjust its demand policy in response to the domestic contraction, a lower exchange rate will still improve its output-inflation tradeoff by improving the balance between its export and nontraded goods sectors.

We assume that prices of goods are sticky in the currency of the country producing the good in question, and adjust only gradually over time to conditions of excess supply or demand. The key assumption in deriving our results is that the inflation/output tradeoff within each of the two sectors, domestic goods and exportables, is concave upward. J. M. Fleming (p.471), for example, claimed that

> the inverse relationships between unemployment and price inflation···are typically curvilinear, at least in the vicinity of full employment. As unemployment approches zero successive percentage declines in unemployment must impart increasingly powerful stimuli to inflation.[2]

2. Fleming used this fact to argue that the average Phillips curve tradeoff among a group of countries will be more favorable under floating exchange rates than fixed exchange rates, However, this is not the same as showing that each country individually will be better off under floating rates, which is the object of the present paper.

Empirical support for the concavity of the curve lies in the familiar observation that at high levels of unemployment and excess capacity, changes in output come more easily than changes in inflation, whereas the reverse is true closer to full employment and peak capacity utilization.[3] Theoretical support for concavity lies in the rationale that the aggregate supply curve gets its slope from neoclassical firm optimization aubject to a production function with a low elasticity of substitution between the factors of production that are fixed in the short run, say capital, and whatever other few factors of production are variable in the short run, say unskilled labor. For example, if (1) output is given by a CES production function with λ the elasticity with respect to the variable factor, unskilled labor, and σ the elasticity of substitution, (2) the firm produces where the marginal product of labor is equal to the real wage, and (3) the nominal wage is proportionate to last period's price level, or to some predetermined expected price level, then one plus the inflation rate will be proportional to output to the power of $(1-\lambda)/\lambda\sigma$. This number will be greater than one, i.e., the relationship will be concave, if elasticity of substitution is sufficiently low.

We will demonstrate six propositions.

I. To obtain the most Favorable Tradeoff between Aggregate Inflation and Aggregate output, A Country should Expand Equally or Contract Equally in both sectors.

The intuition here is that, with concave supply curves, if the contraction were more severe in the domestic sector than in the export sector, the marginal reduction in inflation gained for a given further loss in output would be greater in the latter sector than the former. Our two supply curves are:

$$\begin{aligned} 1+\pi_N &= (Y_N/\overline{Y}_N)^{\delta} \\ 1+\pi_X &= (Y_X/\overline{Y}_X)^{\delta} \end{aligned} \tag{1}$$

where we have defined

$Y_N \equiv$ output in the nontradable sector

$\overline{Y}_N \equiv$ potential output (the non-inflationary level) in that sector

$\pi_N \equiv$ the inflation rate in that sector (relative to expectations)

3. Robert Gordon (p.194) for example, offers some evidence that the Phillips curve is flat at high levels of unemployment.

$Y_X \equiv$ output in the export sector

$\overline{Y}_X \equiv$ potential (non-inflationary) output in that sector

$\pi_X \equiv$ the inflation rate in that sector (relative to expectations)

$\delta \equiv$ the elasticity of the price level with respect to output, assumed greater than one (this is the concavity assumption), and for simplicity assumed equal in all sectors. In terms of the CES production function, we can think of δ as $(1-\lambda)/\lambda\sigma$.

The two supply curves are illustrated in figure 1.

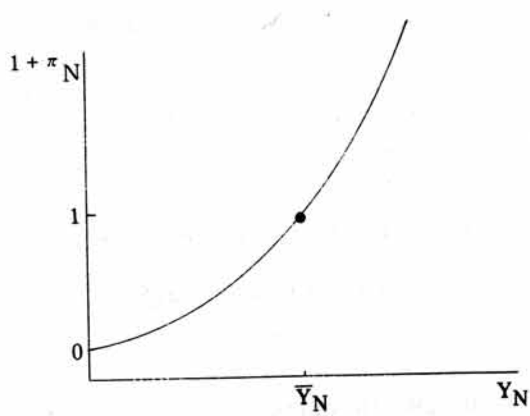
We will focus on the inflation rate π measured by a producer price index, the weighted average of the inflation rates in the two industries:

$$\pi = \alpha\pi_N + (1-\alpha)\pi_X$$
$$1+\pi = \alpha(Y_N/\overline{Y}_N)^{\delta} + (1-\alpha)(Y_X/\overline{Y}_X)^{\delta} \tag{2}$$

The weights are given by $\alpha = \overline{Y}_N/\overline{Y}$ and $1-\alpha = \overline{Y}_X/\overline{Y}$, where $\overline{Y}$ is aggregate potential output.

