Monetary Policy Devaluation and Capital Accumulation in an Open Developing Economy with Fixed Exchange Rates

H. Moussa*

I. Introduction

Tobin has pointed out that in an economy with $n$ assets, Walras Law allows only for $n - 1$ independent market equilibrium equations linking the $n$ corresponding real rates of return. If the real rate of return on money is fixed and the price level is fixed then asset markets can adjust to monetary policy only through changes in the real rates of return on the $(n - 1)$ other assets. However one of the other assets is physical capital. Everything else equal, its rate of return is inversely related to its valuation relative to its replacement cost. Therefore the adjustment to monetary policy requires a departure of the real rate of return from the value of the marginal productivity of capital or a divergence of the market valuation from the reproduction cost of capital. In turn the latter divergence makes the production of new capital more or less profitable and leads to a change in the level of gross investment which in turn may alter real national income and will necessarily alter the rate of accumulation of capital. In the long run this may lead to a change in the demand for real money balances because of the change in the marginal productivity of capital and in the total wealth. Therefore in the long run if the price level is allowed to change, the price level may not change in the way predicted by classical economics (new and old). In this paper we attempt to detail this process in the case of a small open developing economy with fixed exchange rates and two sectors, the sector of traded goods whose international price is given and a home good with a flexible price.

Since the exchange rate is fixed the domestic price of the traded goods is also fixed and we used this price to deflate nominal money balances. This world is not unlike that of a fixed price world. In addition we assume that the economy does not produce capital.

* Department of Economics, Acadia University, Canada
goods and is financially repressed so that domestic asset holders can hold only domestic money balances or physical capital. The flow of investment is determined as in Tobin’s model by the divergence of the domestic price of domestic capital from the international reproduction cost and some transaction costs that keep the flow of investment finite.

In agreement with Tobin’s suggestion and under the assumption of perfect foresight, monetary policy anticipated or unanticipated causes a short run effect which in our model is capable of producing a long term effect in terms of capital accumulation. However in our model monetary policy is partially a disguise for a policy of international reserves in the sense that it must be backed up by a willingness and the ability to make effective changes in the stock of foreign reserves. Unlike Obstfeld’s model the conduct of monetary policy may be more than the reshuffling of foreign assets between the central bank and the private sector. In some cases it can be productive in the sense that a given loss of reserves to finance an increase in money supply may lead to a gain in the capital stock larger than the initial loss in foreign assets. However in most cases the loss in reserves is greater than the value in terms of foreign currency of the additional capital stock acquired. The extent by which the former loss exceeds the latter value may be reduced if the wealth owners can be convinced to have a lower elasticity of consumption of the traded good with respect to wealth for a given disposable income. This particular result is valuable for development strategies since it implies that monetary policy can be used as a tool of promoting economic development.

We also use the model to interpret devaluation as a means of choosing the appropriate path of capital accumulation. Devaluation anticipated or unanticipated does not have an effect on the long run equilibrium but slows down the rate of capital accumulation. Therefore it lowers the present discounted value of the output path and as such it is painful. If demand of the home good depends on private wealth with a small elasticity than the devaluation will also cause a contraction and a decrease in the real wage and employment despite the flexibility of the real wage. This is a second source of pain. However these pains may be necessary because of a past error of judgement or as a cost of achieving a new target of development. Indeed despite the implicit assumption made in this paper that the terms of trade are fixed, the devaluation has a large chance to succeed to improve the trade balance.

This paper is organized as follows: the first section describes the model. In the second section we discuss some short run comparative statics. In the third section we derive some sufficient conditions for stability. In the fourth and fifth sections we describe the long run adjustment of the economy to an open market purchase and a devaluation respectively. The last section contains our conclusions.
II. The Model

In our small open economy, there are two goods: a traded good and a nontraded good that we call the home good. The price of the traded good in terms of foreign currency is determined internationally and is treated as fixed. With a fixed exchange rate the price of the traded good in terms of domestic currency is also fixed.

There are two factors of production capital $K$ and labor $L$. As in many developing economies most capital is not produced domestically and is imported. Capital imports are paid for either by the excess of exports over imports of the traded good or by borrowing or by using accumulated foreign assets. As for labor we think of it as having a utility function between wages and employment $L$. Labor knows that for each level of employment there corresponds an equilibrium wage rate under perfect competition and that ceteris paribus an increase in $L$ will reduce equilibrium wage and an increase in $K$ will increase it. That is it knows that the relationship between total employment and the corresponding equilibrium wage is downward sloping and represents a tradeoff constraint that shifts outward as the capital stock increases. Maximization of its utility subject to this constraint yields the optimal employment level $L^\phi$ and the optimal wage $\phi$. The total income $\phi L^\phi$ is later divided in some way among all the members of the labor force. A plausible way of dividing it is to allow for work sharing in the home good sector.

Assume that the wage and employment are “normal goods.” In this case the optimal wage and employment will change in the same direction as the shift in the employment wage constraint. To keep things simple and at the expense of departing from the first best, we assume that in the face of shocks shifting the employment wage constraint, the employment level remains fixed but the wage changes to keep the economy on the trade off constraint. Knowing what happens to the wage level under this assumption is equivalent to knowing what would happen to the wage and the employment level in the first best world. Using this convention we assume in the rest of the paper a fixed labor force, a fixed unemployment rate, a fixed level of employment $L$ and a flexible wage rate $s$.

We take the price of the traded good in terms of foreign currency as equal to one. All the prices are expressed in terms of the traded good. Let $P_m$ be the price of money, $P$ be the price of the home good, $Q$ be the domestic price of capital whose international price is also set equal to one, $r$ be the rental rate on capital services and $s$ be the wage rate. These definitions imply that $P_m$ is the exchange rate. If $M$ is the stock of money, its value in terms of the traded good is

$$X = P_m M$$  \hspace{1cm} (1)
As is the case in developing countries, there is no financial capital mobility between the domestic economy and the rest of the world. There are no government bonds and the resident asset holders cannot hold foreign currency.

