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Convergence and Divergence of Economic Growth in a Two-Country Model without Externalities

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Abstract

International convergence as well as divergence processes of economic growth are both observable trends in economic development. To explain long-run differences of per capita income growth rates across countries, recent studies in economic growth have incorporated economies of scale and externalities. This paper shows that convergence as well as divergence processes in an integrated world may also be explained without relating to any externalities. (JEL: F21, Q41)

I. Introduction

Traditional neoclassical growth theory predicts an international convergence of per capita income growth. In reality, however, such a trend is not at all obvious. Certainly, a number of countries, e.g. the OECD-countries, have

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experienced a process of convergence over the last decades, but on a world-wide level countries have grown at considerably different rates, resulting in a progressive dispersion of per capita incomes. This is even more surprising as the economies have become more and more integrated during the last decades.

Recent developments in the theory of economic growth, initiated by the seminal papers of Romer [1986] and Lucas [1988], tried to close this theoretically unsatisfactory characteristic of traditional theory by endogenizing the long-run growth rate of per capita income. These approaches were then applied to international aspects, to explain cross-country growth trends.[1] The effects of economies of scale and externalities figure prominently within this literature of endogenous growth. In recent studies, however, Mankiw, Romer and Weil [1992] as well as Backus, Kehoe and Kehoe [1992] did not find empirical evidence for substantial nonconvexities on the aggregate level. Thus, while nonconvexities are interesting from a theoretical point of view, an additional theoretical framework may be appropriate to overcome the convergence hypothesis of traditional growth theory.

A second line of new growth theory relates to convex technologies. A promising approach, provided by Rebelo [1991], assumes two reproducible inputs, physical and human capital. Under this assumption constant returns to scale do not rule out self-sustained long-run growth.[2] The model of Rebelo uses a closed economy. Thus, different cross-country growth rates may be easily explained by international differences in the willingness to accumulate human, respectively, physical capital. Whether long-run cross-country differences in per capita income growth may also occur within this type of model assuming free international capital mobility, or whether a tendency to convergence prevails, is still open to question. The aim of this paper is to

1. See e.g. Grossman and Helpman [1990, 1991a, 1991b], Kohn and Marion [1992], Rivera-Batiz and Romer [1991a, 1991b] or Song [1993].

2. There are further examples of endogenous growth models with convex technologies: Jones and Manuelli [1990] picked up an idea, already proposed by Solow [1956], that diminishing returns to physical capital are limited by a lower bound. Under this assumption capital accumulation leads to sustained long-run per capita income growth. Quite recently, Deardorff [1994] has shown that international differences in the rates of population growth may also allow a country to limit the decline of marginal returns to physical capital.

explore what kind of convergence respectively divergence processes may arise in an integrated world without any externalities.

The paper employs a two-country model, where both countries are integrated through an international capital market.[3] Production in each country takes place according to a constant returns to scale production function that combines physical capital and labor in efficiency units, where the latter is the size of the labor force times human capital per worker. Households in both countries are in a position to accumulate physical and human capital. A key assumption of our analysis is that the households arbitrage international differences in the return to physical capital but fail to arbitrage domestic differences in the return to physical and human capital.[4] Instead, we assume that, while exhibiting the same savings rates, the countries differ in the division of investment per period regarding the respective accumulation activities.[5] In this paper we will show that depending on the differences in accumulation activities international productivity differentials may, but need not persist in the long run.

The rest of the paper is organized as follows: Section II outlines the basic framework. In section III the main results concerning long-run per capita income growth are derived. Finally, section IV provides a brief summary of the results and draws some conclusions.

II. The Basic Framework

A. A Simple Closed Economy

In this section we start with a simple growth model of a closed economy. Output Y is produced using a technology $F(K, hL)$, exhibiting constant returns to scale, where K represents the aggregate stock of physical capital,

3. With regard to international capital mobility the present approach is close in spirit to the models of Hamada [1966], Onitsuka [1974] or Ruffin [1979].
4. This failure may be evoked by, *e.g.* distorting taxation ⟨see *e.g.* Rebelo [1991]⟩, capital market imperfections ⟨see *e.g.* Galor and Zeira [1993] or Perotti [1993]⟩, or certain cultural values ⟨see *e.g.* Easterlin [1981]⟩.
5. Considering the remark in the previous footnote the different accumulation activities may be the result of, *e.g.* different policies by the governments, different income distributions, or different cultural values.

h human capital per worker[6] and L unelastically supplied labor. Income per worker $y = Y/L$ is then given by:

$$y = hf(\kappa), \tag{1}$$

where $\kappa = K/hL$ and f fulfills: $f' > 0$ and $f'' < 0$. Households can accumulate physical and human capital by investing some part of the final output. In what follows we assume that the proportion of final output, which will be invested per period, is given by s. $\sigma_h\, Ly$ units will be used for human capital and $(s - \sigma_h)Ly$ for physical capital accumulation. Of course, this drastically simplifies the household's decision problem. In general, an optimization approach should be used, but since we are mainly interested in the long-run properties of international growth patterns the assumption of constant investment rates may be justified.