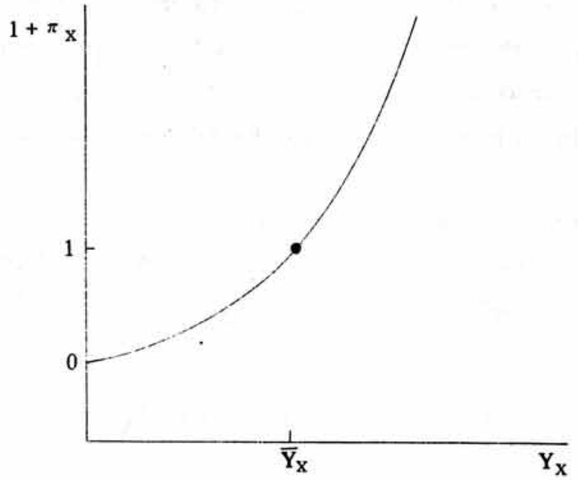
Note that if we used a consumer price index that included the price of imported goods, instead of the producer price index,[4] we would find that the price level, as

Figure 1 Output-inflation tradeoff for nontraded goods



4. Or if we allowed the price of the exportable good to be determined on world markets.

Output-inflation tradeoff for traded goods



opposed to the inflation rate, would fall instantaneously when the exchange rate falls. Buiter and Miller have shown that any gains against inflation of this nature must be given back later when the real exchange rate returns to its long-run level. We would thus be in the difficult position of having to compare the welfare effects of an unambiguous fall in the rate of price change (under a fixed exchange rate), versus a path that features an initial fall in the price level followed by an increased rate of change (under a floating rate). It is easier to leave import prices out altogether.

Let "a" be the share of output that is allocated to nontraded goods.

$$1+\pi=\alpha(aY/\alpha\overline{Y})^{\delta}+(1-\alpha)((1-a)Y/(1-\alpha)\overline{Y})^{\delta}$$

To find the value of "a" that minimizes π for a given level of Y, we differentiate:

$$\frac{d\pi}{da}=\alpha\delta(aY/\alpha\overline{Y})^{\delta-1}Y/\alpha\overline{Y}-(1-\alpha)\delta[(1-a)Y/(1-\alpha)\overline{Y})^{\delta-1}Y/(1-\alpha)\overline{Y}=0$$

$$(a/\alpha)^{\delta-1}=[(1-a)/(1-\alpha)]^{\delta-1}$$

$$a=\alpha$$

Thus the country should allocate output in the same proportions between the two sectors as at full employment. If the government is going to "put the screws" to the construction industry, it should do the same to autos and steel.

A consequence is that the optimal aggregate tradeoff is of the same shape as the individual tradeoffs in the two sectors:

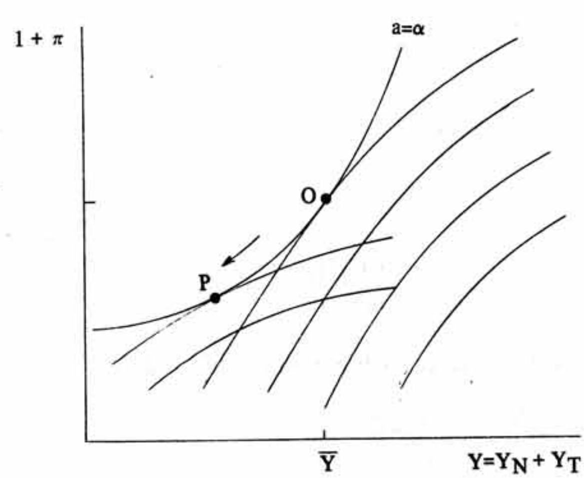
$$1+\pi=\alpha(Y/\overline{Y})^{\delta}+(1-\alpha)(Y/\overline{Y})^{\delta} \\ =(Y/\overline{Y})^{\delta} \tag{3}$$

It is illustrated in figure 2. We draw in upward-sloping social indifference curves to illustrate the preferences between inflation and output. A shift in priority from fighting unemployment to fighting inflation is shown as a decrease in the slopes of the indiffernce curves. The tangency moves down the curve to lower levels of inflation and output.

II. To Contract Equally in the Two Sectors, A Reduction in the level of Expenditure must be Accompanied by an Appreciation of the Currency in order to Switch Expenditure away from Exportable Goods.

If there were no change in the exchange rate or other expenditure-switching policies, a countraction of expenditure would be concentrated relatively more in the output of non-traded goods. (though it would also have some effect on the output of exportables assuming they enter domestic consumption). Export sales would to a large extent be buoyed by foreign expenditure. If output is to fall equiproportionately in the two sectors some policy like a revaluation of the currency is necessary to switch expenditure away

Figure 2 The optimal output-inflation tradeoff with a shift in preferences



from exportables toward non-traded goods. In the case of expenditure by the foreign country, this means a shift in demand away from the export of the domestic country toward its own goods. In the case of expenditure by the domestic country, it means a shift in demand towards its import good, away from its own exportable (and a similar shift away from its non-traded goods, which is assumed to be dominated by the other effects).

We wish to keep output in the two sectors in the same proportions, as we found in equation (3):

$$\frac{Y_X}{Y_N}=\frac{1-\alpha}{\alpha} \tag{4}$$

We define

$A \equiv$domestic expenditure, determined by policy

$A^* \equiv$foreign expenditure, determined by policy

$x \equiv$the share of domestic expenditure falling on the exportable good

$x^* \equiv$the share of foreign expenditure falling on the domestic exportable

$n \equiv$the share of domestic expenditure falling on the nontraded good, and

$E \equiv$the exchange rate defined as units of domestic currency per unit of foreign currency.

x, x^* and n are all increasing functions of the exchange rate. In the case of x and n, if the exchange rate increases, i.e., the domestic currency depreciates, domestic consumers substitute away from the importable good, since its price goes up in terms of domestic currency. In the case of x^*, foreign consumers substitute away from the domestic importable as well as from their own nontraded good, since the price of the domestic exportable falls in terms of foreign currency.