The central bank can do open market operations by buying physical capital. These assumptions are equivalent to assuming that government bonds and equities are very close g-substitutes. (See Walsh for a definition of \(g\)-substitution). Suppose the central bank owns \(z\) units of capital. If it wishes to increase its holdings of capital by \(\Delta z\) then it will have to expand the money supply by

\[
\Delta M = Q_0 \Delta z / P_m
\]

(2)

Asset holders may hold their wealth \(W\) in either of the two assets available, money or capital. We can write total private wealth \(W\) as

\[W = X + Q_0 (K - z)\]

(3)

Let \(i\) be the rate of return on capital equal to the sum of \(r\) plus the expected capital gains on capital \(\pi^c\).

\[i = r + \pi^c\]

(4)

Let \(X^d(i, W)\) and \(K^d(i, W)\) be respectively the stock demand in terms of the traded good of money and capital. Their sum satisfies the wealth constraint

\[X^d(i, W) + K^d(i, W) - W\]

(5)

Assume that capital and money are "normal" assets in the sense that an increase in wealth will increase the demand for both assets. Then the slope of the demand for capital with respect to wealth is a positive fraction. On the other hand it is not unreasonable to assume that the demand for an asset is an increasing function of its own rate of return. Therefore we can write

\[0 < K^d_w \text{ and } K^d_i > 0; \text{ where } K^d_w = \frac{\partial K^d}{\partial W} \text{ and } K^d_i = \frac{\partial K^d}{\partial i}\]

(6)

1 Most discussion of open market purchases assume that the central bank trades only in government bonds. In practice central banks hold also private corporation bonds (e.g. Bank of Canada holds and trades in bankers acceptances which are short term private corporation bonds guaranteed by the chartered banks. Sparkays, p. 50). On the other hand in many countries governments hold equity in semi-public corporations. Some of the governments debt held by the central bank of these countries represent this equity. Therefore government bonds can be construed as sales or purchases of physical capital.

2 Differentiating both sides of (5) with respect to \(W\) and using the assumption of normality we obtain the stated result.
As the equilibrium in the capital market requires

$$Q(K - Z) = K^2(i, W)$$

(7)

(3) and (5) imply that if the capital stock market is in equilibrium then the money market is also in equilibrium.

We assume that physical capital and labor can be shifted freely between the output of the two goods. The production function of the two goods are constant returns to scale. Assume that the traded good is more capital intensive than the home good.\(^3\)

When the two goods are efficiently produced, the capital labor ratios in both industries are independent of the stocks of capital \(K\) and labor \(L\). They will depend only on the price \(P\) of the less capital intensive home good.

Given the stocks of capital and labor, the production possibility frontier is fixed and we assume that it is convex. An increase in the price \(P\) of the home good will raise the output of the home good raising thereby the capital labor ratio in each industry. But then the marginal productivity of capital in both industries will fall and the rental rate on capital will fall. That is,

$$r = r(P), \quad r_p = \frac{dr}{dP} < 0.$$  

(8)

In addition for a fixed \(P\) as \(K\) increases, the production of the more capital intensive good will rise and that of the labor intensive good will fall (Rybczynski). In general the production \(\phi\) of the home good is an increasing function of \(P\) and a decreasing function of \(K\).

$$\frac{\partial \phi}{\partial P} > 0 \quad \text{and} \quad \frac{\partial \phi}{\partial K} < 0.$$  

(9)

Furthermore, the gross domestic product \(y\) in terms of the traded good is an increasing function of \(P\) and \(K\).

$$y = y(P, K), \quad \frac{\partial y}{\partial P} > 0, \quad \frac{\partial y}{\partial K} > 0.$$  

(10)

Let \(\phi^d\) be the demand for the home good. We can write \(\phi^d\) as a function of \(P\) and \(y\) with

---

\(^3\) This would be the case if the traded good is an agricultural commodity or a natural resource and the home good is a service. In general the traded good is a composite commodity of exports and imports and may therefore be thought of as having a higher capital labor ratio.
\[ \frac{\partial \phi}{\partial P} < 0 \quad \text{and} \quad \frac{\partial \phi}{\partial y} > 0 \]  

(11)

At equilibrium we have

\[ \phi(P, K) = \phi^*(P, y) \]  

(12)

Now equilibrium national income requires

\[ S = I + EX - IM \]

where \( S \) = savings, \( I \) = investment, \( EX \) = exports and \( IM \) = imports.

By our assumptions

\[ I = IM \quad \text{and} \quad EX - IM = S - I \]

Therefore

\[ P_nM = S - I \]  

(13)

where a dot on a variable indicates the change in the value of the variable per unit of time change. Let \( \delta \) be the rate of depreciation. We have

\[ \dot{K} + \delta K = I \]

Thanks to Tobin we know that the price \( Q \) of installed capital may not be the same as the price one of newly produced capital. Tobin suggested that investment depends on \( Q \). When \( Q > 1 \), there will be a profit to be made by importing at the international price \( P \) and selling at the domestic price \( Q \) after incurring the adjustment, administrative, transportation and risk costs. We assume that the per unit cost of importing capital goods increases as the investment increases. This makes the flow of new capital goods finite and small compared to the existing stock of capital. If \( Q < 1 \), capital will be reexported and net investment may be negative. The amount of net investment depends on the difference between the domestic price of installed capital \( Q \) and the price of newly produced capital goods which is equal to one by definition. Therefore we can write

\[ I = g(Q - 1) + \delta K \]  

(14)

where \( g \) is a concave increasing function with \( g(0) = 0 \).

Finally we assume that savings is a decreasing function of wealth \( W \) and an increasing function of national income \( y \). That is

\[ \text{cf. Dornbusch, Foley and Sidrauski, and Metzler.} \]
\[ S = S(W, y) \text{ with } S_w = \frac{\partial S}{\partial W} < 0 \text{ and } S_y = \frac{\partial S}{\partial y} > 0. \]  \hfill (15)

Recapitulating we have that equilibrium in the home good, capital and money markets requires

\[ \phi(P, K) = \phi'(P, y(P, K)) \]  \hfill (16)

\[ Q(K - Z) = K^d(r(P) + \pi, X + Q(K - Z)) \]  \hfill (17)

the trade balance is given by

\[ P_m M = \hat{X} = S(W, y(P, K)) - g(Q - 1) - \delta k \]  \hfill (18)

and the rate of accumulation of capital is given by

\[ \dot{k} = g(Q - 1) \]  \hfill (19)

III. Short run equilibrium and comparative statics

In this model of fixed exchange rates, \( P_m \) is fixed in the short and long run. \( K, M, \) and \( Z \) are stocks and are fixed in the short run.

