

Exporting versus Foreign Direct Investment

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Abstract

In this paper we investigate how strategic aspects influence the choice between exporting and servicing foreign markets by setting up a plant in the foreign country. We show that tariffs on imports in conjunction with the size of the set up costs incurred while setting up plants and the size of the foreign market will determine whether domestic firms which face competition from a foreign firm will choose to deter foreign direct investment (FDI), prevent exports or may accommodate either form of penetration of a foreign firm in their market. Our analysis reveals that there is no simple relationship between the size of the tariff and the propensity of foreign firms to engage in foreign direct investment. Higher tariffs may result in exports rather than FDI. Furthermore, due to actual competition among domestic firms while facing potential competition in the form of FDI, a rise in tariffs may lead to a decrease in domestic output. (JEL Classifications: F12, F21) <Key Words: Foreign direct investment; Imperfect competition; Tariff jumping.>

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I. Introduction

A cursory look at the data indicates that during the last two decades the extent of inward foreign direct investment (FDI) in the U.S., Canada and the EU has varied dramatically. During the same period these countries' governments imposed an array of trade restrictions which include tariff and in particular non tariff barriers to trade.¹ These changes in FDI and heightened overall trade barriers raises questions about whether there is a relationship between them. In the past many theoretical studies examined the relationship between foreign direct investment and trade policy. These studies however failed to take into account a crucial characteristic of the industries where most FDI takes place: these industries tend to be highly concentrated and dominated by a few large firms, some domestic and others foreign MNC's. Consequently, strategic considerations are expected to play an important role in such industries. Hence, to arrive at meaningful conclusions about the determinants of FDI in concentrated industries one needs to develop a theoretical framework which accounts for strategic behavior of domestic and foreign firms.

In this paper we show that tariffs in conjunction with strategic behavior of the players in the industry have important repercussions with regard to the decisions of international firms about the choice of the location of their production facilities. Firms which engage in international trade may find it attractive to set up plants in countries where they sell their products in order to circumvent tariffs and local content requirements imposed by foreign governments.² Domestic firms which face competition from foreign firms in their home markets, have a stake in the actions that foreign firms may take with regard to the mode of supplying these markets. Foreign firms may supply foreign markets either by exporting goods produced abroad, or make Foreign Direct Investment (FDI) by setting up plants near the markets they intend to penetrate. The nature of competition between

1. See for example Graham and Krugman [1991] for a comprehensive survey of foreign direct investment in the U.S.

2. For an earlier discussion on the effects of tariffs on the decision to export or to get behind the tariff wall and produce abroad, in the context of imperfectly competitive markets, see Horst [1971], Corden [1974] and Caves [1982].

domestic and foreign firms will depend on the foreign firms choice of location of its production facilities.

Some recent and closely related papers Horstman and Markusen [1987], Smith [1987], Motta [1992] and Markusen [1997] examined the conditions which lead to the existence of a multinational enterprise (MNE). The similarity between their investigation and our lies in the strategic aspects of the analysis. Based on the combination of three factors, plant scale economies, firm-specific costs and tariff and transportation costs they predict that MNE will be found in industries where tariff and transportation costs are large relative to plant scale economies.

In this paper we further explore how the interplay between the level of tariffs levied on imports, the size of the costs involved in setting up plants in foreign countries and the size of the market affect the behavior of domestic firms and foreign firms. When facing potential competition from foreign firms, the domestic firms may engage in strategic behavior by preempting entry of foreign firms in their domestic markets. There are two kinds of entry prevention that domestic firms might consider: deterring exports by the foreign firm to their market and preempting foreign direct investment. Both types of entry preemption are costly to the domestic firms since both involve over production, relative to the production levels that they would choose when entry is accommodated. The strategy chosen by the domestic firms with regard to entry of the foreign firm will ultimately determine how the foreign firm will service the domestic market.³ Although our paper focuses on a situation which is somewhat different from the one examined by Horstman and Markusen [1987], Smith [1987] and Motta [1992], a comparison is instructive. In these papers the common assumption is that the

3. For a recent and more comprehensive examination of alternative modes of supply of new goods to a foreign market, i.e., via exporting, or licensing or setting up production facilities abroad, see Ethier/ Markusen [1991] and Ethier [1992]. Although their model is more general they focus on the conditions which influence the firm's decision about the dissemination of information about new products across countries. We focus on the effects of various entry deterrence strategies by domestic firms towards the attempts of a foreign firm to penetrate their domestic market. See also Levinsohn [1989] who examined the implications of the possible occurrence of FDI for the equivalence between tariffs and quotas in imperfectly competitive markets.

foreign firm, i.e., the MNE is established already, while the domestic firm is a potential entrant. We however, consider the case where the domestic firms are established already whereas the foreign firm contemplates entry in the foreign market. Put differently, their case fits better competition between US multinationals and host countries domestic firms and our case is closer to competition between domestic US firms and Japanese and European multinationals which contemplated entry in the US market, during the last two decades.

We show that changes in the tariff levels, will affect the strategies of all firms, domestic and foreign. Due to competition between domestic and foreign firms, lower tariffs may entail outcomes which differ from the common wisdom. We show that depending on the level of the tariffs, the value of the set up cost and the size of the market, there are five possible equilibrium outcomes: exports accommodation, FDI accommodation, FDI deterrence, exports deterrence and blockaded entry. The conventional wisdom, which is consistent with the prediction of Horstman and Markusen [1987] and [1992], is that low tariffs encourage exports and higher tariffs lead to FDI. Our main result indicates that low tariffs may be conducive to more extensive foreign investment which is contrary to the conventional wisdom. Consequently, lower tariffs can in fact lead to a decrease in unemployment in sectors where domestic firms are protected from foreign competition.

The contribution which our paper makes is twofold: At the theoretical level we extend the industrial organization literature on entry deterrence that typically deals with one mode of entry in a particular market. We consider and examine two modes of market penetration; via exports and via FDI. The second contribution is in relation to the literature on international trade and investment. Building upon a plausible sequence of moves, where established domestic firms simultaneously move first and potential foreign entrant moves second, we are able to examine two concurrent types of competition; actual competition among established domestic firms and potential competition between a non-cooperative domestic oligopoly and a potential entrant, the foreign firm. To our knowledge this approach is novel in view of the existing international trade and investment literature. Thus, not surprisingly perhaps we are able to reach unconventional conclusions regarding the non monotonic relationship between tariffs and foreign direct invest-

ment. The paper is organized as follows. In the next section we present the basic model. In Section III we examine the exports and the foreign direct investment games. Section IV is devoted to equilibrium analysis and in section V we discuss our main results. The paper concludes with a summary and additional remarks about the optimal tariff in the presence of FDI. Technical details are relegated to the appendix.