Output in the two sectors is determined by demand:

$$Y_X=x(E)A+x^*(E)A^* \qquad Y_N=n(E)A \tag{5}$$

So our condition (4) is

$$\frac{x(E)A+x^*(E)A^*}{n(E)A}=\frac{1-\alpha}{\alpha} \tag{6}$$

We wish to demonstrate the relationship between E and A:

$$\frac{dA}{dE}=\frac{\partial[(1-\alpha)/\alpha]/\partial E}{\partial[(1-\alpha)/\alpha]/\partial A}$$

$$=\frac{(x'A+x^{*\prime}A^*)/nA-(xA+x^*A^*)n'/n^2A}{-x^*A^*/nA^2}$$

where x', $x^{*\prime}$ and n' are the positive derivatives with respect to E. This expression will be positive if

$$(x'A+x^{*\prime}A^*)-(xA+x^*A^*)\ n'/n>0 \tag{7}$$

Intuitively the question is whether an increase in E raises the numerator of (6) more than the denominator; we already know that an increase in A does the reverse.

Using (6) in (7), the question is whether

$$x'A+x^{*\prime}A^*>\frac{1-\alpha}{\alpha}\ \ n'A \tag{8}$$

Define the elasticity of domestic demand for nontrades goods $\varepsilon_N \equiv n'A/Y_N$, the elasticity of domestic demand for exportables $\varepsilon_X \equiv x'A/wY_X$, and the elasticity of foreign demand for exportables $\varepsilon_X^* \equiv x^{*\prime}A^*/(1-w)Y_X$, where w is whatever share of Y_X happens to be sold to domestic consumers. Then our condition is

$$\varepsilon_X wY_X+\varepsilon_X^*(1-w)Y_X>\frac{1-\alpha}{\alpha}\varepsilon_n Y_N$$

Using (4),

$$\varepsilon_X w+\varepsilon_X^*(1-w)>\varepsilon_n \tag{9}$$

Thus the question comes down simply to whether a weighted average of the domestic and foreign elasticities of demand for the exportable exceeds the elasticity of demand for nontraded goods.

We cannot prove that (9) holds, but it seems likely. It says that exportables are closer substitutes for the importable good whose price has changed than are nontraded goods. It is often observed that countries tend to trade similar products. A common model, sometimes called the dependent economy model, even assumes that exportables and importables are perfect substitutes. We shall simply assume condition (9). The reader may find the proposition that a devaluation shifts relative expenditure into exportable goods, and that a revaluation shifts relative expenditure out of exportable goods, sufficiently plausible that Proposition (2) can be taken directly.

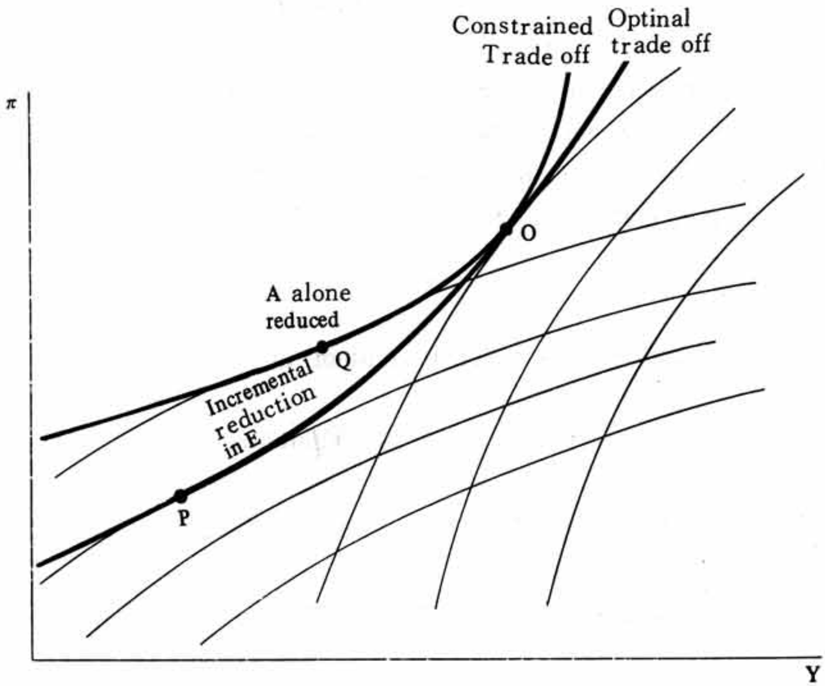
As long as (9) holds, there will exist some size decline in the exchange rate that will allocate a decline in expenditure in the desired equal proportions between the two sectors. Of course there is no guarantee that the size of the decline in the exchange rate that actually takes place will be of the correct magnitude. It depends obviously on what kind of exchange rate model is assumed and what parameter values. But it also depends on what is done with other policy variables besides expenditure A. First, we must allow for the foreign country responding by changing its level of expenditure A^*. Second, we must recognize that either government can and does affect the exchange rate. In a portfolio-balance model, the central banks can intervene on the foreign exchange market to affect the exchange rate without changing the money supply. In a monetary model, à la Mundell-Fleming, Dornbusch (1976) or Buiter-Miller, the government can affect the exchange rate by varying the monetary/fiscal policy mix, even if effective sterilized foreign exchange intervention is precluded by the assumption of pure floating, or of perfect substitutability between domestic and foreign bonds.