In the short run the capital, the home good and the traded good markets must be in equilibrium. The traded good market is in equilibrium since its price is determined internationally when the exchange rate is fixed.

Therefore the short run equilibrium is achieved when \( P \) and \( Q \) are determined by (16) and (17). Clearly (16) determines \( P \). In the \( (P, Q) \) plane of Figure 1 (16) is represented by a vertical line \( HH \) at the value of \( P \) that is equal to its solution. When \( K \) increases and \( P \) is fixed, the supply of the home good falls and the national income rises. The rise in the national income increases the demand for the home good. That is an increase in \( K \), when \( P \) is fixed, creates an excess demand. To keep the home good market in equilibrium, we assume that the price of the home good must rise.\(^5\) Hence an increase in \( K \) will shift \( HH \) to the right to \( H'H' \) in Figure 1.

In turn (17) can be represented by a downward sloping curve \( KK \). Due to (6) an increase in \( Q \) will increase the righthand side by less than it will increase the lefthand side of (17) creating an excess supply of capital. Hence \( r \) must rise to increase the demand for capital and reduce its excess supply. However this requires a fall in \( P \). That is the curve

\(^5\) This would be the case if the income elasticity of the home good is small and its price elasticity is large. See (36) below and the adjoining text.
$KK$ representing (17) is downward sloping and $dQ/Q < 0$. (See Figure 1).

Due to (6) an increase in $K$, keeping $P$ and $Q$ constant, will create an excess supply of capital. When $Q$ is decreased proportionately to the increase in $K$, there is still an excess supply for capital (again due to (6)). Therefore to keep the capital market in equilibrium, an increase in $K$, keeping $P$ fixed, requires a more than proportional fall in $Q$. That is as $K$ increases the curve $KK$ shifts downward more than proportionately to $K''K''$ in Figure 1.\(^7\)

For given values $K_0$, $X_0$, and $Q_0$ the short run equilibrium price vector is given by the solution $(P_0, Q_0)$ to (16)–(17). In Figure 1 this equilibrium price vector is given by the intersection of $HH$ and $KK$ at point $A = (P_0, Q_0)$. The equilibrium interest rate is then given by $r(P_0) + \pi$ and the level of national income is $\gamma(P_0, K_0)$ which in turn will deter-

\(^4\) Differentiate (17) totally with respect to $P$ and $Q$. Then

$$dQ/Q = \frac{\partial K^P \partial P}{\partial P}$$

Due to (5) the denominator of the RHS of this expression is positive. Due to (5) and (8), the numerator is negative. Hence $dQ/Q < 0$.

\(^7\) Differentiate totally (17) with respect to $Q$ and $K$. Then

$$(K - \zeta) \frac{dQ}{dK} = -Q. \text{ Therefore } \frac{dQ}{dK} < 0 \text{ and } \frac{d}{dK} \log (QK) = \frac{K \frac{dO}{dK} + Q}{QK} = \frac{\zeta}{QK} \frac{dO}{dK} < 0.$$
mine along with wealth the level of savings. The value of $Q_3$ will determine the level of investment. The difference between savings and investment will determine the current account balance. The investment will determine the rate of accumulation of capital and the current account balance will, in the absence of sterilization by the central bank, determine the rate of change of the money supply.

Figure 1 shows that when $K$ increases the equilibrium moves from $A$ to $C = (P_1, Q_1)$. Since the movement from A to B represents a decline in $Q$ that is more than proportional to the increase in $K$, it follows that the equilibrium price of capital falls more than proportionately to the increase in $K$ and the real value of the capital stock $QK$ will fall. By our assumptions the home good price will rise which implies that the national income will rise both because of the rise in $K$ and the induced rise in $P$ and the interest rate on holding capital will fall.

A change in $X$ or $\pi^e$ will not affect $HH$. Therefore the equilibrium price $P$ of the home good is independent of $X$ and $\pi^e$. An increase in $X$ will create an excess demand for capital. The required correction to keep the capital market in equilibrium is an increase in the price $Q$ of capital. An increase in $\pi^e$ will also create an excess demand for capital. To keep the capital market in equilibrium the price $Q$ of capital must rise.

Recapitulating we have: If $(P^e, Q^e)$ is the short run equilibrium price vector then

$$\frac{\partial Q^e}{\partial K} < 0, \frac{\partial (Q^e \cdot K)}{\partial K} < 0, \frac{\partial Q^e}{\partial X} > 0, \frac{\partial Q^e}{\partial \zeta} > 0, \frac{\partial Q^e}{\partial \pi^e} > 0.$$  \hfill (20)

$$P_K = \frac{\partial P^e}{\partial K} > 0, \frac{\partial P^e}{\partial X} = \frac{\partial P^e}{\partial \zeta} = \frac{\partial P^e}{\partial \pi^e} = 0.$$  \hfill (21)

$$\gamma_K = \frac{d\gamma}{dK} = \frac{d\gamma}{dP^e} \frac{\partial P^e}{\partial K} + \frac{d\gamma}{d\pi^e} > 0.$$  \hfill (22)

**IV. Dynamics and the conditions for stability of the long run equilibrium**

The condition of perfect foresight, valid in the long run, means when applied to the capital market that the expected rate of change ($\pi^e$) in the price of capital must equal the actual rate $\dot{Q}/Q$. We can use (16)–(17) to express $\pi^e$ as a function $H$ of $K$, $Q$, $X$, and $\zeta$. Assuming perfect foresight we can write:

$$\dot{Q} = QH (K, Q, X, \zeta) = h(K, Q, X, \zeta).$$

Recapitulating, the system is driven by the following differential equations.
\[
\dot{K} = g(Q - 1) \\
\dot{Q} = h(K, Q, X, \zeta) \\
\dot{\zeta} = S(X + Q(K - \zeta), y(K)) - g(Q - 1) - \delta K
\]  
(23.a) (23.b) (23.c)

The long run equilibrium is obtained by setting \( \dot{K} = \dot{Q} = \dot{\zeta} = 0 \). That is in the long run \( K, Q \) and \( X \) satisfy the following equations.

