Adjustment Dynamics under Dual Exchange Rates

Ching-chong Lai
Yun-peng Chu

I. Introduction

The erratic movement of capital across national borders, often a result of foreign exchange speculation, has contributed a great deal to the instability of the international economy and the fall of the Bretton Woods system. In place of the pegged exchange rate system, many countries have adopted a flexible rate regime but even more have established a so-called dual rate system,¹ under which the markets for current and capital account transactions are separated, and the price in the former market, called the commercial exchange rate, is often fixed, while that in the latter market, called the financial exchange rate, is allowed to adjust freely.² By so doing it is hoped that the current account will be immune from the irregular international capital movements and the resultant volatility of the exchange rate.³

The existing literature on the regime of dual exchange rates had mainly focused on the problem of its policy effects and insulation function [Argy and Porter (1972), Dornbusch (1976a), Marion (1981), Flood and Marion (1982)], and its relative advantages or disadvantages compared with other methods of capital mobility control [Fleming (1974),

---

¹ According to Flood (1978), at the end of 1970, only 9 countries adopted the dual rates system but by the beginning of 1973, 26 countries have done so. Recently, Argentina (1981), Bolivia (1982), Mexico (1982) and Venezuela (1983) have also joined.
² Some countries keep the two markets apart but let the exchange rates in both markets adjust freely. See Flood and Marion (1982, 1983).
³ Our model below assumes an open capital market which may not be the case for many of the countries adopting dual exchange rate systems. However, it can be easily shown that if restrictions on capital movement can be described by the Arellano (1982) method of multiplying the capital flow by a fixed parameter, the results of our model remain intact.
Lanyi (1975), Phylaktis and Wood (1984)], and with other exchange rate regimes [Argy and Porter (1972), Swoboda (1974)]. As for the problem of dynamic adjustment of the (financial) exchange rate following changes in the monetary policy, it seems that only Cumby (1984), Aizenman (1985) and Gardner (1985) have offered systematic analysis.4 For operational convenience, however, both Aizenman and Cumby have assumed that the price level adjusts freely to clear the goods market instantaneously, while Gardner assumes complete capital mobility and perfect foresight by the market participants.5 This paper will relax these assumptions and see how the results differ. In particular, following Dornbusch (1976b), Bhandari (1981) and Frenkel and Rodriguez (1982), we assume that while the money and (financial) foreign exchange markets are cleared instantaneously in our economy, the price level adjusts to excess demand or supply in the goods market with a lag. In addition, capital mobility does not have to be complete,6 nor do the market participants enjoy perfect foresight.7

In section II below a macroeconomic model will be presented. Sections III and IV then describe the long-run equilibrium and the short- to intermediate-run adjustment path respectively. Finally section V summarizes the findings.

II. The Model

Our model is similar to that of Argy and Porter (1972) in its basic structure, but it will highlight the dynamic adjustment process of the financial exchange rate via changes in the price level and the current account imbalance. It assumed that

(i) the domestic economy is in full-employment equilibrium, given flexible wage rates in the labor market;

(ii) expectations are regressive;8

(iii) the effects of current account surplus and deficit on money supply are not

4 Gardner (1984) has revised the Kouri (1976) model to analyze the dual exchange rate system. But that paper only discusses the problem of dynamic stability and is based on the purchasing power parity assumption (see the next footnote). Flood (1978) has also discussed the dynamic stability of dual exchange rates system under rational expectations, but he is not concerned with changes in monetary policy.

5 Aizenman assumes purchasing power parity which implicitly implies an instantaneously cleared goods market.

6 But we do assume what Frankel (1983) calls the "perfect asset substitutability" between domestic and foreign assets, as is clear from our equation (4) below, where the exchange rate in the capital account market will always adjust to equate the returns on domestic and foreign assets.

7 In particular, it is assumed that market participants form their expectations regressively. For an empirical justification of this expectation form, see, e.g., Frankel and Froot (1985).

sterilized, and
(iv) domestic price adjusts with a lag, not instantaneously.