If the domestic country were small, so that it alone cared about its exchange rate, we might content ourselves with the observation that it can obtain the optimum outcome by the proper revaluation, if it so desires. But the necessity to consider the policy options of the rest of the world inspires us to consider some further propositions, beginning with the welafre effects of a decrease in the exchange rate that is smaller than the optimum.

III. When it Reduces Expenditure in order to Fight Inflation, Even if the Country is constrained from Discretely Decreasing the Exchange Rate, it is still true that an incremental decrease in the Exchange Rate (Appreciation) will Improve its Welfare.

The basic intuition here is the same as for proposition (2) : under a fixed exchange rate the reduction in expenditure falls disproportionately on non-traded goods, so that an incremental appreciation to shift expenditure away from exportable goods moves the economy closer to a balanced contraction.The situation is illustrated by Figure 3. The optimal tradeoff pictured in Figure 2 held when the country was free to vary both E and A at will. If the country is constrained from varying E, it will necessarily have a less attractive opportunity set. We assume that we start from a point O on the optimal tradeoff curve, where output in the two sectors is proportional to their full-employment capacities, and that the exchange rate is then fixed at that level. Now society's indifference curves shift. With E fixed, the new optimal tangency point P is no longer attainable, and the economy must settle for the tangency with a more

Figure 3 Output-inflation tradeoff with constrained exchange rate



concave constrained tradeoff, at Q. Since the constrained tradeoff is flatter at lower levels of output, Q lies above and to the right of P. An incremental decrease in E will incrementally lower π and Y, which is a movement southwestward, so it seems likely that this will improve welfare. But the proposition needs to be proven.

We repeat equations (5)

$$Y_X = x(E)A + x^*(E)A^* \qquad Y_N = n(E)A \tag{5}$$

We substitute them into equation (2) for aggregate inflation, and the equation $Y = Y_N + Y_X$ for aggregate output, to see how these variables depend on E and A:

$$\begin{gathered} 1+\pi = \alpha[n(E)A/\overline{Y}_N]^{\delta} + (1-\alpha)[(x(E)A + x^*(E)A^*)/\overline{Y}_N]^{\delta} \\ Y = n(E)A + x(E)A + x^*(E)A^* \end{gathered} \tag{10}$$

We are interested in the slope of the constrained curve in Figure 3, the terms of the tradeoff between inflation and output as A alone is varied:

$$\frac{d\pi}{dY/\overline{Y}}\bigg|_{E=\overline{E}}=\frac{\partial\pi(A,\overline{E})/\partial A}{\partial Y(A,\overline{E})/\partial A\cdot 1/\overline{Y}}$$
$$=\overline{Y}\frac{\alpha\delta[n(E)A/\overline{Y}_N]^{\delta-1}n(\overline{E})/\overline{Y}_N+(1-\alpha)\delta[(x(\overline{E})A+x^*(\overline{E})A^*)/\overline{Y}_X]^{\delta-1}x(E)/\overline{Y}_X}{n(\overline{E})+x(\overline{E})} \quad (11)$$

At points of tangency like O and Q, the slope is equal to society's marginal rate of substitution between π and Y. There is no way to know what the society indifference curves look like, even whether they are convex or concave. We assume for simplicity that they are linear, that welfare W is given by

$$W=c(Y/\overline{Y})-d(1+\pi) \quad (12)$$

Thus the marginal rate of substitution is constant[5] at

$$\frac{d\pi}{d(Y/\overline{Y})}=-\partial W/\partial(Y/\overline{Y})/\partial W/\partial\pi=\frac{c}{d} \quad (13)$$

Equating to the slope given by (11), and using $\overline{Y}_N=\alpha\overline{Y}$ and $\overline{Y}_X=(1-\alpha)\overline{Y}$,

$$\frac{c}{d}=\frac{\delta}{n(\overline{E})+x(\overline{E})}\{[n(\overline{E})A/\overline{Y}_N]^{\delta-1}n(\overline{E})+[(x(\overline{E})A+x^*(\overline{E})A^*)/\overline{Y}_X]^{\delta-1}x(\overline{E})\} \quad (14)$$

We can see from (14) how a decrease in the slope c/d of the indifference curves will require a reduction in the only free policy variable, A. Given the non-linearity of equation (14), it is impossible to slove explicitly for A. Nor is it necessary to slove for A in order to demonstrate Proposition (3). However, it will help to make things more concrete if we take a moment out to consider the example $\delta=2$, which makes (14) linear and allows us to slove for A:

$$\frac{c}{d}=\frac{2}{n(\overline{E})+x(\overline{E})}\{n(\overline{E})^2A/\overline{Y}_N+(x(\overline{E})A+x^*(\overline{E})A^*)x(\overline{E})/\overline{Y}_X\}$$

$$A=\frac{\frac{c}{d}\,\frac{n(\overline{E})+x(\overline{E})}{2}-\frac{x^*(\overline{E})x(E)}{\overline{Y}x}A^*}{\frac{n^2(\overline{E})}{\overline{Y}_N}+\frac{X^2(\overline{E})}{\overline{Y}_X}} \quad (15)$$

5. Even if the indifference curves are not in reality linear, the propositions derived here will be valid in the neighborhood of the point O, i.e., for small policy changes (assuming of course the indifference curves are differentiable).