\[
Q = 1
\]  
(24)

\[
K - \zeta = K^4(r(p(K), X + K - \zeta)
\]  
(25)

\[
S(X + K - \zeta, y(K)) = \delta K
\]  
(26)

Let \( (\bar{K}, \bar{X}) \) be the solution to (25)–(26). Define \( q = Q - 1, k = K - \bar{K}, x = X - \bar{X}, \)

\[
h_k = \partial h/\partial k, h_q = \partial h/\partial q, h_x = \partial h/\partial x, S_w = \partial S/\partial W, S_y = \partial S/\partial y,
\]
where the partial derivatives are evaluated at the long run equilibrium. In addition, let

\[
A = \begin{pmatrix}
0 & h_q & 0 \\
h_k & h_q & h_x \\
S_w + S_y y_k - \delta & S_w (K - \zeta) - g' S_w
\end{pmatrix}
\]

We can approximate the system (24)–(26) around the long run equilibrium by the following linear system:

\[
(\dot{\kappa}, \dot{q}, \dot{x})' = A(k, q, x)'.
\]  
(27)

Let \( \eta_k \) (\( \eta_y \)) be the absolute value of the elasticity of savings with respect to wealth (income) and \( \eta_K \) be the absolute value of the total elasticity of income with respect to capital.

We can write

\[
a_{31} = S_w + S_y y_k - \delta = -\delta \left[ 1 + \frac{K}{K + \bar{X} - \zeta} \eta_w - \eta_y \eta_K \right].
\]  
(28)

Assume that

\[
1 + \frac{K}{K + \bar{X} - \zeta} \eta_w - \eta_y \eta_K > 0
\]  
(29)

(29) is satisfied if \( \eta_y = 0 \) or if \( \eta_y \) is small.

Assumption (29) is needed for stability as we shall see and implies that \( a_{31} < 0 \).
Clearly \( a_{33} < 0, a_{33} < 0, g' > 0, h_y > 0, h_y > 0, \) and \( h_y < 0. \)

(30)

Let \( F(\lambda) = -\lambda^3 + (S_w + h_y)\lambda^2 + (g'h_K + a_{33}h_x - S_yh_y)\lambda + g'(a_{33}h_x - h_yS_w) \)

(31)

where \( \lambda \) is a scalar. If \( I \) is the identity matrix then \( F(\lambda) = |A - \lambda I| \). To investigate the stability of the system we must find the signs of the three roots \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) of the characteristic equation \( F(\lambda) = 0 \). Now (30) implies that \( F(0) > 0 \) and \( \lim F(\lambda) = -\infty \) when \( \lambda \to -\infty \).

Therefore there exists a positive root to the characteristic equation. Call this root \( \lambda_1 > 0 \).

We can also write \( F(\lambda) = (\lambda - \lambda_1) (-\lambda^2 + b\lambda + c) \) where \( b \) and \( c \) are constants whose sign we wish to determine. By identification with coefficients of the powers of \( \lambda \) in (31) we may write:

\[ -\epsilon \lambda_1 = g'(a_{33}h_x - h_yS_w) > 0 \]

(33)

and

\[ \epsilon - b\lambda_1 = g'h_K + a_{33}h_x - S_yh_y \equiv LH > 0. \]

(34)

(33) implies that \( \epsilon < 0 \) and (34) implies that

\[ -b\lambda_1 = -\epsilon + LH > 0. \] That is \( b < 0 \).

Now \( \lambda_2 \) and \( \lambda_3 \) are the roots of the equation \( -\lambda^2 + b\lambda + c = 0 \). But then \( \lambda_2 + \lambda_3 = -b < 0 \) and \( \lambda_2\lambda_3 = c' + 1 \Rightarrow \epsilon > 0 \). This implies that the real parts of \( \lambda_2 \) and \( \lambda_3 \) are negative. Since \( \lambda_1 > 0 \) the dimension of the converging subspace is 2 equal to the number of predetermined stock variables \( K \) and \( X \). Since the third variable of the differential equations (27) is the price \( Q \), it follows that the economy is stable (see Begg) provided that \( S_y \) is small.

---

\( ^8 \) Clearly \( h_K = \frac{\partial H}{\partial K} = \frac{d\pi^f}{dk} = \frac{(1 - K^f)}{K^f} > 0 \)

(32.a)

\( h_x = \frac{\partial H}{\partial X} = \frac{d\pi^f}{dx} = \frac{-K^f}{K^f} < 0 \)

(32.b)

\( h_y = H(\hat{K}, 1, \hat{\lambda}, \zeta) + \frac{\partial H}{\partial Q} = \frac{d\pi^f}{dQ} = (\hat{K} - \zeta) \frac{(1 - K^f)}{K^f} > 0 \)

(32.c)

These results are obtained by total differentiation of (11) and the evaluation of derivatives at the long run equilibrium.
V. Monetary policy: The case of an open market purchase

An open market purchase at time $t_0$ requires changing $M$, $X$ and $\zeta$ at the same time so that (2) holds. (2) implies that $Q \Delta \zeta = \Delta X^t$ where $\Delta X^t$ is the immediate discrete change in $X$ caused by the central bank purchase. Taking limits as $\Delta \zeta$ goes to zero we have

$$\frac{dX^t}{d\zeta} = Q$$  (35)

An open market purchase means that the central bank owns from $t_0$ henceforth a bigger proportion of the capital stock. However the change in ownership at time $t_0$ will not change disposable income as we assume that the income from the rent of the capital stock owned by the central bank is redistributed through transfers. At time $t_0$ the adjustment of the economy to this change in ownership depends on whether the open market operation was expected or unexpected. However as we shall see the effect of the open market operation on the long run equilibrium does not depend on whether it was expected or unexpected. It depends on the amount of foreign reserves the central bank is willing to part with.