Following are the equations:

\[ \dot{p} = k \left[ I(r - \frac{2(\dot{p} - \ddot{p})}{\ddot{p}}) - S(\ddot{y}) + \bar{G} + B \left( \frac{\ddot{y}, \ddot{p}^e}{\ddot{p}} \right) \right] \]  \hspace{1cm} (1)

\[ L(\ddot{y}, r) = \frac{D}{\ddot{p}} + R \]  \hspace{1cm} (2)

\[ \dot{R} = B \left( \ddot{y}, \frac{\ddot{p}^e}{\ddot{p}} \right) \]  \hspace{1cm} (3)

\[ K \left( r - \frac{\ddot{p}^e}{\epsilon_f} - \frac{\theta(\epsilon_f - \epsilon_f)}{\epsilon_f} \right) = 0^{10, 11} \]  \hspace{1cm} (4)

where

\( k \) = speed of adjustment in the goods market,

\( \ddot{p} \) = domestic price level,

\( I \) = investment expenditure,

\( r \) = domestic nominal rate of interest,

\( \ddot{y} \) = full-employment output,

\( S \) = savings,

\( \bar{G} \) = government expenditure (deficit),

\( B \) = current account balance,

\( \ddot{p}^e \) = (fixed) commercial exchange rate, defined as the price of foreign currency in terms of domestic currency,

\( \epsilon_f \) = (flexible) financial exchange rate,

\( L \) = demand for real balance,

\( D \) = domestic credit,

---

9 See Swoboda (1974) for discussion on the dual exchange rate system with sterilization.

10 Following Bhandari (1981) and Frenkel and Rodriguez (1982), this paper adopts the flow approach to international capital movements. Lately some models adopt the stock approach instead, for examples, Kouris (1976), Marion (1981), Turnovský (1981) and Aizenman (1985). For the difference between the two, see Sinn (1982), Bhandari, Driskill and Frenkel (1984) and Bhandari (1984). In particular, Bhandari (1984) argues that there is virtually no qualitative difference between the two approaches as far as over- and under-shooting of the exchange rate are concerned, although quantitative differences do exist.

11 This equation in effect implies that the uncovered interest rate parity will hold even though capital mobilities between national borders are imperfect in the sense that their response to interest rate differences is not perfectly elastic. If the economy in question adopts a single exchange rate regime instead, the models in Frenkel and Rodriguez (1982) or in Bhandari (1981) will prevail. In those models, only the sum of the current and capital accounts will be zero, so imperfect capital mobility coexists with the possible disparity between uncovered interest rates.
\( R = \) foreign exchange reserves,
\( K = \) capital account balance,

and that a "**" indicates foreign, a "*" indicates the long-run equilibrium level, a "*" indicates the rate of change with respect to time, and a "-" denotes a fixed or exogenous amount.

Among these equations, (2) and (4) describe the money market and the foreign exchange market for the capital account respectively, equation (1) indicates how price changes in response to excess demand in the goods market, and equation (3) shows how foreign exchange reserves change as a result of trade surplus or deficit, given the fixed commercial exchange rate. It is worth noting that in the goods market, investment is a function of the real interest rate, \( i \), and

\[
i = r - \frac{p^* - p}{\bar{p}}
\]

where \( p^* \) is the expected price level, and it is assumed

\[
p^* = \lambda \hat{p} + (1 - \lambda)p
\]

Similarly, in the foreign exchange market of the capital account, \( K \) is a function of the difference between the return on domestic assets, \( r \), and that on foreign assets, \( r^* \hat{e}_f/\hat{e}_f + (\hat{e}_r^* - e_r)/\epsilon \), where \( \hat{e}_r \) is the expected exchange rate and it is assumed

\[
\hat{e}_r = (1 - \theta)\hat{e}_f
\]

### III. Long-run Equilibrium

At long-run equilibrium, \( \dot{p} = \dot{\hat{p}} = \dot{K} = 0 \), as \( p, R, r \) and \( e_f \) are all at their equilibrium levels, \( \bar{p}, \hat{p}, \hat{r} \) and \( \hat{e}_f \). Equations (1)–(4) become

\[
I(\hat{r}) - S(p) + \bar{g} + B(\hat{y}, \hat{e}_f p) = 0
\]