We thus see explicitly how the fall in $\frac{c}{d}$, say from $[\frac{c}{d}]_0$ to $[\frac{c}{d}]_1$ caused the government to reduce A, say from A_0 at O to A_1 at Q. The question is, what is the effect on welfare of an incremental decline in E from point Q? From the expression for welfare (12),

$$\frac{\partial W(A_1,\bar{E})}{\partial E} = c_1\frac{\partial Y(A_1,\bar{E})/\bar{Y}}{\partial E} - d_1\frac{\partial \pi(A_1,\bar{E})}{\partial E} \tag{16}$$

Taking our derivatives from (10),

$$\begin{aligned}\partial W(A_1.\bar{E})/\partial E = {} & c_1/Y_0(n' A_1+x' A_1+x^{*\prime} A^*) - d_1\{\alpha\delta[n(\bar{E})A_1/\bar{Y}_N]^{\delta-1}n' A_1/\bar{Y}_N \\ & +(1-\alpha)\delta[x(\bar{E})A_1+x^*(\bar{E})A^*)/\bar{Y}_X]^{\delta-1}(x' A_1+x^{*\prime} A^*)/\bar{Y}_X\}\end{aligned}$$

We want to show that a decrease in E increases welfare, i.e., that the expression is negative. This will be true if

$$(\frac{c}{d})_1 < \delta\frac{[n(\bar{E})A_1/\bar{Y}_N]^{\delta-1}n' A_1+[(x(\bar{E})A_1+x^*(\bar{E})A^*)/\bar{Y}_X]^{\delta-1}(x' A_1+x^{*\prime} A^*)}{(n'+x')A_1+x^{*\prime} A^*} \tag{17}$$

From equation (14), $[\frac{c}{d}]_1$ is a weighted average of two terms

(a) $\delta[n(\bar{E})A_1/\bar{Y}_N]^{\delta-1}$

(b) $\delta[(x(\bar{E})A_1+x^*(\bar{E})A^*)/\bar{Y}_X]^{\delta-1}$

where the weights are

(14a) $\dfrac{n(\bar{E})}{n(\bar{E})+x(\bar{E})}$ and

(14b) $\dfrac{x(\bar{E})}{n(\bar{E})+x(\bar{E})}$, respectively.

The righthandside (RHS) of (17) is a weighted average of the same two terms, (a) and (b), with weights

(17a) $\dfrac{n' A_1}{n' A_1+x' A_1+x^{*\prime} A^*}$ and

(17b) $\dfrac{x' A_1+x^{*\prime} A^*}{n' A_1+x' A_1+x^{*\prime} A^*}$, respectively.

Now $x' + x^{*'} A^*/A_1 > x' + x^{*'} A^*/A_0$ because $A_1 < A_0$

$$> \frac{1-\alpha}{\alpha} \; n' \quad \text{by equation (8)}$$

$$= \frac{(1-\alpha)Y(A_0,E)}{\alpha Y(A_0,E)} \quad n'$$

$> \dfrac{x(\bar{E})}{n(\bar{E})} n'$ because we saw in Proposition (1) that outputs in the two sectors were originally proportionate to their full-employment levels at a point like O :

$$\alpha Y(A_0,\bar{E}) = n(\bar{E})A_0,$$

and

$$(1-\alpha)Y(A_0,\bar{E}) = x(\bar{E})A_0 + x^*(\bar{E})A^* > x(\bar{E})A_0.$$

This means that the ratio of the weights (14a) and (14b) is greater than the ratio of the weights (17a) and (17b)

$$\frac{n(\bar{\bar{E}})}{x(\bar{\bar{E}})} > \frac{n' A_1}{x' A_1 + x^{*'} A^*} \tag{18}$$

Again by virtue of Proposition (1), the two terms (a) and (b) would be equal at point O, i.e. with A_0 substituted for A_1. (There the slope in equation (14) reduces to $[\frac{c}{d}]_0 = \delta[Y/\bar{Y}]^{\delta-1}$, as can be seen by differenentiating (3).) But since A has fallen to A_1, both terms have fallen, with (a) falling by more. Thus our finding that equation (14) puts relatively more weight on the first term (a), implies that $[\frac{c}{d}]_1$ is less than the RHS of (17), which is precisely what we needed to show. this inequality was our condition for $\frac{\partial W}{\partial E} < 0$: an incremental fall in the exchange rate improves welfare.

In the foregoing we have taken foreign expenditure A^* as given. We now consider the foreign country's reaction to the change in international circumstances.

IV. If the exchange rate is not allowed to fall, the foreign country should react to the domestic contraction by expanding its expenditure.