V.a The response of the long run equilibrium to an open market purchase

The long run equilibrium is determined by (24)–(26). The new long run equilibrium will have the same value (one) for $Q$. We can rewrite (25)–(26) as

$$K - \zeta - K^0(r(P(K)), X + K - \zeta) = 0$$

$$S(X + K - \zeta, y(K)) - \delta K = 0$$

Taking total differential of both equations we have:

---

9 The short run impact of an open market purchase and the ensuing dynamic adjustment, starting from a long run equilibrium, may be summarized as follows:

When the purchase is unexpected there will be a jump in real money balances and an excess supply of money and an excess demand for capital causing its price $Q$ and real wealth to rise. Because of perfect foresight, $Q/Q = \pi^*$ will have to jump downwards to a negative value. The rise in $Q$ and the fall in $\pi^*$ will reequilibrate the asset markets in the short run but will cause $S$ to fall and $I$ to rise resulting in an immediate balance of payments deficit. The process will continue by decreasing the real stock of money and raising the stock of capital.

When the purchase is expected at date $t_0$, happen at a later date $t_1$, speculation will cause the price $Q$ of capital to jump upward immediately causing an excess supply in the capital market and $\pi^*$ to jump upward in order to reequilibrate the asset markets. The rise in $Q$ will cause a rise in wealth, a fall in savings and a rise in investment resulting in a balance of payment deficit. Again the capital stock will start rising and real money balance falling. The adjustment processes that we have sketched are not necessarily monotonic and may generate cyclical fluctuations.
\[
\begin{pmatrix}
1 - K^*_d & -K^*_d \\
S_u - \delta + S_y r_K & S_u
\end{pmatrix}
\begin{pmatrix}
(dK/d\zeta) \\
(dX/d\zeta)
\end{pmatrix} = \begin{pmatrix}1 - K^*_d \\
S_u \end{pmatrix}
\]

Let \( J \equiv -\left(1 - K^*_d + K^*_d (-r_p) P_K\right) S_u - K^*_d \left(S_u - \delta + S_y r_K\right) \).

The stability condition (29) implies that \( J > 0 \).

The solution to the above system of equations is given by

\[
\frac{dK}{d\zeta} = -\frac{S_u}{J} \quad \text{and} \quad \frac{dX}{d\zeta} = \frac{(-S_u) K^*_d (-r_p) P_K - (1 - K^*_d) (\delta - S_y r_K)}{J}
\]

Therefore

\[
\frac{dW}{d\zeta} = \frac{dK}{d\zeta} + \frac{dX}{d\zeta} - 1 = -\frac{\delta + S_y r_K}{J} = -\frac{\delta (1 - \eta^*_r \eta^*_K)}{J}
\]

Let \( \eta^*_r \) be the elasticity of the demand for capital with respect to the interest rate, \( \eta^*_K \) be the elasticity of the rental rate on capital with respect to the price of the home good and \( \eta^*_K \) be the elasticity of the price with respect to the capital stock.

At the initial long run equilibrium with \( K = \bar{K} \) and \( X = \bar{X} \) we can write

\[
\frac{dK}{d\zeta} = \frac{1}{1 + \frac{K - \bar{K}}{\bar{K}} \eta^*_r \eta^*_K + \delta \left(\frac{K}{S_u}\right) (1 - \eta^*_r \eta^*_K)}
\]

and

\[
\frac{dX}{d\zeta} = 1 - \frac{\delta (1 - \eta^*_r \eta^*_K)}{J} = \delta \left[ \frac{\bar{K} - \bar{X} \eta^*_K \eta^*_r \eta^*_K - (1 - K^*_d) (1 - \eta^*_r \eta^*_K)}{J} \right]
\]

The stability condition implies that \( dK/d\zeta > 0 \) and \( dX/d\zeta < 1 \).

Using these formula we can set up the following table.

The results in Table 1 can be justified in the following way. In the long run and every-

<table>
<thead>
<tr>
<th>( 1 - \eta^<em>_r \eta^</em>_K )</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta^*_r )</td>
<td>( dK/d\zeta &gt; 0 )</td>
<td>( dK/d\zeta &lt; 0 )</td>
</tr>
<tr>
<td>small</td>
<td>(&lt; 1 )</td>
<td>(&lt; 0 )</td>
</tr>
<tr>
<td>large</td>
<td>(&lt; 1 )</td>
<td>( &gt; 0 )</td>
</tr>
</tbody>
</table>

10 In constructing table 1, we assumed the stability condition

\[
\eta^*_r \eta^*_K < 1 + \frac{K}{X + K - \bar{X}} \eta^*_r
\]
thing else equal a rise in $\zeta$ will create an excess demand for capital. That is the long run equilibrium capital stock must rise when $\zeta$ increases. As a consequence the real rate of interest $r$ must fall.

Suppose that $\eta_k^x/\eta_k < 1$. In this case a 1% rise in $K$ will increase savings (through an increase in income) by less than 1%. If private wealth $W$ remains the same or increases, savings will surely rise by less than 1%. However the replacement investment ($\delta K$) will rise by 1%. At the new long run equilibrium savings will fall short of the replacement investment contradicting (26). Therefore $W$ must fall. (See column 4 of Table 1).

When the interest elasticity, $\eta_k^x$ is small, the fall in the rental rate, $r$, caused by the rise in the capital stock will have little dampening effect on the demand for capital and stimulation effect on the demand for money. As a consequence, the fall in the private wealth $W$ requires a fall in the private capital stock $K - \zeta$ and in the private real money holdings $X$. That is the rise in $K$ will be less than the rise in $\zeta$. These results confirm the first three entries of the first row of Table 1. When the interest elasticity of the demand for capital is large, the private real money holdings will rise. This is confirmed by the first three entries of the second row of Table 1.

Suppose that $\eta_k^x/\eta_k > 1$. That is, ceteris paribus, a 1% increase in $K$ will increase savings by more than 1%. If private wealth, $W_0$ remains the same or falls, savings will surely rise by more than 1%. However the replacement investment will rise by 1%. At the new long run equilibrium savings will exceed the replacement investment contradicting (26). Therefore $W$ must rise. (See last column of Table 1).

When $\eta_k^x$ is small, the dampening effect of the fall in the equilibrium rental rate is small and the rise in $W$ will dominate raising the private demand for capital and so both $K - \zeta$ and $X$ will rise. This is confirmed by the second half of the first row of Table 1. When $\eta_k^x$ is large the dampening effect of the fall in $r$ will dominate the stimulating effect of the rise in $W$ on the demand for capital. In this case the private demand for capital will fall and $X$ will rise. This is confirmed by the second half of the second row in Table 1.