(5)

\[
L(\hat{y}, \hat{r}) = \frac{D}{\bar{p}} + \hat{R}
\]

(6)

\[
B(\hat{y}, \hat{e}_f p) = 0
\]

(7)

\[
K(\hat{r} - \hat{r}^* \hat{e}_f) = 0
\]

(8)

\[\text{The first term is interest income which will be paid through the current account, and the second term is expected capital gain due to exchange rate changes. See, e.g., Flood (1978) or Marion (1981).}\]
Letting $\tilde{c}'$, $\tilde{p}^*$ and the initial values of $\tilde{p}$ and $\tilde{e}_f$ all be unity, differentiation of equations (5)–(8) gives

$$
\begin{bmatrix}
-B_q & I_i & 0 & 0 \\
D & L_r & -1 & 0 \\
-B_q & 0 & 0 & 0 \\
0 & K_d & 0 & K_d \tilde{R}^*
\end{bmatrix}
\begin{bmatrix}
\frac{d\tilde{p}}{dD} \\
\frac{d\tilde{r}}{dD} \\
\frac{d\tilde{R}}{d\tilde{e}_f} \\
\frac{d\tilde{e}_f}{d\tilde{e}_f}
\end{bmatrix} =
\begin{bmatrix}
0 \\
dD \\
0 \\
0
\end{bmatrix}
$$

(9)

where $q = \sigma(p^*/p)$ is the terms of trade, and $\Delta = r - (r^* + \epsilon_f) = \sigma(\epsilon_f - e_f) + e_f$ is the difference between returns on domestic and foreign assets; and that it is assumed as usual that $B_q = \partial B/\partial q > 0$, $I_i = \partial I/\partial i < 0$, $L_r = \partial L/\partial r < 0$, and $K_d = \partial K/\partial \Delta > 0$.

By Cramer’s rule,

$$\frac{\partial \tilde{p}}{\partial D} = \frac{\partial \tilde{r}}{\partial D} = \frac{\partial \tilde{e}_f}{\partial D} = 0$$

(10)

$$\frac{\partial \tilde{R}}{\partial D} = -1$$

(11)

showing that domestic monetary policy will not have any effects on the prices in the long run. By buying (selling) domestic bonds in the open market, the monetary authorities would be merely giving up their foreign (domestic) assets in exchange for domestic (foreign) ones in the long run. This property is similar to the one implied by the Mundell (1963) model with fixed exchange rate and perfect capital mobility. The only difference is that, there the official reserves decrease as a result of over-all balance of payments deficit, here the transmission channel is confined to the current account.

IV. Dynamic Adjustment

Since the asset markets are cleared instantaneously by assumption, equations (2) and (4) must hold at any point in time. Differentiation of the two equations yields

$$
\begin{bmatrix}
L_r & 0 \\
K_d & K_d(\tilde{R}^* + \theta)
\end{bmatrix}
\begin{bmatrix}
\frac{dr}{d\tilde{e}_f} \\
\frac{d\tilde{R}}{d\tilde{e}_f}
\end{bmatrix} =
\begin{bmatrix}
\frac{dD}{d\tilde{e}_f} + \frac{dR}{d\tilde{e}_f} - \frac{D\tilde{p}}{d\tilde{e}_f} \\
K_d \theta \frac{d\tilde{e}_f}{d\tilde{e}_f}
\end{bmatrix}
$$

(12)

Substituting equation (10) into (12), we can now establish two reduced form equations:

$$
\frac{dr}{d\tilde{e}_f} = \sigma(p, R, D)
$$

(13)