If there were no change in the exchange rate, the foreign country would bear part of the burden, in the form of lost exports, of the domestic contraction. This fact in itself supplies one reason why the foreign country should want its currency to depreciate: to help insulate it from an externally imposed movement down the Phillips curve. But here we begin the analysis by seeing how the foreign country will adjust its expenditure policies. Given the exchange rate, it will want to fight the push down the Philips curve by following expansionary policies.

We model the foreign country symmetrically to the domestic country. Foreign welfare is a function of foreign income and inflation, which are in turn functions of foreign output of non-traded goods and export goods:

$$W^*=c^*\frac{Y^*}{\bar{Y}^*}-d^*(1+\pi^*)$$
$$=c^*\frac{Y_N^*+Y_X^*}{\bar{Y}^*}-d^*[\alpha^*[\frac{Y_N^*}{\bar{Y}_N^*}]^{\delta*}+(1-\alpha^*)[\frac{Y_X^*}{\bar{Y}_X^*}]]^{\delta*} \tag{19}$$

Foreign outputs are in turn functions of expenditure shares and expenditure levels

$$Y_N^*=n^*(E)A^*$$
$$Y_X^*=m(E)A+m^*(E)A^*,$$

where $n^*\equiv$the share of foreign expenditure falling on their nontraded good

$m\equiv$the share of domestic expenditure falling on the foreign export (which is of course the domestic import; $m\equiv 1-n-x$)

$m^*\equiv$the share of foreign expenditure falling on their own exportable ($m^*\equiv 1-n^*-x^*$),

all of them *decreasing* functions of the exchange rate.

We assume that the foreign country is starting from a point on its optimal output- inflation tradeoff, i.e., that output is allocated between the two sectors in proportion to their full-employment levels

$$Y_N^*=\alpha^*Y^* \qquad Y_X^*=(1-\alpha^*)Y^*,$$

and that the government then chooses the level of expenditure such that the society's marginal rate of substitution between output and inflation is equal to the terms of the tradeoff.

Analogously to equation (14),

$$\frac{c^*}{d^*}=\frac{\delta^*}{n^*(\bar{E})+m^*(\bar{E})}\{[n^*(\bar{E})A^*/\bar{Y}_N^*]^{\delta^*-1}n^*(\bar{E}) \\ +[(m^*(\bar{E})A^*+m(\bar{E})A)/\bar{Y}_X^*]^{\delta^*-1}m^*(\bar{E})\} \tag{20}$$

It can be seen from equation (20) that when A falls, the foreign country will have to raise A^* if it wants to maintain optimality.[6]

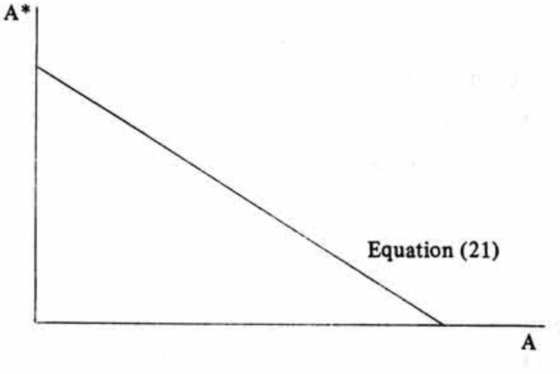
Figure 4 graphs the inverse dependence of foreign expenditure on domestic expenditure. The curve might be concave or convex. In the graph we choose to show the case where $\delta^*=2$ so that the relationship is linear.

In this case we can slove explicitly for A^* in terms of A, analogously to equation (15):

$$A^*=\frac{\frac{c^*}{d^*}\cdot\frac{n^*(\bar{E})+m^*(\bar{E})}{2}-\frac{m(\bar{E})m^*(\bar{E})}{\bar{Y}_X^*}\cdot A}{\frac{n^{*2}(\bar{E})}{\bar{Y}_N^*}+\frac{m^{*2}(\bar{E})}{\bar{Y}_X^*}} \tag{21}$$

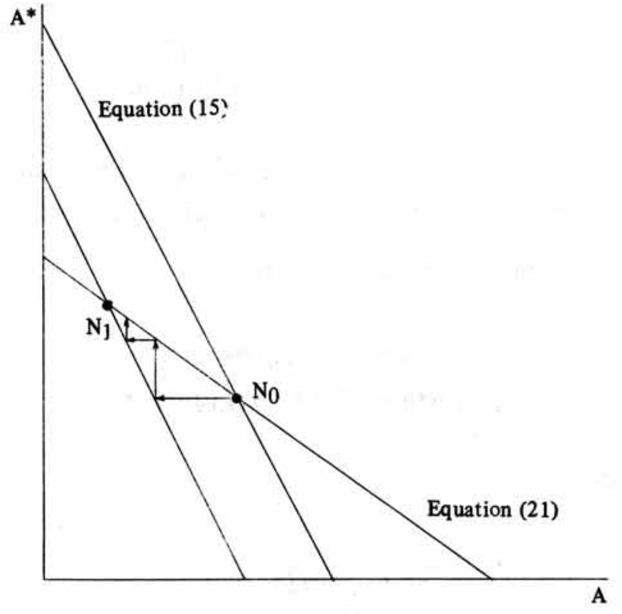
The absolute value of the slope is almost certainly less than 1.0; it is at any rate less than m/m^*.