Table 1 implies that the economy's long run equilibrium stock of capital will rise. The equilibrium stock of real money holdings will rise except when the interest elasticity of the stock demand of real money balances and the income elasticity of saving are both small. The capital stock held by private asset holders will fall except when the interest elasticity of the stock demand of real money balances is small and the income elasticity of savings is large. The total private wealth will increase (decrease) when the income elasticity of savings is large (small) enough but not too large to prevent stability.

These results show that money is not neutral. Sargent (pp. 65–66) has already argued along the lines suggested by Metzler that the addition of the wealth effect of Pigou strips
the classical model of its famous proposition of the naturality of money. Our results confirm this conclusion for the long run equilibrium of a dynamic economy. Indeed the effect of the open market purchase on the capital stock depends on the elasticity of savings with respect to wealth since \(dK/d\zeta = -S_{\zeta}/f\). If this elasticity is zero then there is no effect on the capital stock and the effect is confined to real money balances and the stock of reserves. Furthermore the short run analysis of footnote 9 indicates that even if the elasticity of savings with respect to wealth is zero, the fixity of the exchange rate is enough to guarantee the existence of short run effects of changes in the money stock. The assumption of fixed exchange rates is responsible for the result than an open market purchase whether anticipated or not is capable of producing short run fluctuations in real economic activity in agreement with ideas advanced by Friedman in his monetary and fiscal framework for economic stability and other writings.

Suppose as is probably the case that \(\gamma^x\) is small and \(\gamma^w\) is large enough then table 1 indicates that

\[
\frac{dK}{d\zeta} > 1 > \frac{dX}{d\zeta} > 0 \text{ and } \frac{dW}{d\zeta} > 0.
\]

In this case an open market purchase by the central bank will increase the capital stock held by the central bank the capital stock, the stock of real money balances and the total wealth held by the private asset holders. However the long run increase in real money holdings is less than the initial jump in these holdings. Therefore the central bank must eventually suffer a loss of foreign reserves and the economy must go through a period of balance of trade deficits.

Suppose that \(\gamma^w\gamma^x = 1\). In this case the long run equilibrium private wealth will not change and the long run equilibrium private capital stock will fall while the long run equilibrium real money holdings will rise. Starting at a long run equilibrium value of \(S = \delta K_0\), savings will continue to be equal to \(\delta K\) as \(K\) rises provided that private wealth is constant. This is represented by line \(AB\) in Figure 2 below. However we know that at time \(t_0\), private wealth jumps upwards and then gradually returns back to its long run equilibrium value. As a consequence savings will immediately jump downwards and continue to be below the level of replacement investment as long as wealth is higher than its long run value. As \(K\) converges to its higher long run value, savings converge to the corresponding higher long run replacement investment. This behavior of savings is represented by curve \(CB\) in Figure 2. On the other hand, replacement investment will continue to be equal to \(\delta K\) but net investment, \(ni\), will immediately jump upwards at time \(t_0\) as \(Q\) does, then falls towards zero as \(Q\) falls towards 1. Hence gross investment will immediately jump
The effects of an open market purchase on capital accumulation and balance of trade

The behavior of gross investment is represented by curve $DB$ in Figure 2. The balance of trade deficit is $-(S - \delta K - n) > n$. It is represented by the vertical distance between the two curves $CB$ and $DB$. The accumulated deficit is represented by the shaded area between the two curves. The accumulated increase in the capital stock is represented by the area between the gross investment curve $DB$ and the replacement investment curve $AB$.

Clearly in this case, the accumulated balance of trade deficit is larger than the increase in the capital stock. That is the loss in reserves is bigger than the increase in the capital stock. By continuity this result will hold in a neighborhood of the $\eta_{RK} = 1$. That is if $\eta_{RK}$ is in the neighborhood of 1, then the loss in reserves is larger than the increase in the capital stock and the open market purchase may be said to be unproductive.

If $\eta_{R}$ increases so that $\eta_{R} > 1$ the final long run equilibrium capital stock and private wealth will be higher. This implies that the price of capital at time $t_0$ will jump upwards to a higher level. The curve $DB$ will shift upwards to $D'B'$ and the curve $CB$ will shift downwards to $C'B'$. The balance of payments deficit will increase and the excess of the loss of reserves over the accumulated capital will increase by area $CB'B'C'$ in Figure 2.

The excess of the loss of reserves over the accumulated capital stock is due to the fail-
ure of savings to keep up with replacement investment during the transition period as a result of an increase in private wealth. To lower the extent of the loss in reserves, a lower elasticity of savings with respect to wealth for a given income is needed. Equivalently a lower wealth elasticity of consumption of the traded good for a given income is needed to lower the excess of the loss of reserves over the accumulated capital. This result is to be contrasted with that obtained by Obstfeld using a model with fixed capital stock and fixed income. In his model open market purchases are equivalent in the long run to a transfer of foreign assets from the central bank to the private sector: a dollar for a dollar. In our model, the central bank has to part with more than a dollar worth of foreign reserves in order to induce a dollar increase in the domestic capital stock. The difference between the two results is due to the difference in the treatment of the effects of a change in wealth on consumption during the transition period.

Finally we note that the open market purchase has an effect on the real sector because the central bank is willing to part with the necessary loss of reserves. If the central bank was not willing to loose reserves then a devaluation must take place immediately. If savings do not depend on wealth then the devaluation will reduce real money holdings and wealth and the previous capital stock will be maintained with an appropriate devaluation. In this case the open market purchase will have no real effect. This would vindicate Tobin's claim that changes in the money supply have real effects because some price level is fixed and the nominal rate of return on money is fixed by fiat. Tobin's claim is vindicated with vengeance when savings depend on wealth. In this case as long as the nominal rate of return on money is zero the devaluation does not preclude a real effect of the open market purchase provided that it is unanticipated. This matter is left to a future research on open market purchases with flexible exchange rates.

V.b The Open Market Purchase and the Long Run Composition of the labor force

There is another way of justifying the assumption that the traded good is more capital intensive. We can think of the home good as a composite good for all goods of those sectors which harbour disguised unemployment such as traditional agriculture, the service sector, other family business. In this case a shift of labor out of the home good sector is desirable. The goal of this section is to show that an open market purchase can cause such a shift if the demand for the home good is largely inelastic with respect to national income.