II. The Model

There are n domestic firms and one foreign firm which produce an identical product sold in the home country H . All $n+1$ firms produce under constant marginal cost. We consider prices net of marginal cost and assume, without loss of generality, that the marginal cost equals zero.⁴

The inverse market demand for the product is given by $P(X)$ where X is the total quantity sold, which satisfies the following: The function $P(\cdot)$ is twice continuously differentiable, strictly decreasing and concave on the interval $[0, \bar{X}]$ with $P(\bar{X}) = 0$.

The foreign firm has three options:

(i) **Export:** Exporting the good to the home country's market and be subjected to a per unit tariff t .

(ii) **Foreign Direct Investment (FDI):** Produce the good in a plant located in country H , after incurring a fixed cost I . By serving the market with goods produced in this plant the foreign firm can circumvent the tariff.

(iii) Stay out of the country H market.

The sequence of moves is the following: First all domestic firms simultaneously select their level of production, x_1, \dots, x_n . After observing the output produced by the domestic firms the foreign firm decides whether to sell in

4. The assumption that marginal cost are the same for all firms, domestic and foreign, is not necessary, but it greatly simplifies computations. What is important is that the foreign firm's total marginal cost (including tariffs) when it is exporting is higher than the domestic firms' marginal cost. The consequence of this assumption is that the foreign firm is at a competitive disadvantage relative to the domestic firms, when it exports to the home country's market rather than engaging in FDI by setting up a plant in the home country.

the home country's market and then chooses the mode of servicing the market, i.e., via exporting or via FDI. In either case, after the foreign firm determines its level of output x_f , the profits of the domestic firm $d=1,...,n$ are

$$\pi_d(x_1, \dots, x_n, x_f) = x_d P(X_H + x_f), \quad (1)$$

where $X_H = \sum_{i=1}^n x_i$ is the combined output of the domestic firms. Depending on the level of X_H , firm f may choose either to export or to engage in FDI or to stay out of the home country's market.⁵ The profits of the foreign firm when it exports are given by

$$\pi_f^E(X_H, x_f, t) = x_f P(X_H + x_f) - tx_f \quad (2)$$

and when it sets up a plant in country H its profits are

$$\pi_f^I(X_H, x_f, I) = x_f P(X_H + x_f) - I. \quad (3)$$

We now examine the behaviour of firm f . In particular, we shall determine the foreign firm's best response, given the domestic firms level of combined output X_H , in the exporting regime and in the case of FDI.

Exporting Regime: The domestic firm may prevent exports of the foreign firm by choosing the combined level of output X_H satisfying $P(X_H) = t$. Thus, the exporting regime is viable only if the value of tariffs t is less than $P(X_H)$. Denote by $X_H^{ED}(t)$ the **export-deterring** value of domestic output which satisfies

$$P(X_H) = t. \quad (4)$$

Then for $X_H < X_H^{ED}(t)$ the optimal quantity of exports of the foreign firm is given by

$$\pi_f^E(X_H, t) = \arg \max_{x_f} \pi_f^E(X_H, x_f, t), \quad (5)$$

5. It is important to note that the construct that firms move sequentially generates results which differ from those that would obtain if the firms were to make their output decisions simultaneously. This is not to say that the assumption adopted in this paper is always to be preferred. There are circumstances, such as in the context of entry in foreign markets, that sequential moves may be as appropriate an assumption as its alternative.

and is determined as the solution of the following equation

$$x_f P(X_H + x_f) + P(X_H + x_f) - t = 0 \quad (6)$$

The profits generated by firm f when it responds optimally, is denoted by $\hat{\pi}_f^E$, where

$$\hat{\pi}_f^E(X_H, t) = \pi_f^E(X_H, x_f^E(X_H, t), t). \quad (7)$$

FDI Regime: In this regime, for any value of the domestic firms combined output X_H , the foreign firm optimal output produced in the plant which is located in country H is

$$x_f^I(X_H, I) = \arg \max_{x_f} \pi_f^I(X_H, x_f, I). \quad (8)$$

where $x_f^I(X_H, I)$ is the solution of the following equation

$$x_f P(X_H + x_f) + P(X_H + x_f) = 0 \quad (9)$$

and obviously, is independent of I . The profits generated by firm f when it responds optimally is denoted by $\hat{\pi}_f^I$, i.e.,

$$\hat{\pi}_f^I(X_H, t) = \pi_f^I(X_H, x_f^I(X_H, I), I). \quad (10)$$

Then there exists a level of domestic output, denoted $X_H^{ID}(I)$, satisfying

$$\hat{\pi}_f^I(X_H^{ID}, I) = 0, \quad (11)$$

such that firm f makes positive profits in the FDI regime if $X_H < X_H^{ID}(I)$, whereas FDI is deterred when $X_H \geq X_H^{ID}(I)$.

So far, we characterized the optimal strategy of the foreign firm given t and I , whenever it has at most one viable option, either exporting or engaging in FDI.

It remains to consider the situation where both exports and FDI are viable, i.e., when the domestic firms combined output $X_H < \min[X_H^{ED}(t), X_H^{ID}(I)]$. In order to find out firm's f optimal strategy we need to compare the profits of the foreign firm in both regimes, $\hat{\pi}_f^E(X_H, t)$ and $\hat{\pi}_f^I(X_H, I)$. It turns out that if, for given t and I , the value $X_H^{ED}(t)$, the combined domestic output which deters the exports of firm f, is less than $X_H^{ID}(I)$, the combined domestic output which prevents FDI, then firm f will choose the FDI option for all

$X_H < X_H^{ED}(t)$. However when $X_H^{ED}(t) > X_H^{ID}(I)$, then firm f will choose the FDI option if the value of X_H is small but will prefer to export if the combined domestic output X_H is large.⁶

The proposition below, the proof of which is relegated to the Appendix, characterizes the optimal response of the foreign firm to various levels of the domestic firms combined output.⁷

Proposition 1: Let the tariff t and the value of fixed cost I be given.

(i) If $X_H < \max[X_H^{ED}(t), X_H^{ID}(I)]$ the foreign firm stays out of the home country's market.

(ii) If $X_H^{ID}(I) < X_H^{ED}(t)$ then for all $X_H < X_H^{ID}(I)$ the foreign firm will exercise the FDI option.

(iii) If $X_H^{ID}(I) < X_H^{ED}(t)$ then there exists a cut-off value of combined domestic output denoted $X_H^0(t, I)$ such that the foreign firm will exercise the exporting option whenever $X_H^0(t, I) < X_H < X_H^{ED}(t)$ and the FDI option whenever $0 < X_H < X_H^0(t, I)$.