Given s and σ_h, the increase of human capital per worker h and the stock of physical capital K will be $\dot{h} = \sigma_h y$ and $\dot{K} = (s - s_h)Ly$. Without delving into detail, the steady state κ is then given by $(s - \sigma_h)/\sigma_h$, and the long-run growth rate of per capita income g of the closed economy may be written as:

$$g = \sigma_h f\left(\frac{s - \sigma_h}{\sigma_h}\right). \tag{2}$$

If the total investment rate s is given, *i.e.* if a decision about the amount of consumption to be sacrificed for accumulation per period has already been reached, one may ask which share of investment should be devoted to human capital accumulation in order to maximize long-run growth of per capita income. Maximizing g with respect to σ_h yields:

$$\left(\frac{\sigma_h}{s}\right)^{opt} = 1 - \frac{f'\left(\frac{s - \sigma_h}{\sigma_h}\right)}{f\left(\frac{s - \sigma_h}{\sigma_h}\right)} \frac{s - \sigma_h}{\sigma_h} = \varepsilon, \tag{3}$$

where $(\sigma_h/s)^{\text{opt}}$ is the relative share of the investment rate s devoted to human

6. As is common in the literature of endogenous growth, see e.g. Lucas [1988] or Rebelo [1991], the term human capital refers to a measure of human productivity and may be accumulated over time without bound.

capital accumulation, which maximizes g for a given investment rate s. ε is the production elasticity of labor in efficiency units, if κ approaches its steady state value.[7] Equation (3) contains a simple endogenous growth version of the well known golden rule of accumulation.[8] Rearranging (3) yields:

$$f\left(\frac{s-\sigma_h}{\sigma_h}\right)-f'\left(\frac{s-\sigma_h}{\sigma_h}\right)\frac{s-\sigma_h}{\sigma_h}=f'\left(\frac{s-\sigma_h}{\sigma_h}\right). \tag{4}$$

Equation (4) states that for a given investment rate s per capita income growth is at its highest level if the returns to labor in efficiency units (left hand side) and physical capital (right hand side) are equalized. This condition is intuitively plausible. Because no other allocation of investment yields a higher return of one asset without lowering the return of the other to a greater extent, growth of per capita income cannot be augmented further through the choice of a different accumulation structure.

B. A Two-Country Model

In the following, we consider two countries, country 1 and 2, each described by the model of the previous section. Both countries have access to a common capital market, *i.e.* households in each country can hold shares of the stock of physical capital of the home and the foreign country. Both countries produce with the same constant returns to scale technology F. Let W_i denote the physical capital possessed by households of country i, called the real wealth of country i, h_i average human capital, L_i unelastically supplied labor, A the amount of physical capital owned by households of country 1 and used for production in country 2, and r the interest rate of physical capital. Income in country 1, respectively, country 2 is then given by:

$$Y_1 = F(W_1 - A, h_1L_1) + rA, \tag{5}$$

$$Y_2 = F(W_2 + A, h_2L_2) - rA. \tag{6}$$

7. In the special case of a Cobb-Douglas production function $f = \kappa^{1-\alpha}$ equation (3) reduces to $\sigma_h/s = \alpha$.
8. In the Solow-model per capita consumption per period reaches a maximum, if the savings rate is chosen, such that it equals the elasticity of production of physical capital, see *e.g.* Phelps [1966].

Per capita income $y_i = Y_i/L_i$ in both countries may be written as:

$$y_1 = h_1[f(w_1 - a) + ra], \tag{7}$$

$$y_2 = h_2[f(w_2 + \lambda pa) - \lambda pra], \tag{8}$$

where W_i/h_iL_i represents the ratio of real wealth and labor in efficiency units of country i, which will be termed as wealth intensity. $p = h_1/h_2$ is the ratio of average human capital of both countries and represents the level of relative human productivity of country 1. a is defined as A/h_1L_1 and λ as L_1/L_2. In the following we assume that the population of each country is stationary, so λ is constant.

Under the assumption of free capital mobility the following no-arbitrage condition must hold:

$$f'(w_1 - a) = f'(w_2 + \lambda pa) = r. \tag{9}$$

Considering (9), a and r may be written as functions of w_1, w_2 and p:

$$a(w_1, w_2, p) = \frac{1}{1+\lambda p}(w_1 - w_2) \tag{10}$$

and

$$r(w_1, w_2, p) = f'\left(\frac{1}{1+\lambda p}(\lambda p w_1 + w_2)\right). \tag{11}$$

To keep the notation simple, the argument of f will be written as κ with:

$$\kappa = \kappa(w_1, w_2, p) = \frac{1}{1+\lambda p}(\lambda p w_1 + w_2). \tag{12}$$