Differentiating (16) totally we have

---

31 See footnote 3 and the corresponding text for other justifications.
\[
(\eta^h_{P} - \varepsilon^h_y) \frac{dP}{\rho} = (\varepsilon^h_{K} + \eta^h_{K}) \frac{dK}{K} 
\]

where \(\eta^h_{P}(\varepsilon^h_{K})\) is the absolute value of the elasticity of output (demand) of the home good with respect to variable \(V\), and \(\eta^h_{K}\) is the elasticity of real national income with respect to variable \(V\).

Assume \(\varepsilon^h_{K} = 0\). Then (36) shows that \(dP/dK > 0\). That is an open market purchase which increases the capital stock in the long run will cause the price of the home good to rise. In addition if \(\varepsilon^h_{K} = 0\), the demand for the home good does not depend on national income \(y\). Therefore an open market purchase will cause the demand and hence the long run equilibrium output of the home good to fall. On the other hand the increase in the price \(P\) will cause the rate of return on capital \(r\) to fall (see (8)) which in turn will cause the capital labor ratio in both sectors to rise.

Assume that the production functions are quasi-concave. Figure 2 describes the isoquants of the home good production function. The original long run equilibrium is represented by point A. In response to the increase in the total capital stock the output of the home good falls from \(\phi_0\) to \(\phi_1\) and the capital-labor combination moves from A to B. In addition the capital labor ratio increases and hence the capital labor combination moves to C. Therefore the amount of labor working in the home good sector falls definitely from

**Figure 3**

The effect of an open market purchase on the employment in the home good sector
Suppose we have a developing country with a small starting capital stock that is converging to a long run equilibrium with a higher capital stock but is experiencing a chronic balance of trade deficit that is $\dot{X} < 0$. One of the common solutions to this problem is a devaluation. In this model a devaluation is equivalent to a decrease in $P_m$ or a downward jump in $X$. In this section we study two kinds of devaluation: An unexpected and an expected devaluation.

VI.a An unexpected devaluation

Equation (16) shows that an unexpected devaluation has no impact on the price of the home good and hence no impact on the national income. That is the nominal price of the home good in domestic currency, $P/P_m$, will jump upwards in the same proportion as the devaluation. In the usual non monetary approach to the balance of payments problems [e.g. Corden], the balance of payments (trade) will not improve as there is no switching and the demand and supply of all goods remain the same. However in our model as in the monetary approach devaluation causes an immediate increase in the price level and an immediate fall in real money balances and wealth. As in the monetary approach this causes a temporary decline in the demand for capital and a downward jump in the price of capital $Q$. In addition it causes a fall in the demand for the traded good as it causes an increase in savings. The downward jump in $Q$ is cushioned by an increase in $\pi^*$ since the price $Q$ has to converge downwards to the lower long run equilibrium level of $1$.

---

12 At every long run equilibrium profit maximization requires that the amounts of capital and labor used minimize the total expenditures of producing the corresponding output $\phi$. The conditional demand for labour may be written as:

$$L_1 = L(\alpha, \phi) \quad \text{with} \quad L_1 = \frac{\partial L}{\partial \alpha} < 0, \quad L_2 = \frac{\partial L}{\partial \phi} > 0.$$ 

The capital labor ratio $k$ is an increasing function of $(\alpha, \phi)$. Profit maximization requires that $\frac{dK}{dK} < 0$.

Therefore:

$$\frac{dL}{dK} = L_1 \cdot \frac{d(\alpha)}{dK} - \frac{dK}{dK} \cdot \frac{d\phi}{dK} - \frac{d\phi}{dK} \cdot \frac{dK}{dK}$$

Using the assumption that $\frac{\partial \phi}{\partial K} = 0$, we have $\frac{dL}{dK} < 0$ as asserted in the text.

13 By continuity this conclusion will still hold if $\frac{\partial \phi}{\partial K}$ is small.

14 The increase in $\pi^*$ (which is assumed to be negative) reflects a slow down in the rate of decline in $Q$ since the long run increase in capital has to be achieved by lower investments. The capital loss on real money balances is unexpected when the devaluation is unanticipated and therefore does not cause by itself a shift away from money balances.
Therefore the exports of the traded good will immediately jump upwards and the imports of the capital good downwards causing an immediate improvement of the balance of trade. The effect on exports is predicted by the monetary approach but the effect on imports of investment is not.

The devaluation solves the problem of deficit in the trade balance by reallocating over time the use of the existing scarce foreign resources and or by giving the economy enough time to earn more foreign resources using a larger capital stock and a higher national income. The devaluation will not change the long run equilibrium \((K, \hat{X}, 1)\) solution of (24)–(26). Its impact is to lower the price of capital \(Q\) and to shift downward the path of \(Q\). As a consequence the path of investment will shift downward. The downward shift in investment is partly responsible for the improvement in the trade account. The downward shift in investment slows down the rate of accumulation of capital and the path of the capital stock shifts downward which in turn slows down the rate of growth of income.

The devaluation is a powerful tool that allows the monetary authority a choice of the most appropriate path for capital accumulation. A greater devaluation implies a less acute problem of trade deficit but lowers the rate of capital accumulation and hence the growth rate of output. It shifts forward the periods of higher output giving them a lesser weight in the present discounted value of the output path.

Suppose that the demand for the home good depends also on wealth. The equilibrium condition (16) may be rewritten as

\[
\dot{\phi}(P, K) = \phi^*(P, x(K, P), X + Q(K - Z)) \tag{16'}
\]

For a fixed capital stock a fixed \(X\) and a fixed \(Z\), it can easily be seen that (16') can be represented by an upward sloping curve such as \(HH\) in Figure 4. Equation (17) can be represented by a downward sloping curve such as \(QQ\) in Figure 4. The short run equilibrium is represented by \((P^0, Q^0)\).

An increase in \(X\) will create an excess demand for capital which can be eliminated by an increase in \(P\). Hence an increase in \(X\) will shift \(QQ\) to the right. However an increase in \(X\) creates an excess demand for the home good. A rise in \(P\) will reequilibrate it provided that the income elasticity of the home good is small. Therefore an increase in \(X\) will also shift \(HH\) to the right. If the wealth elasticity is small then an increase in \(X\) will raise the equilibrium prices \(P\) and \(Q\) to \(P_1\) and \(Q_1\) as is illustrated in Figure 4.