Figure 4 Dependence of foreign expenditure policy on domestic expenditure policy



6. The proposition that the optimal response is for the foreign country to expand depends on the posited fixity of the exchange rate. If the foreign currency instead depreciated sufficiently, the optimal response would be a contraction.

Figure 5 Shift in Nash equilibrium when domestic priorities shift against inflation



V. Given the Domestic Contraction, an Incremental Decrease in the Exchange Rate will Improve Foreign Welfare.

The foreign country is the converse situation from that of the domestic country in Proposition (3). There the domestic country had contracted as much as it wanted to, but the contraction was concentrated disproportionately in the non-traded goods sector, so an appreciation of its currency was needed. Here the foreign country has expanded as much as it wants to, but the expansion is concentrated disaproportionately in the non-traded goods sector, so a depreciation of its currency is needed. The world is indeed lucky that both countries want the same exchange rate to move in the same direction !

Let $A_1{}^*$ be the level that foreign expenditure rises to, according to equation (19), or its linear form (20), in response to the decrease in domestic expenditure to A_1. Then we want to show that

$$\frac{\partial W^*(A_1, A_1{}^*, \bar{E})}{\partial E} < 0 \tag{22}$$

If we differentiate equation (19), we find that (22) is true if a condition analogous to condition (17) for the domestic country holds:

$$\frac{c^*}{d^*} > \frac{\delta^*[n^*(\bar{E})A_1^*/\bar{Y}_N^*]^{\delta^*-1}(-n^*)A_1+[(m^*(\bar{E})A_1^*+m(\bar{E})A_1)/\bar{Y}_X^*]^{\delta-1}(-m^{*\prime}A^*-m^{\prime}A_1)}{(-n^{*\prime})A_1^*+(-m^{*\prime})A_1^*+(-m^{\prime})A_1} \quad (23)$$

(Recall that the derivatives n^*, m^* and $m^{\prime}$ are negative.) From equation (20), we know that, once the foreign country has raised its expenditure to the optimizing level A_1^*, $\frac{c^*}{d^*}$ is equal to a weighted average of two terms:

$$\text{(a*)} \quad \delta^*[n^*(\bar{E})A_1^*/\bar{Y}_N^*]^{\delta^*-1}$$

$$\text{(b*)} \quad \delta^*[(m^*(\bar{E})A_1^*+m(\bar{E})A_1)/\bar{Y}_X^*]^{\delta^*-1}$$

where the weights are

$$\text{(20a)} \quad \frac{n^*(\bar{E})}{n^*(\bar{E})+m^*(\bar{E})} \quad \text{and}$$

$$\text{(20b)} \quad \frac{m^*(\bar{E})}{n^*(\bar{E})+m^*(\bar{E})} \quad \text{, respecitively.}$$

The RHS of condition (23) is a weighted average of the same two terms, (a^*) and (b^*), with weights:

$$\text{(23a)} \quad \frac{-n^{*\prime}A_1^*}{-n^{*\prime}A_1^*-m^{*\prime}A_1^*-m^{\prime}A_1} \quad \text{and}$$

$$\text{(23b)} \quad \frac{-m^{*\prime}A_1^*-m^{\prime}A_1}{-n^{*\prime}A_1^*-m^{*\prime}A_1^*-m^{\prime}A_1}, \quad \text{respectively.}$$

Now $-m^{*\prime}-m^{\prime}A_1/A_1^*>\frac{1-\alpha^*}{\alpha^*}\ (-n^{*\prime})$ by the analogous version of assumption (8) for the foreign country,

$$>\frac{m^*(\bar{E})}{n^*(\bar{E})}(-n^{*\prime}) \quad \text{because}$$

by Proposition (1) outputs in the two sectors were originally proportionate to their full-employment levels:

$$\alpha^* Y^*(A_0,A_0^*,\bar{E})=n^*(\bar{E})A_0^*$$

and

$$(1-\alpha^*)\overline{Y}^*(A_0,A_0^*,\overline{E})=m^*(\overline{E})A_0^* + m(\overline{E})A_0 > m^*(\overline{E})A_0^*$$

This means that the ratio of the weights (20a) and (20b) is greater than the ratio of the weights (23a) and (23b):

$$\frac{n^*(\overline{E})}{m^*(\overline{E})} > \frac{-n^{*\prime}A_1^*}{-m^{*\prime}A_1^* - m^{\prime}A_1}$$

The two terms, (a^*) and (b^*), would be equal to each other if A_0 and A_0^* were substituted for A_1 and A_1^*, again by Proposition (1). But since A has decreased to A_1 and A^* has increased to A_1^*, the first term (a^*) is now greater than the second (b^*). Since the relative weight on the first term is greater in equation (20), $\frac{c^*}{d^*}$ is indeed greater than the RHS of condition (23). Thus (22) holds: a decrease in the exchange rate raises foreign welfare.