---

13 This means that the Marshall-Lerner condition will hold.

14 See Aizenman (1985).
\[ \epsilon_f = \epsilon_f(p, R, D) \]  \hspace{1cm} (14)

\[ (-) \, (+) \, (+) \]

where the parenthesized signs are those of the partial derivatives, as it is clear from equations (10) and (12) that

\[ r_p \equiv \frac{\partial r}{\partial p} = -\frac{D}{L_r} > 0 \]  \hspace{1cm} (15)

\[ r_R \equiv \frac{\partial r}{\partial R} = r_D \equiv \frac{\partial r}{\partial D} = \frac{1}{L_r} < 0 \]  \hspace{1cm} (16)

\[ (\epsilon_f)_p \equiv \frac{\partial \epsilon_f}{\partial p} = \frac{D}{L_r(p^* + \theta)} < 0 \]  \hspace{1cm} (17)

\[ (\epsilon_f)_R \equiv \frac{\partial \epsilon_f}{\partial R} = (\epsilon_f)_D \equiv \frac{\partial \epsilon_f}{\partial D} = \frac{-1}{L_r(p^* + \theta)} > 0 \]  \hspace{1cm} (18)

A comparison between (10) and (18) indicates that

\[ \frac{\partial \epsilon_f}{\partial D} - \frac{\partial \epsilon_f}{\partial R} = \frac{-1}{L_r(p^* + \theta)} > 0 \]

i.e., while \( \epsilon_f \) remains unchanged as \( D \) rises, the short-run \( \epsilon_f \) will rise on impact, exhibiting a kind of "overshooting" phenomenon.

To trace how this over-reacting short-run \( \epsilon_f \) moves over time towards its long-run equilibrium, substitute equations (13) and (14) into (1) and (3):

\[ \dot{p} = k \left[ I^r(p, R, D) - \frac{\lambda(p - \bar{p})}{\bar{p}} \right] - S(y) + \bar{G} + B \left( y, \frac{\epsilon_f p^*}{\bar{p}} \right) \]  \hspace{1cm} (20)

\[ \dot{R} = B \left( y, \frac{\epsilon_f p^*}{\bar{p}} \right) \]  \hspace{1cm} (21)

Using equation (10), we can re-write them as

\[ \dot{p} = J(p, R, D) \]  \hspace{1cm} (22)

\[ (-) \, (+) \, (+) \]

\[ \dot{R} = H(p) \]  \hspace{1cm} (23)

\[ (-) \]

where

\[ J_p \equiv \frac{\partial J}{\partial p} = k[I^r(p + \lambda) - B_r] < 0 \]  \hspace{1cm} (24)

\[ J_R \equiv \frac{\partial J}{\partial R} = k[I^r R_R] > 0 \]  \hspace{1cm} (25)
Ching-chong Lai and Yun-peng Chu

\[ J_D = \frac{\partial J}{\partial D} = K_I \tau_D > 0 \]  \hspace{1cm} (26)

\[ H_p = \frac{\partial H}{\partial \eta} = -B_q < 0 \]  \hspace{1cm} (27)

It is then clear that the slopes of the equations \( \dot{p} = 0 \) and \( \dot{K} = 0 \) in Fig. 1 are

\[ \frac{\partial \dot{p}}{\partial \dot{K}} \bigg|_{\dot{p}=0} = -\frac{J_R}{J_p} > 0 \quad \text{and} \quad \frac{\partial \dot{p}}{\partial \dot{K}} \bigg|_{\dot{K}=0} = \frac{0}{H_p} = 0 \]  \hspace{1cm} (28)

respectively. Now let \( \beta \) be the characteristic root of the dynamic system and form the following characteristic equation

\[ \dot{\beta}^2 - J_p \beta - H_p J_R = 0 \]

Since \(-J_p > 0, -H_p J_R > 0\), the system is stable. In addition, it is straightforward that the adjustment path will be non-cyclical if

\[ \Omega = J_p^2 + 4H_p J_R > 0 \]

as shown by the solid curve \( \alpha \beta \) in Fig. 1. Conversely, if \( \Omega < 0 \), the adjustment path will be the dotted cyclical curve in the same figure.