III. Exports and FDI Games

Thus far we have characterized the best response of the foreign firm to the domestic firms combined output in various regimes. To examine the equilibrium outcomes we turn now to focus on a domestic firm d best response to the output of other domestic firms $X_{-d} = \sum_{i \neq d} x_i$, while taking into account the foreign firm's best response to their combined output X_H . We will present the domestic firms best response for the case when exports and FDI are accommodated and for the case when they are deterred. To conduct this analysis it will be useful to consider the following three types of games:

Exports Game: For any tariff t we consider the n -player game in which each domestic firm maximizes its profit given the output of other domestic

6. We assume that in the case where the foreign firm is indifferent between the exports and FDI, it chooses exports.

7. In the next section we derive explicit expressions for the foreign firm's best response for the case of linear demand functions.

firms and correctly anticipating the exports of the foreign firm. Thus, each domestic firm d maximizes the profit function $\pi_d(x_d, X_{-d}, x_f^E(X_H, t))$ by choosing the output

$$x_d^E(X_{-d}, t) = \arg \max_{x_d} \pi_d(x_d, X_{-d}, x_f^E(X_H, t)), \quad (12)$$

where $X_H = x_d + X_{-d}$. The assumptions on the demand function imply that this game has a unique equilibrium denoted $(\bar{x}_1^E(t), \dots, \bar{x}_n^E(t))$. The domestic firms profits in equilibrium are denoted by:

$$\pi_d^E = \pi_d(\bar{x}_d^E(t), \bar{X}_{-d}^E(t), x_f^E(\bar{X}_H^E(t), t)) \quad (13)$$

where $\bar{X}_H^E(t) = \bar{x}_d^E(t) + \bar{X}_{-d}^E(t)$.

FDI Game : For any value of fixed costs I we consider the n -player game in which each domestic firm maximizes its profit given the output of the other domestic firm and correctly anticipating direct investment by the foreign firm. Thus, each domestic firm d maximizes profits $\pi_d(x_d, X_{-d}, x_f^I(X_H, I))$, by choosing its output

$$x_d^I(X_{-d}, I) = \arg \max_{x_d} \pi_d(x_d, X_{-d}, x_f^I(x_d + X_{-d}, I)). \quad (14)$$

The domestic firms' equilibrium profits are given by

$$\pi_d^I = \pi_d(\bar{x}_d^I, \bar{X}_{-d}^I, x_f^I(\bar{X}_H^I, I)), \quad (15)$$

where $\bar{X}_H^I = \bar{x}_d^I + \bar{X}_{-d}^I$.

Cournot Game : Let $(\bar{x}_1^C, \bar{x}_2^C)$ be the Nash-Cournot equilibrium levels of production chosen by the domestic firms for the case where the foreign firm stays out of the market. In this case the equilibrium profits of a domestic firm d are

$$\pi_d^C = \pi_d(\bar{x}_d^C, \bar{X}_{-d}^C), \quad (16)$$

where $\bar{X}_H^C = \bar{x}_d^C + \bar{X}_{-d}^C$.

We turn now to examine the strategies that each domestic firm will select in equilibrium. As one may expect the equilibrium strategies are affected by the interplay between tariffs, the level of fixed costs and the size of the country H market and, consequently, by the export deterring value of output

$X_H^{ED}(t)$ and the FDI-deterring level of output $X_H^{ID}(I)$ determined by (4) and (11), respectively. The assumption we made about the demand function implies that $X_H^{ED}(t)$ is decreasing in t and $X_H^{ID}(I)$ is decreasing in I , and, moreover, for each I there exists a value $t^*(I)$ such that

$$X_H^{ED}(t^*(I)) = X_H^{ID}(I). \quad (17)$$

Thus for each pair t and I we have $X_H^{ED}(t) > X_H^{ID}(I)$ whenever $t < t^*(I)$ (the tariffs are relatively low) and $X_H^{ED}(t) < X_H^{ID}(I)$ whenever $t > t^*(I)$ (the tariffs are relatively high). Proposition 1 implies that if $t > t^*(I)$, then from firm f vantage point the exports option is dominated by the FDI option. Since in this case the only viable option is FDI the model becomes very similar to that studied by Gilbert/Vives [1986]. They examined the issue of entry deterrence in the context of a oligopolistic market where several incumbents face the threat of potential entry, while competing against each other.⁸ Since the purpose of this paper is to study the interplay between values of tariffs and fixed investment costs, we focus on the more interesting case when both export and FDI options are viable for the foreign firm. The expression in (17) implies that this will happen when

$$t < t^*(I) \quad (18)$$

The existence of both FDI and exporting options leads to the emergence of several interesting distinctive equilibrium outcomes which we examine in the next section.

IV. Equilibrium Analysis

In this section we examine the equilibrium strategies of each domestic firm. To provide a complete characterization of the equilibrium we shall assume throughout this section that the market demand is linear. Specifically,

8. Gilbert and Vives analysis is based on the presumption that there is a single mode through which a potential entrant can enter a market. We however, consider two alternative modes of entry into a market. Consequently, in our framework the entrant and the incumbents strategies are richer and hence our analysis entails a larger array of results.

$$P(X) = A - X \quad (19)$$

where the intercept A represents the size of the market in country H .

Equations (4) and (11) imply that the export-detering domestic output $X_H^{ED}(t)$ is equal to $A - t$ and the FDI-detering total domestic output $X_H^{ID}(I)$ is equal to $A - 2\sqrt{I}$.

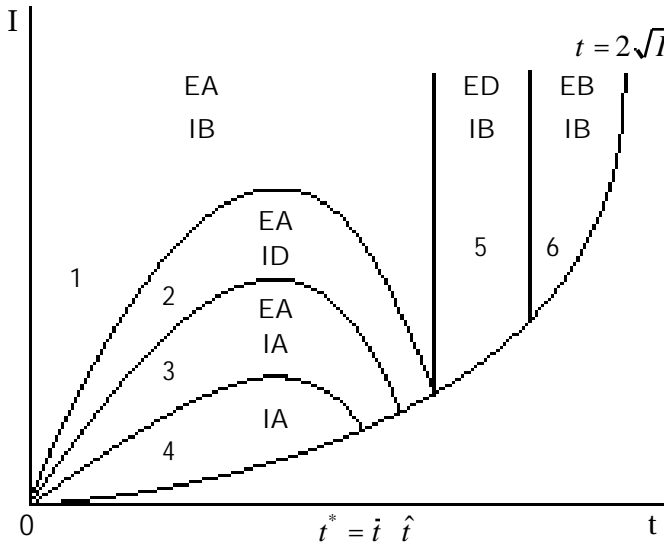
This implies that (17) can be rewritten as

$$t = 2\sqrt{I}, \quad (20)$$

which determines the pairs of tariffs rates and setup cost that will result in a level of total domestic production that will deter exports as well as FDI. The expression in (20) is depicted by the convex curve in Figure 1.