We originally proved Proposition (3) on the assumption that foreign expenditure A^* could be taken as given. Now that we have recognized that, at the given exchange rate, the foreign country will respond to the domestic contraction be expanding its expenditure, we must take this into account. Equation (14), and its linear from equation (15), tell us that the domestic country, in order to achieve its desired point on the output-inflation tradeoff, will react to the increase in A^* by reducing further its own expenditure A. We could show that at this new point it is again true that domestic welfare would benefit from an incremental fall in the exchange rate. However there is no reason to assume that the process will stop there. Equation (20), and its linear form equation (21), tell us that the foreign country will in turn react to the further contraction by undertaking a further expansion. Then the domestic country will contract further, and so on. The logical thing to do is to take up the question when the process converges.

VI. In the Nash Equilibrium in which both Countries are Simultaneously setting Expenditure taking into Account the Other Country's Expenditure, an Incremental Decrease in the Exchange Rate would Benefit each Country.

Indeed given the further decreases in domestic expenditure and increase in foreign expenditure which are necessary to reach Nash equilibrium, domestic output becomes even more skewed away from nontraded goods than it was under Proposition (3), so the appreciation of its currency is even more needed; and similarly foreign output becomes even more skewed toward nontraded goods, so the depreciation of its currency is even more needed.

Figure (5) graphs the dependence of domestic expenditure on foreign expenditure on the same axes as the graph showing how foreign expenditure depends on domestic expenditure.The Nash equilibrium occurs at the intersection, point N_0. It is clear from equation (14), or its linear from equation (15), that when the domestic country's marginal rate of substitution between inflation and unemployment, c/d, falls, its policy reaction schedule shifts inward in Figure (5). The two countires can then be thought of as taking turns in adjusting their policies in reaction to each other until the new Nash equilibrium is reached.

We can slove equations (15) and (21) algebraically for the equilibrium point. The solution is

$$\hat{A}=\frac{\frac{c}{d}\frac{Y^2}{2}(\alpha+x)-[\alpha+\frac{x}{1-\alpha}][\frac{c^*}{d^*}\frac{Y^{*2}}{2}(\alpha^*+m^*)\frac{(1-\alpha)}{m^*}]}{[\alpha+\frac{x}{1-\alpha}][-(\alpha^*\frac{1-\alpha^*}{m^*}+m^*)\frac{1}{m}]+\frac{x^*}{1-\alpha}m}$$

and similarly for $\hat{A}^*$.

The derivation of the welfare effects proceeds along the same lines as before. For the domestic country, because the Nash equilibrium point represents an optimal setting of A, equation (14) holds with $A=\hat{A}$ and $A^*=\hat{A}^*$. The condition necessary for

$$\frac{\partial W(\hat{A},\hat{A}^*,\bar{E})}{\partial E}<0$$

is the same as condition (17), but with $\hat{A}$ substituted for A_1, and $\hat{A}^*$ for A_1, and $\hat{A}^*$ for A_1^*. We can again think of two terms, the first less than the second because $\hat{A}<A_0$ and $A^*>A_0$, of which the RHS of (14) is a weighted average with relatively more weight on the first (14a) than the second (14b), and of which the RHS of (17) is a weighted average with relatively more weight on the second (17b) than the first (17a). It follows that the inequality holds. An appreciation benefits the domestic country.

For the foreign country, because the Nash equilibrium point represents an optimal setting of A^*, equation (20) holds with $A^*=\hat{A}^*$ and $A=\hat{A}$. The condition necessary for

$$\frac{\partial W^*(A,A^*,\bar{E})}{\partial E} < 0$$

is the same as condition (23), but with $\hat{A}$ and $\hat{A}^*$ substituted. Of the two terms, the first is greater than the second. The RHS of (20) is a weighted average that puts relatively more weight on the first (20a) than the second (20b), and the RHS of (23) is a weighted average that puts relatively more weight on the second (23b) than the first (23a). It again follows that the inequality holds. A depreciation of its currency benefits the foreign country.

Conclusion

We have demonstrated one argument why, given a domestic monetary contraction, an appreciation of the currency is beneficial to both countries in that it allows them each to achieve the best possible tradeoff between aggregate output and inflation. In the absence of any change in the exchange rate, in the domestic country the burden of lost output would fall disproportionately on the nontraded goods sectors, and in the foreign country the burden would fall disproportionately on the traded goods sector. Allowing the domestic currency to appreciate causes the domestic traded sector to share in the contraction, as well as reducing the burden on the foreign traded sector. As a result, the composition of output in both economies is better balanced.

We have chosen to concentrate on an incremental change in the exchange rate. The finite change in the exchange rate that actually takes place when there is a change in monetary policy could be greater than or less than the change, described in Propositions 1 and 2, that is optimal for the domestic country. If the actual change were larger than the optimal change by a wide enough margin, the country could theoretically be worse off than if the exchange rate had not moved at all.

We could have chosen to model explicitly the exchange rate, and each country's level of expenditure, as functions of the countries' monetary policies and fiscal or debt policies, in order to see the welfare effects of the actual exchange rate change.

But this approach would have complicated the Nash equilibrium solution considerably. More importantly, the results would have been very dependent on the particular model used. The approach followed here, working directly in terms of the exchange rate and expenditure levels, has allowed us to keep the argument as general and model-free as possible.

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