Now consider an unexpected devaluation. It will cause a downward jump in \(X\) and hence a downward jump in \(P\) and \(Q\) and an increase in \(\pi^\tau\). The decrease in \(P\) will be relatively small but it will cause a fall in the output of the home good and an increase in the
output of the traded good. The downward jump in \( X \) will also cause a decrease in the domestic demand for the traded good. Therefore the net exports of the traded good will jump upwards and the imports of the investment good downwards causing an immediate improvement in the balance of trade. Since the relative price of the home good has fallen the trade balance improves because of the resulting switch in demand towards the home good and because of the fall in investment. The switch, however, is not enough to dominate the effects of the fall in income and wealth so the output of the home good falls. Since the home good is labor intensive, the wage rate will jump downwards as the gain in employment in the capital intensive traded good sector is not enough to absorb the loss of employment in the home good sector unless the wage falls. Using our choice model for labor, the fall in the equilibrium wage rate with fixed employment means that optimally the wage and the employment will be lower. A devaluation causes an improvement in the trade balance despite the fact that the terms of trade remain fixed but at a cost of causing a contraction despite the assumption that wages are flexible.

It is interesting to compare the results of our analysis to those obtained by other writers. In this respect it is convenient to divide the existing literature on the effects of a devaluation in two schools of thought. The old school, following Meade and the elasticity approach, emphasizes the positive effects of a devaluation on real income and employment
under the assumption of excess capacity and fixed wages and prices. The new school, following Diaz and Cooper [See Bruno, Krugman and Taylor, and Taylor] has challenged this view emphasizing instead the insensitivity of the supply of exports to changes in the terms of trade. Krugman and Taylor show that within a fixed price (and wage) model, a devaluation may reduce real output when, as is most likely the case in developing countries, (1) imports initially exceed exports, (2) the propensity to save out of capital income is higher than out of labour income and (3) government income is raised by a devaluation. Pulling together the two schools of thought within the context of a keynesian model, Gyfason and Schmid and Gyfason and Risager show using a simulation technique that for a number of countries the expansionary effects on demand are probably stronger than the contractionary effects of a devaluation.

In terms of our analysis, the results obtained in the current literature may best be described as describing the very short run effects of devaluation. In contrast the results obtained in this paper throw light on the short term and long run aspects of a devaluation. In the long term the downward shift of the path of capital accumulation reduces the growth rate of real income. In the short run the devaluation lowers real national income and real wages as it depresses the relative price of the home good. If the real wages are prevented from absorbing the whole weight of the devaluation, say by indexation, then unemployment will increase. These conclusions contrast very sharply with those of the current literature and they represent another novel aspect of devaluation which hitherto was unknown.

VI.b An expected devaluation

An unexpected devaluation causes capital losses on money and capital holdings. Since these losses are unexpected they do not change the preferences of the asset holders. It is not clear a priori what are the relative magnitudes of the capital losses on capital and money holdings caused by an unexpected devaluation.

Suppose that at a time $t$ it was announced that at a future date $t_0$ there will be a devaluation. Under perfect foresight asset holders know that a devaluation will cause at $t_0$ capital losses on capital and money holdings. If the relative capital losses are equal the expected devaluation will not change the path of the economy until time $t_0$ where the analysis of the previous section applies.

If the relative capital losses are not equal there will immediately be a jump in $Q$ at $t$ in such a way that the relative capital losses in money and capital at time $t_0$ will be equalized. The jump in $Q$ at $t$ will lift the economy above the convergent plane to allow it to jump back down into the plane at $t_0$ in such a way that the capital losses on capital
and money will be the same. The upward jump in $Q$ will cause an immediate decline in savings, an increase in investment, an immediate deterioration in the balance of trade, a decline in the growth rate of real money balances (which may be already negative) and the shifting of higher output periods towards the present. Therefore the anticipation of a devaluation reduces and may nullify its desirable effects. This result should be taken seriously because in a world of perfect foresight a devaluation can hardly be unexpected. Furthermore real world devaluations are almost certainly anticipated. To help make an anticipated devaluation more productive, the rate of devaluation must be chosen carefully so as if it were unanticipated, it would entail at the date of its implementation the same capital loss on real money balances and on physical capital.

VII. Conclusions

In this paper we have considered a small open economy with a fixed exchange rate in which capital accumulation is possible and the financial sector is repressed in the sense that there are limited possibilities for financial intermediation and the domestic price and rate of return of physical capital may depart substantially from their international counterparts in the short run.

In this model an open market purchase has no impact effects on output or employment. In contrast it has short and long run effects. First it leads to a loss of foreign reserves. Second if the economy is stable and foreign reserves pay an interest rate lower than the rate of return on domestic physical capital, then the open market purchase may be productive in the sense that it may induce an increase in the capital stock that is higher than the value of the loss in reserves. This is possible if the interest elasticity of the demand for real money balances is small and the current income elasticity of savings is relatively large but not too large to prevent stability. Most likely, however, the open market purchase is unproductive in the sense that the loss of reserves is greater than the value (in terms of foreign currency) of the capital stock acquired because of the transitory increase in consumption caused by the higher wealth achieved initially as a consequence of the open market purchase. Furthermore, if the income elasticity of the home good is small, an open market purchase induces a shift of labor out of the labor intensive home good into the capital intensive traded good sector. If the labor is more efficient in the latter sector we may interpret labor intensiveness as disguising unemployment and the shift caused by the open market purchase must be a welcome effect of monetary policy.

We have also used the model to analyze the effects of a devaluation. Essentially we may interpret a devaluation as a tool of shifting forward the periods of higher output to
solve a problem of a balance of trade deficit and/or a gain in foreign reserves that are crucial to economic development. In addition if the demand of the home good has a small positive elasticity with respect to wealth then a devaluation will cause a contraction and a decline in the real wage and employment.

The model indicates that neither monetary policy nor devaluation is painless. They are useless as short run stabilization tools. However they may be used as tools for implementing economic development programs. This is consistent with the new view of keynesian economists (Fischer) that economic policy should not be used for fine tuning but for correcting persistent large shocks affecting the economy.

References