Figure 1
The reason why $\rho$ and $R$ will adjust in the way they do is not hard to comprehend. Suppose point $\alpha$ in Fig. 1 is an initial position and suppose $\dot{\rho} = 0$ shifts the new intersection is now point $\beta$ perhaps as a result of expansionary monetary policies. From equation (13), when $D$ rises, $r$ will fall instantaneously to maintain equilibrium in the money market. The decreased $r$ stimulates investment in equation (20) and the resultant excess demand bids up the price. Then, through equation (23), the higher domestic price creates trade deficit which runs down the reserves, $R$. This explains why initially $(R, \rho)$ moves northwestwards from $\alpha$ in Fig. 1.

But the price level cannot rise forever. Before soon the decreases in reserves lower the money supply and drive up the nominal interest rate, which along with a reduced expected rate of future inflation and a shrinking trade balance as results of the price rise, eliminates the excess demand in the goods market and create an excess supply instead. Price starts to fall as shown in Fig. 1, but before it goes back to its original level $R$ would not stop falling, therefore $(R, \rho)$ is moving towards the southwest. It will then reach point $\beta$ directly if $Q > 0$ and cyclically if $Q < 0$, as discussed earlier.

Having shown the behavior of $\rho$ and $R$, it is time to see how $\dot{\epsilon}_f$, our major concern, adjusts over time. Differentiate equation (14) with respect to time:

$$\dot{\epsilon}_f = (\epsilon_f)_p \dot{\rho} + (\epsilon_f)_R \dot{R} + (\epsilon_f)_D \dot{D}$$  \hspace{1cm} (31)

Then use equations (22) and (23) to obtain

$$\dot{\epsilon}_f = (\epsilon_f)_J (\rho, R, D) + (\epsilon_f)_H (\rho) + (\epsilon_f)_D \dot{D}$$ \hspace{1cm} (32)

The slope of the curve $\dot{\epsilon}_f = 0$ is

$$\frac{\partial \rho}{\partial R} \bigg|_{\rho = 0} = \frac{-(\epsilon_f)_J}{(\epsilon_f)_R J + (\epsilon_f)_D H}$$  \hspace{1cm} (33)

which will be positive (negative) if

$$A = (\epsilon_f)_R J + (\epsilon_f)_D H > 0 \ (\ < 0)$$

Moreover, since $(\epsilon_f)_J < 0$, $(\epsilon_f)_R > 0$, $J > 0$, $J < 0$ and $H < 0$, and by comparison with equation (28), the slope of $\epsilon_f = 0$ will be larger than that of $\dot{\rho} = 0$ if it is positive.15

Three representative $\dot{\epsilon}_f = 0$ curves, $\hat{\epsilon}_f$, $\check{\epsilon}_f$ and $\hat{\epsilon}_f$ are displayed in Fig. 1. Regardless of

$$\frac{\partial \rho}{\partial R} \bigg|_{\rho = 0} = \frac{(\epsilon_f)_J}{(\epsilon_f)_R J + (\epsilon_f)_D H} \cdot \frac{\partial \rho}{\partial R} \bigg|_{\rho = 0}$$

from equations (28) and (33). So the slope of $\dot{\epsilon}_f = 0$ will approach that of $\dot{\rho} = 0$ if $(\epsilon_f)_H / ((\epsilon_f)_J)$ approaches zero.
their slopes, since in equation (32) \( (\varepsilon_f)_p < 0 \) and \( J_R > 0 \), any point \((R, \rho)\) off and to the right (left) of \( \hat{\varepsilon}_f = 0 \) will give negative (positive) values of \( \hat{\varepsilon}_f \). Knowing this, it is possible to identify the following cases of the dynamic adjustment in \( \varepsilon_f \) following an exogenous shock:

(i) \( \Omega > 0 \) and the \( \hat{\varepsilon}_f = 0 \) curve is either \( \hat{\varepsilon}_f^* = 0(A > 0) \) or \( \hat{\varepsilon}_f^* = 0(A < 0) \) in Fig. 1.\(^{16}\)

In this case, the time path of \( \varepsilon_f \) over time would look like the curve numbered 1 in Fig. 2. I.e., following an expansionary monetary shock (rise in \( D \)), the \( \varepsilon_f \) will first rise according to equation (18). Then in Fig. 1, as \((R, \rho)\) goes along the non-cyclical solid time path the entire length of which lies to the right of \( \hat{\varepsilon}_f^* = 0 \) (also \( \hat{\varepsilon}_f = 0 \)), \( \varepsilon_f \) will adjust downwards continuously until it reaches the long-run equilibrium in Fig. 2.