When the foreign firm is facing a sufficiently high level of domestic output, $X_H = A - t$, it stays out of the market. (Proposition 1 part (i).) If the foreign firm is confronted with an "intermediate" level of combined output X_H satisfying $A - 2\sqrt{I} < X_H < A - t$, only the exports option is viable. When the foreign firm is facing a sufficiently low level of combined output $X_H < A - 2\sqrt{I}$, both exports and FDI options are viable. In the latter case

Figure 1
Switches of Equilibrium Outcomes



one has to compare the profits that the foreign firm will make in each regime. Proposition 1 implies that the foreign firm will choose the exports option if and only if the combined level of domestic output X_H satisfies the inequalities $X_H^0(t, I) - X_H < A - t$ where $X_H^0(t, I)$ is the solution of equation (A1) in the appendix. Recall that $X_H^0(t, I)$ is the level of the domestic output which renders the foreign firm indifferent between FDI and exports. By (17) and (20), $X_H^0(t, I)$ is

$$X_H^0(t, I) = \begin{cases} A - \frac{t}{2} - \frac{2I}{t} & \text{if } \frac{t}{2} + \frac{2I}{t} < A \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

To summarize, the optimal response of the foreign firm is determined as follows:

- If $X_H > A - t$, the foreign firm stays out of the market.
- If $X_H^0 - X_H < A - t$, the foreign firm will exercise the exporting option.
- If $0 < X_H < X_H^0$ the foreign firm will choose the FDI option.

In Figure 2 we portray the foreign firm's best response for a given level of set up cost. From many possible set up cost values we choose a level that entails a rich array of best responses encompassing several switches in regimes; exports, FDI and back to exports. When a switch in regime occurs there is a discontinuity in the best response function.

Now we are ready to examine the strategies that the domestic firms will choose in equilibrium.

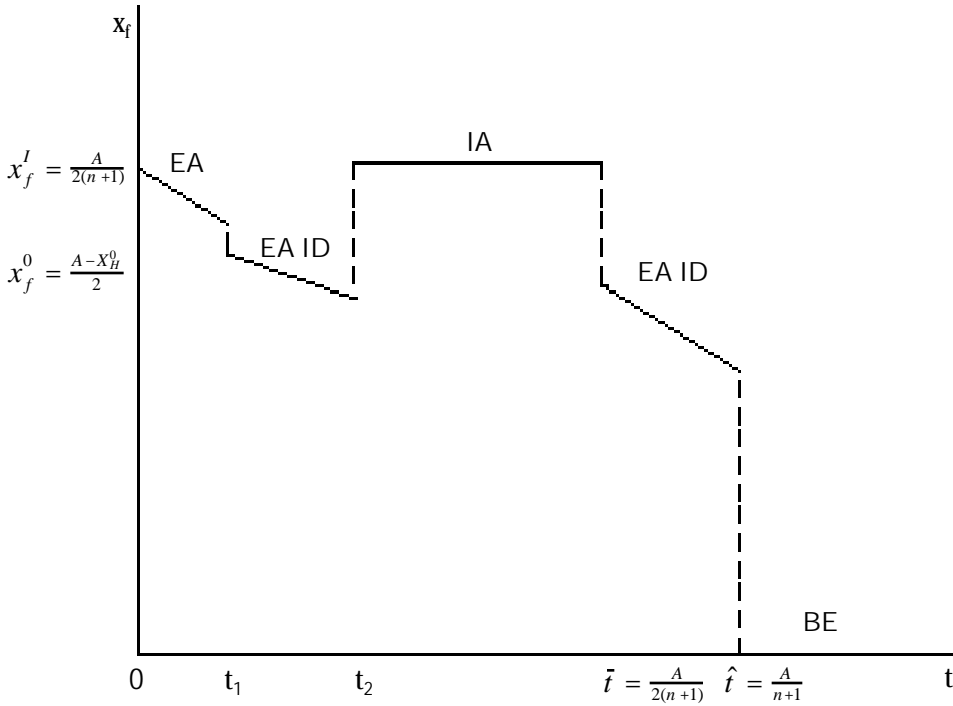
The Nash-Cournot levels of output, \bar{x}_d^C , $d = 1, \dots, n$, will be the equilibrium outcome if domestic firms total output $\bar{X}_H^C = \sum_{i=1}^n \bar{x}_i^C$ is sufficiently large to induce the foreign firm to stay out of the market. This requirement is satisfied when

$$\bar{X}_H^C > X_H^{ED}(t). \quad (22)$$

Since $\bar{X}^C = \frac{An}{n+1}$ and $X_H^{ED}(t) = A - t$, it follows that (22) holds when $t > \frac{A}{n+1}$, i.e., tariffs are prohibitively high. Thus, we have

Proposition 2 : Blockaded Exports and Blockaded FDI: If $t > \frac{A}{n+1}$, the domestic firms produce the Nash-Cournot output whereas the foreign firm stays out of the market.

Figure 2
The Foreign Firm Best Response



We turn now to examine the case when the tariff levels are not prohibitive and the Cournot equilibrium level of output \bar{X}^C is not sufficient to deter exports. That is $\bar{X}_H^C < X_H^{ED}(t)$ which is equivalent to $t < \frac{A}{n+1}$. The best response of a domestic firm to other domestic rivals output in the Exports and the FDI games, as given by (12),(14) are:

In the **Exports** game, if the total output of all domestic firms except d is X_{-d} , then the best response of firm d is $x_d = \frac{1}{2}(A - X_{-d} + t)$, while the foreign firm exports are $x_f^E = \frac{1}{2}(A - X_H - t)$ units. It is easy to verify that the each domestic firm output is $\bar{x}_d^E = \frac{A+t}{n+1}$, whereas the foreign firm exports are $\bar{x}_f^E = \frac{1}{2(n+1)}[A - (2n+1)t]$ units. Let $\bar{X}_H^E = n\bar{x}_d^E = \frac{n(A+t)}{n+1}$ denote the total output of the domestic firms in Exports equilibrium.

In the **FDI** game, the best response of a domestic firm firm d is $x_d = \frac{1}{2}(A - X_{-d})$, while the foreign firm output is $x_f^I = \frac{1}{2}(A - X_H)$ units.

Thus, the equilibrium level of output of each domestic firm is $\bar{x}_d^I = \frac{A}{n+1}$, whereas the foreign firm produces $\bar{x}_f^I = \frac{A}{2(n+1)}$ units.

When tariffs are not prohibitive and the domestic firms would have selected the Cournot equilibrium level of output, the foreign firm will find it profitable to export to the home country's market. Whether exports will actually occur will depend on whether the domestic firms accommodate or deter exports. This in turn depends on how the incumbents' output in the exports game, \bar{X}^E relates to two critical quantities: the exports deterrence quantity of output, $A-t$ and the combined level of output that induces the foreign firm to be indifferent between exports and FDI, $X_H^0(t, I)$. To proceed with the analysis we shall distinguish between three cases:

(a) $\bar{X}_H^E - A - t > X_H^C$: We now show that although the level of output that is chosen in the Exports game, \bar{X}^E is sufficient to deter exports, it is not part of the equilibrium if the inequalities stated in (a) hold. In this case both choices, \bar{X}_H^E and X_H^C are inconsistent with the best response of the foreign firm, namely, choosing the Cournot output allows the exports, whereas the combined level of domestic output of the Exports game prevents exports. Hence, in equilibrium each domestic firm will end up producing only the minimal level of output, $\frac{A-t}{n}$, that is sufficient to prevent exports. We thus have

Proposition 3 : Exports Deterrence and Blockaded FDI: $\frac{A}{n+1} > t - \frac{A}{2(n+1)}$ the domestic firms will deter exports by jointly producing $\bar{X}^{ED} = A - t$ units of output. The set of equilibrium levels of output is given by:

$$\{(x_1, \dots, x_n) \mid \sum_{d=1}^n x_d = A - t \text{ \& } x_d \leq 2t \text{ for each } d = 1, \dots, n\}.$$

The proof of Proposition 3 is relegated to the Appendix.

(b) $X_H^0(t, I) - \bar{X}_H^E < A - t$: If the domestic incumbents would have selected a combined level of output equal the equilibrium output of the Exports game, then even though both modes of entry are viable, based on the best response, the foreign firm prefers exports over the FDI option. (Proposition 1 part (iii).) For intermediate levels of tariffs, it will be too costly for the

domestic incumbents to deter exports, and consequently the equilibrium outcome is to accommodate exports.

To formally state this result, note that the combined equilibrium domestic output in the Exports game is $\bar{X}_H^E(t) = \frac{n(A+t)}{n+1}$, and since we know that $\bar{X}_H^E(t) < A - t$ it follows that $t < \frac{A}{2n+1}$. For each $t < \frac{A}{2n+1}$ denote by $\hat{I}(t)$ the value of the set up cost I , such that the level of domestic output, in Exports equilibrium \bar{X}_H^E , renders the foreign firm indifferent between exports and FDI. That is

$$X_H^0(t, \hat{I}(t)) = \bar{X}_H^E. \quad (23)$$

Since the function $X_H^0(t, I)$ as defined in (21), is decreasing in I , it follows that the inequality $\bar{X}_H^E < X_H^0(t, I)$ is satisfied for all $I > \hat{I}(t)$. Thus, we have

Proposition 4 : Exports Accommodation and Blockaded FDI: If $t < \frac{A}{2n+1}$ and $I > \hat{I}(t)$, the domestic firms will accommodate exports and each will produce $\bar{x}_d^E = \frac{A+t}{n+1}$ whereas the foreign firm will produce $\bar{x}_f^E = \frac{1}{2(n+1)} [A - (2n+1)t]$ units of output.

(c) $\bar{X}_H^E < X_H^0(t, I)$: In this case the total domestic output in the Exports game, again will render exports and FDI options viable. Now however, the foreign firm prefers the FDI option over exports. (Proposition 1 part (iii).)

Suppose that the FDI option is an equilibrium outcome. In this equilibrium each domestic firm produces $x_d^I = \frac{A}{n+1}$ units of output. In order to determine whether FDI accommodation is an equilibrium outcome, we inquire whether it is worthwhile for any domestic firm to unilaterally deviate (given that all other domestic firms play the FDI accommodation strategy) from accommodating FDI to preempting FDI. This will require that the deviating firm increases its output to contribute enough to total domestic production so that it reaches the critical level $X_H^0(t, I)$. The minimal level of output that the deviating firm d would need to produce in order to achieve this is $\tilde{x}_d(t, I) = X_H^0(t, I) - \frac{N-1}{n+1} A$. We will show that such a deviation is not profitable.

For each $t < \frac{A}{2n+1}$ we define by $\tilde{I}(t)$ the value of setup cost I which renders each domestic firm indifferent between: FDI accommodation on one hand and the minimal unilateral increase in output that leads to FDI deterrence on the other hand. That is

$$_d(\tilde{x}_d(t, \tilde{I}(t)), \bar{X}_{-d}^I(t), x_f^E(\bar{X}_{-d}^I + \tilde{x}_d)) = _d(x_d^I, X_{-d}^I, x_f^I(X_{-d}^I + x_d^I)). \quad (24)$$

The next proposition, the proof of which is relegated to the Appendix, states that if the value of the fixed cost is lower than $\tilde{I}(t)$, then FDI accommodation emerges as an equilibrium outcome:

Proposition 5 : FDI Accommodation: If $t < \frac{A}{2n+1}$ and $I < \tilde{I}(t)$, the domestic firms will accommodate FDI and each will produce $x_d^I = \frac{A}{n+1}$ units whereas the foreign firm will produce $x_f^I = \frac{A}{2(n+1)}$ units of output.

We turn now to the case where the domestic firms opt for exports accommodation and at the same time prevent FDI. As before since the total domestic output in Exports equilibrium \bar{X}_H^E is smaller than $X_H^0(t, I)$, the domestic output in the exports equilibrium is not large enough to deter FDI. To deter FDI the total domestic output needed to be raised to $X_H^0(t, I)$. Naturally, the domestic firms would have preferred to prevent FDI if the entry cost I , were not too low. We thus have

Proposition 6 : Exports Accommodation and FDI Deterrence: Assume that $t < \frac{A}{2n+1}$. Then there exists a value of set up costs $\tilde{I}(t)$, such that whenever $I < \tilde{I}(t)$, the domestic firms will deter FDI but accommodate exports, by jointly producing $X_H^0(t, I)$ units and the foreign firm exports $\frac{A - X_H^0}{2}$ units. Moreover, there exist a lower and an upper bound, x^l and x^h , such that in equilibrium the output of each domestic firm is no more than x^h and is at least x^l units.

The proof of this proposition is relegated to the appendix.

The properties of the functions $\hat{I}(t)$, $\tilde{I}(t)$, $I^*(t)$, of which we made use to prove Propositions 2-6, are examined in the Appendix. Based on these properties it follows that $\hat{I}(t) > \tilde{I}(t) > I^*(t)$ for all relevant tariffs, t . This leads to the conclusion that there are values of set-up costs I , for which both FDI accommodation and exports accommodation can co-exist in equilibrium.