This result is similar to the ones obtained by Cumby (1984) and Aizenman (1985) following an unanticipated rise in the money supply. However, as discussed earlier, the restrictive assumptions adopted in those two models make this kind of “simple” overshooting the only possible route of adjustment in \( \varepsilon_f \). Our model will offer two more below.

(ii) \( \Omega > 0 \) and the \( \hat{\varepsilon}_f = 0 \) curve is \( \hat{\varepsilon}_f^* = 0(A > 0) \) in Fig. 1.\(^{17}\) Here the adjustment

\[ A = \frac{D}{I_R(I^* + 6)} \left( -\frac{D}{I_R} + \hat{\lambda} \right) + \frac{B_1}{I_R(I^* + 6)} (Dk - 1) \]

and the first term at the RHS is positive, while for the second term, \( B_1[I_R(I^* + 6)] > 0 \). So if \( Dk > 2, \Omega > 0 \) and \( A > 0 \), but one cannot of course exclude the possibility that \( \Omega > 0 \) but \( A < 0 \).

\[^{16}\] It can be shown from the definition of \( \Omega \) that

\[ \Omega = kI \left( -\frac{D}{I_R} + \hat{\lambda} \right) + kB_1 - 2kB_1I_R + \frac{2B_1}{I_R} (Dk - 2) \]

and all of the terms in the square brackets except the last one is positive. So a sufficient condition for \( \Omega > 0 \) is \( Dk > 2 \), given \( 2B_1[I_R/I_R > 0] \). Also, it can be shown that

\[^{17}\] \( \hat{\varepsilon}_f = 0 \) is more likely to be the \( \hat{\varepsilon}_f = 0 \) curve if \((\varepsilon_f)_p \approx \hat{\varepsilon}_f \) is closer to zero, see footnote (15).
path of \((R, \rho)\) in that figure will cross the \(\dot{e}_f = 0\) curve, and therefore along the way \(\dot{e}_f\) is first negative then positive, giving rise to the path number 2 in Fig. 2. So \(e_f\) will rise to \(e_f^*\) on impact, then undershoots before it settles at the long-run equilibrium.

(iii) \(\Omega < 0\) and the \(\dot{e}_f = 0\) is either \(\dot{e}_f^c = 0\), \(\dot{e}_f^p = 0\) or \(\dot{e}_f = 0\) in Fig. 1. Now \((R, \rho)\) will follow a cyclical adjustment pattern, sending \(\dot{e}_f\) oscillate between negative and positive values, so curve number 3 in Fig. 2 becomes the adjustment path of \(e_f\) in this case.

V. Conclusion

Based on a simple dual exchange rate model, this paper explores the dynamic adjustment paths of the financial exchange rate, under the assumption that the price level adjusts to excess demand and supply in the goods market with a lag and that market participants form their expectations of the future exchange rate regressively. It is found that the adjustment path of the financial exchange rate can assume three different patterns. The first pattern by which the \(e_f\) first overshoots following a rise in money supply then gradually decreases is identical to the Aizenman or Cumby results. The second pattern is that \(e_f\) first overshoots then understages before it reaches the long-run equilibrium. Finally the third pattern shows that \(e_f\) will oscillate between values higher and lower than the long-run equilibrium level before it settles down. Among these patterns, the latter two cases are not possible under the Cumby (1984) of Aizenman (1985) framework, and the last one is not possible in the Gardner (1985) model, but they may be very important in explaining the volatility of the financial exchange rate in the real world.

References