Corollary: When the setup cost satisfy the inequality $\hat{I}(t) > I > I^*(t)$ there are two different types of equilibrium: one is FDI accommodation and the other is Exports accommodation-cum-FDI prevention.

In Figure 1 we present the various regimes that will arise in equilibrium and in Figure 2 we depict the foreign firm equilibrium best response.

V. Discussion

The analysis in the preceding section reveals that there is no simple relationship between the level of tariffs, the foreign firm's cost of entry and the size of the home country's market and predictions about the foreign firm's mode of supplying markets presently dominated by domestic oligopolistic firms. For ease of exposition of various outcomes that may arise, as stated in Propositions 2–6, we refer to Figures 1 and 2. It will be convenient to consider the effects of variations in the tariffs (setup costs) while keeping the setup costs (tariffs) fixed at some prespecified level.

Low setup costs in conjunction with any tariff levels are conducive to FDI. This is depicted by the area below the curve $\bar{t} = 2\sqrt{I}$ in Figure 1. At intermediate levels of setup cost and "low" tariffs the foreign firm will serve the market by exporting. In this regime (area 1 in Figure 1) the higher the tariff is, the larger is the output produced by domestic firms. The combined domestic equilibrium output in the exports regime blocks FDI in the sense that even when faced with a lower level of combined domestic output the foreign firm will refrain from direct investment since will not be able to recover its setup cost.

Maintaining the setup cost fixed at the intermediate level and letting the tariff rise still leads to the exports regime, (area 2 in Figure 1). When tariffs lie in this range the domestic firms become more concerned with the possibility that the foreign firm might circumvent the tariff by setting up a plant in their country. Hence FDI deterrence requires an increase in domestic output to render FDI by the foreign firm unprofitable. Lack of cooperation among domestic firms results in a combined level of domestic output $X_H^0(t, I) = A - \frac{t}{2} - \frac{2I}{t}$ that deters FDI even though is lower than the level that would have been produced if no domestic firm would contemplate to deviate from the most desirable entry deterring level of output. To ensure that no such deviation occurs each domestic firm will increase its output in response to rising tariffs, but by less than in the case where the threat of FDI was absent. Further deviation from $X_H^0(t, I)$ the level of output which deters FDI and accommodates exports becomes less profitable; if such deviation will occur it would lead to a switch in regime, from exports to FDI and each domestic firm will face the foreign firm as a competitor from behind

the tariff wall. The consequence is the total output (domestic output and the foreign firm's exports) declines and thus yields profits to all firms, domestic and foreign, that are higher than in the FDI regime. In Figure 2 we depict the foreign firm's equilibrium best response to $X_H^0(t, I)$.

It is noteworthy that although an increase in tariff within the range $t \in [t_1, t_2]$ provides domestic firms more protection from import competition, they will respond with only a moderate expansion in production in comparison with a situation where strategic considerations are absent. This result stems from the fact that domestic firms prefer competition from exports rather than head on competition from a foreign plant located in their country, FDI. Tariffs at any level endow the domestic firms with a competitive advantage, since they raise the foreign rival total marginal cost relative to the case where the foreign firm supplies the market with goods produced behind the tariff wall. Ensuring that the competitive advantage is preserved entails costs since the domestic firms end up producing less output than they would have produced if they could coordinate output decisions, in their effort to deter FDI.

As tariffs become even higher, for the same level of intermediate setup cost, two types of equilibria co-exist: FDI deterrence-cum -exports accommodation and FDI accommodation (area 2 in Figure 1). This result was stated in the corollary of Propositions 5 and 6.⁹ In the former case tariffs in the range $[\tilde{t}, \hat{t}]$ are conducive to FDI deterrence whereas in the latter case, tariffs in the range $[t^*, \tilde{t}]$ will lead to FDI accommodation. This is the **tariff-jumping** phenomenon. As tariffs continue to rise $t > \hat{t}$ (area 5 in Figure 1) for the same set up cost as before the incentives for each domestic firm to engage in exports deterrence are reinforced and concomitantly FDI is effectively blockaded. Finally, when tariff levels are very high and the home market is small relative to the foreign firm's fixed cost of entry, the domestic firms may ignore the foreign firm's threat of entry and produce the Cournot

9. The implications of lack of coordination among incumbents while facing competition from a potential entrant, for the co-existence of different types of equilibria, was previously noted by Gilbert and Vives [1986] and Donnenfeld and Weber [1995]. These authors however, dealt with the effects actual competition among incumbents while facing a single mode of (potential) entry in their market. Here (potential) entry encompasses two alternatives: exports and FDI.

equilibrium levels of output. That is, as stated in Proposition 2, both exports and FDI are effectively blockaded, (area 6 in Figure 1).

The upshot of this analysis is that one needs not expect to find a monotonic relationship between tariffs and the propensity to engage in foreign direct investment. As indicated above for some range of tariffs, set up cost and the home country market size, as tariff rise and enter the range $\tilde{t} > t > t^*$, FDI will occur; as tariffs rise further, $t \in [\tilde{t}, \hat{t}]$ exports will occur. That is depicted by switches in regimes that occur when we move from area 2 to area 3 in Figure 1.

VI. Concluding Comments

In this paper we investigated the effects of barriers to trade and foreign investment in industries dominated by a few domestic producers. We have shown that the foreign firm's decision between exports and FDI is influenced by the height of tariffs, the size of the set up costs in relation to the size of the market and by the strategies selected by the domestic firms. For this purpose we constructed the simplest model which captures the aforementioned features. Despite its simplicity the model generates a rich array of equilibrium outcomes.

The conclusions that emerge from our investigation is that in markets where strategic considerations play a significant role, there is no simple relationship between tariffs, exports and foreign direct investment. Varying the height of the tariff leads to switches in regimes. Low and high tariff levels sustain exports by the foreign firm, whereas intermediate levels of tariffs sustain FDI as the mode by which foreign firms serve foreign markets. Furthermore, actual competition among domestic producers while facing potential foreign entry via FDI will dampen the protective effects that tariffs have on domestic production.

Although the main purpose of this paper was to examine the impact of exogenous tariffs (and set up cost) on the equilibrium configuration of the import competing industry, the framework that we developed can be used to derive the optimal tariff. Obviously the optimal tariff will depend on the level of set up cost associated with FDI. Preliminary results indicate that the optimal tariff, i.e., the tariff that maximizes national welfare (the sum of con-

sumers surplus, domestic firms profits and tariff revenues) will be set at a level that will lead to an equilibrium outcome where the domestic output is sufficiently large and thus renders the foreign firm to be indifferent between exports and FDI. The optimal tariffs entails a balance of the following trade offs: (i) inducement of the domestic firms to expand production that is beneficial to domestic consumers (ii) induce the foreign firm to refrain from FDI and thus endow the domestic firms with the competitive advantage when they compete with imports and (iii) generate tariff revenues. A complete investigation of welfare consequences and the optimal tariff is on our agenda for future work.

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Appendix

The Properties of the Functions $\hat{I}(t)$, $\tilde{I}(t)$, $I^*(t)$.

(i) $\hat{I}(t) > \tilde{I}(t) > I^*(t)$ for all $t < \frac{A}{2n+1}$.

(ii) Each of the three curves $\hat{I}(t)$, $\tilde{I}(t)$ and $I^*(t)$ intersects the curve $I = \frac{t^2}{4}$ at two points: one is at the origin $t = 0$ and the other is at \hat{t}, \tilde{t} and t^* , respectively, where $\frac{A}{2n+1} = \hat{t} > \tilde{t} > t^* > 0$.

(iii) All three functions $I = \hat{I}(t)$, $I = \tilde{I}(t)$, and $I = I^*(t)$ are concave and have an interior maximum in the intervals $[0, \hat{t}]$, $[0, \tilde{t}]$ and $[0, t^*]$.

Proof: (i) Equation (23) can be rewritten as

$$A - \frac{t}{2} - \frac{2I}{t} = \frac{n(A+t)}{n+1},$$

yielding

$$I(t) = \frac{2At - (3n+1)t^2}{4(n+1)}. \quad (\text{A.1})$$

The equality in (24) yields the following equation

$$[X_H^0(t, \tilde{I}(t)) - \frac{A(n-1)}{n+1}] \frac{A - X_H^0(t, \tilde{I}(t)) + t}{2} = \frac{A^2}{2(n+1)^2}$$

which yields two roots:

$$X_H^0(t, \tilde{I}(t)) = \frac{2nA + (n+1)t \pm \sqrt{t^2(n+1)^2 + 4At(n+1)}}{2(n+1)}.$$

Since $X_H^0(t, \tilde{I}(t)) > X_H^E(t) = \frac{n(A+t)}{n+1}$ it follows that

$$X_H^0(t, \tilde{I}(t)) = \frac{2nA + (n+1)t \pm \sqrt{t^2(n+1)^2 + 4At(n+1)}}{2(n+1)}.$$

By using (21), we obtain

$$\tilde{I}(t) = \frac{2At - 2(n+1)t^2 - t\sqrt{(n+1)^2t^2 + 4At(n+1)}}{4(n+1)}. \quad (\text{A.2})$$

It is easy to see that $\hat{I}(t) = \tilde{I}(t)$ for all t . Moreover, the equation $\tilde{I}(t) = I^*(t)$ has two roots, $t=0$ and $t = \frac{A(9n^2+6n+1)}{4An(n^2-1)} > \frac{A}{2n+1}$. Since $\tilde{I}(0) > I^*(0)$, it follows that $\tilde{I}(t) > I^*(t)$ for all

(ii) To establish this we make use of (i) and the fact that the equation $\hat{I}(t) = \frac{t^2}{4}$ has a smallest positive root at $\hat{t} = \frac{A}{2n+1}$.

(iii) Concavity of the three functions is verified by simple algebra. It remains to observe that all three functions \hat{I}, \tilde{I} and I^* are increasing at $t=0$ and decreasing at the points \hat{t}, \tilde{t} and t^* , respectively.

Proof of Proposition 1: First, consider the case where $X_H^{ED}(t) < X_H^{ID}(I)$. Then $\hat{f}_f^E(X_H^{ED}(t), t) = 0$. Since the function $\hat{f}_f^I(., t)$ decreases in X_H it follows that $\hat{f}_f^I(X_H^{ED}(t), I) > 0$. Moreover, since the difference $\{\hat{f}_f^E(., t) - \hat{f}_f^I(., I)\}$ increases in X_H , it follows that $\hat{f}_f^I(X_H, t) > \hat{f}_f^E(X_H, I)$ for any $X_H < X_H^{ID}(I)$; thus the foreign firm will choose the FDI option.

Second, consider the case where $X_H^{ED}(t) > X_H^{ID}(I)$. Thus $\hat{f}_f^E(X_H^{ID}(I), t) > 0$, whereas $\hat{f}_f^I(X_H^{ED}(t), I) > 0$. Since the difference $\{\hat{f}_f^E(., t) - \hat{f}_f^I(., I)\}$ increases in X_H , there exists a cut-off value of the domestic output, $X_H^0(t, I) < X_H^{ID}(I)$

such that the foreign firm will prefer exports if $X_H > X_H^0(t, I)$ and FDI otherwise. Specifically, if $\hat{\pi}_f^E(0, t) < \hat{\pi}_f^I(0, I)$, then there exists a positive cut-off value of combined domestic output, $X_H^0(t, I)$ given by the solution of the following equation:

$$\hat{\pi}_f^E(X_H, t) = \hat{\pi}_f^I(X_H, I). \quad (\text{A.3})$$

Hence, the foreign firm will exercise the exporting option if $X_H > X_H^0(t, I)$ and the FDI option if $X_H < X_H^0(t, I)$.

If $\hat{\pi}_f^E(0, t) > \hat{\pi}_f^I(0, I)$ then firm f will prefer exports for all values of X_H satisfying $0 < X_H < X_H^{ED}(t)$. In this case we put $X_H^0(t, I) = 0$.

Obviously, if $X_H > \max[X_H^{ED}(t), X_H^{ID}(I)]$ the foreign firm stays out of the home country's market. This completes the proof of Proposition 1.

Proof of Proposition 3: Since the combined equilibrium domestic output in the Exports game, $\bar{X}_H^E(t)$ is equal to $\frac{n(A+t)}{n+1}$ the inequality $\bar{X}_H^E(t) > A - t$ yields $t > \frac{A}{2n+1}$. We shall show that in this case in equilibrium the domestic firms deter exports by producing total output

$$X_H = \sum_{d=1}^n x_d = A - t.$$

Consider an n -tuple of the domestic firms' outputs (x_1, \dots, x_n) . If $X_H = \sum_{d=1}^n x_d > A - t$ then $t < \frac{A}{2n+1}$ implies that the n -tuple (x_1, \dots, x_n) is not a Cournot equilibrium. Thus, a slight change in the output of at least one of the firms would still deter exports and increase the profits of the deviating firm. If $X_H < A - t$ exports are accommodated and, since domestic output in the exporting equilibrium $(\bar{x}_1^E, \dots, \bar{x}_n^E)$ satisfies $\sum_{d=1}^n \bar{x}_d^E = A - t$, it follows that the n -tuple (x_1, \dots, x_n) is not an equilibrium in the Exports game. Thus, a slight change in the output of, at least, one of the firms would still allow exports and increase the profits of the deviating firm. It follows, therefore, that in equilibrium the total output of the domestic firms X_H is equal to $A - t$. Assume now that $X_H = A - t$. The n -tuple (x_1, \dots, x_n) will constitute an equilibrium if no domestic firm finds it beneficial either to increase its output while still keeping the exports out or to allow exports by reducing its output. Since the best response of firm d to output X_{-d} in the Cournot game is

it follows that firm d will benefit by increasing its output if and only if $x_d + X_d = A - t$. Since $x_d + X_d = A - t$, the last inequality amounts to $x_d < t$. Thus, no domestic firm will increase its output if and only if

We now consider the possibility when one of the domestic firms reduces its output and thus allows the foreign firm to enter. Since the best response of firm d to the combined output X_d in the Exports game is it follows that firm d will benefit by allowing entry if or equivalently, Thus, the condition

is necessary and sufficient for domestic firms to have no benefit from accommodating exports. This completes the proof of Proposition 3.

Proof of Proposition 4: Let and as defined by equation

$$\frac{2A - 2nt^2}{n+1} - \frac{(A+t)^2}{2(n+1)^2},$$

which is equivalent to

$$0 - A^2 - (4n+2)At + (4t^2 + 4n+1)t^2 = [A - (2n+1)t]^2.$$

Thus, the n -tuple of the domestic firms' outputs $(\bar{x}_1^E, \dots, \bar{x}_n^E)$ is, indeed, an equilibrium. \square

Proof of Proposition 5: Let $t < \frac{A}{2n+1}$ and $I < \hat{I}(t)$ as defined by equation (23). We shall derive the conditions under which FDI accommodation emerges as an equilibrium outcome. As we mentioned above, the equilibrium of the FDI game yields the output of $\frac{A}{n+1}$ for each domestic firm. The only threat of deviation from this n -tuple would be the willingness of one of the firms to produce a larger amount of output in order to deter FDI. The minimal level of output that would guarantee the FDI deterrence is $\tilde{x}_d(t, I) = X_H^0(t, I) - \frac{A(n-1)}{n+1}$. Let us first show that the profits of a deviating firm d are decreasing for all $x_d > \tilde{x}_d(t, I)$, which would imply that if firm d decides to preempt FDI it should choose the level of output equal to $x_d = \tilde{x}_d(t, I)$ we have

$$\pi_d(x_d, \bar{X}_{-d}^I(t), x_f^E(\bar{X}_{-d}^I + x_d)) = \frac{x_d}{2} \left[\frac{2A}{n+1} + t - x_d \right],$$

which is decreasing for $x_d > \frac{A}{n+1} + \frac{1}{2}$. However since $\tilde{x}_d = X_H^0(t, I) - \frac{A(n-1)}{n+1} > \frac{A+nt}{n+1}$, it follows that if firm d decides to deter FDI, its optimal choice should be $\tilde{x}_d(t, I)$. To complete the proof of the proposition, it remains to compare the profit of the domestic firm d at the FDI equilibrium and in the case when it unilaterally raises the total domestic output to $X_H^0(t, I)$ in order to preempt FDI. However, equation (24) implies that if $I < \tilde{I}(t)$, then each domestic firm will choose to accommodate FDI, which is the equilibrium outcome. \square

Proof of Proposition 6: The same consideration as in the proof of Proposition 3 leads us to the conclusion that total domestic output X_H satisfies

$$X_H = \sum_{d=1}^n x_d = X^0 = X_H^0(t, I).$$

The n -tuple (x_1, \dots, x_n) will constitute an equilibrium if no domestic firm finds it beneficial either to increase its output while still preventing FDI or to accommodate FDI by reducing its output. Since the best response of firm d to output X_{-d} in the Exports game is $\frac{1}{2}(A - X_{-d} + t)$, it follows that firm d will benefit by increasing its output if and only if $\frac{1}{2}(A - X_{-d} + t) > x_d$. Since $x_d + X_{-d} = X^0$, the last inequality amounts to $x_d < A - X^0 + t$. Thus, no domestic firm will increase its output if and only if

$$\min_{d=1, \dots, n} x_d \leq A - X^0 + t.$$

We now consider the possibility that one of the domestic firms reduces its output and thus allows the foreign firm to enter. Since the best response of firm d to the combined output X_{-d} in the FDI game is $\frac{1}{2}(A - X_{-d})$, it follows that firm d will benefit from accommodating FDI if its profits are greater than when it prevents FDI. That is

$$\frac{x_d(A - X^0 + t)}{2} < \frac{(A - X^0 + x_d)^2}{8}$$

A necessary and sufficient condition for firm d to have no benefit from accommodating exports is

$$x^l = A - X^0 + 2t - 2\sqrt{t(A - X^0)^2 + t^2} \leq x_d \leq A - X^0 + 2t + 2\sqrt{t(A - X^0)^2 + t^2}. \quad (\text{A.5})$$

which together with the previously derived constraint on the value of x_d yields

$$x^l = A - X^0 + t \leq x_d \leq x^h = A - X^0 + 2t + 2\sqrt{t(A - X^0)^2 + t^2}. \quad (\text{A.6})$$

Thus,

$$\sum_{i=1}^n x_i = X^0 \leq n(A - X^0 + 2t + \sqrt{4t(A - X^0)^2 + 4t^2}).$$

The equation $X^0 = n(A - X^0 + 2t + 2\sqrt{t(A - X^0)^2 + t^2})$ has two roots

$$X_{1,2}^0 = \frac{An(n+1) + 2nt \pm 2n\sqrt{At(n+1) + t^2}}{(n+1)^2}$$

Since we consider the case where $X^0 > X_H^E = \frac{An}{n+1}$, we have

$$X^0 = \frac{An(n+1) + 2nt + 2n\sqrt{At(n+1) + t^2}}{(n+1)^2} \quad (\text{A.7})$$

Recalling that $X^0 = A - \frac{2I}{t} - \frac{t}{2}$, we can derive $\tilde{I}(t)$ as a solution (for a given t) of this equation

$$\tilde{I}(t) = \frac{2At(n+1) - (n^2 + 6n + 1)t^2 - 4nt\sqrt{At(n+1) + t^2}}{4(n+1)^2} \quad (\text{A.8})$$

To complete the proof of this proposition, it remains to observe that for all $I \in I^*(t)$, the n -tuple (x_1, \dots, x_n) with $\sum_{d=1}^n x_d = X^0$ and $x^l = x_d = x^h$ for all d , constitutes an equilibrium in which the domestic firms allow the exports.