We assume that both countries devote the same proportion of income per period for accumulation: $s = s_1 = s_2$. As already mentioned in the introduction, both countries differ only in the utilization of investment for real wealth and human capital. The increase of human capital per worker, or respectively the stock of real wealth in country i is then given by: $\dot{h}_i = \sigma_{hi} y_i$ and $\dot{W}_i = (s - \sigma_{hi}) L_i y_i$. Using equations (7) to (12), the development of w_1, w_2 and p can be written as follows:

$$\begin{aligned} \dot{w}_1 &= \phi_1(w_1, w_2, p) \\ &= [s - (1+w_1)\sigma_{h1}]\,[f[\kappa(w_1, w_2, p)] + r(w_1, w_2, p)a(w_1, w_2, p)], \end{aligned} \tag{13}$$

$$\dot{w}_2 = \phi_2(w_1, w_2, p)$$
$$= [s-(1+w_2)\sigma_{h2}]\,[f[\kappa(w_1, w_2, p)] - \lambda p r(w_1, w_2, p) a(w_1, w_2, p)], \quad (14)$$
$$\dot{p} = \phi_p(w_1, w_2, p)$$
$$= p[\sigma_{h1}[f[\kappa(w_1, w_2, p)] + r(w_1, w_2, p) a(w_1, w_2, p)]$$
$$- \sigma_{h2}[f[\kappa(w_1, w_2, p)] - \lambda p r(w_1, w_2, p) a(w_1, w_2, p)]]. \quad (15)$$

Equations (13) to (15) constitute a three dimensional dynamical system. First, the dynamic properties of the model will be briefly discussed. In the next section, this will be followed by considerations on the comparative dynamics concerning long-run growth of per capita income in both countries.

Table 1

Dynamic Behaviour of System (13) to (15)

	regime I	regime II	
accumulation parameters	$\sigma_{h1}/s < s_{h2}/s < \varepsilon_2$	$s_{h1}/s < \varepsilon_1$ and $s_{h2}/s > \varepsilon_2$	
roots	$(w_1^*, w_2^*, p) = \left(\frac{s-\sigma_{h1}}{\sigma_{h1}}, \frac{s-\sigma_{h2}}{\sigma_{h2}}, 0\right)$	$(w_1^*, w_2^*, p) = \left(\frac{s-\sigma_{h1}}{\sigma_{h1}}, \frac{s-\sigma_{h2}}{\sigma_{h2}}, 0\right)$	$(w_1^*, w_2^*, p) = \left(\frac{s-\sigma_{h1}}{\sigma_{h1}}, \frac{s-\sigma_{h2}}{\sigma_{h2}}, p^{**}\right)$
dynamic behavior	globally stable	saddle point	locally stable

The dynamic behavior of the model may be divided into two regimes depending on the size of σ_{h1} and σ_{h2}. Table 1 summarizes the dynamic behavior of the system and the steady state values of w_1, w_2 and p in the respective regimes.[9] The variables ε_1 and ε_2 in Table 1 correspond to ε from the previous section. Table 1 indicates that under regime I both countries

9. The results, summarized in Table 1, are derived in Appendix A of this paper. There exists a third long-run outcome within the dynamic system. For $\varepsilon_2 < \sigma_{h1}/s < \sigma_{h2}/s$ the level of human productivity tends to infinity. This is the case, where both countries

would underaccumulate human capital in autarky, *i.e.* both countries could experience a higher long-run growth rate of per capita income without reducing consumption in any period, but by investing more in human and less in physical capital. In regime II, on the other hand, country 1 would underaccumulate and country 2 would overaccumulate human capital. Table 1 shows that under regime I the level of relative human productivity p approaches zero. This implies that the productivity of country 1, while increasing in absolute terms, becomes infinitely small relative to the productivity of country 2. Thus, the relative position of country 1 deteriorates more and more and results in complete divergence of both countries in course of time. As Table 1 shows, this result holds true for any accumulation parameters fulfilling $\frac{\sigma_{h1}}{s} < \frac{\sigma_{h2}}{s} < \varepsilon_2$. These are the cases where both countries underaccumulate human capital, but where country 2 accumulates at a rate closer to the optimal level. It will be seen that this includes the case where country 1 is in the position of a small country in the long run and country 2 in the position of an autarkic economy. Under regime I only one steady state exists, which is globally stable.

Increasing the accumulation parameter σ_{h2} continuously, a transcritical bifurcation arises, the globally stable equilibrium converts to a saddle point, and another long-run equilibrium emerges.

Under regime II, a bilateral relationship between both countries still exists in the long run. In the steady state of regime II the level of human productivity p exceeds zero. The equilibrium value p^{**} cannot be given explicitly, but in the appendix it is shown that p^{**} is unique, and implicitly defined by:

$$p^{**} + \frac{f[\kappa(w_1^*, w_2^*, p^{**})] - f'[\kappa(w_1^*, w_2^*, p^{**})](1 + w_2^*)}{\lambda[f[\kappa(w_1^*, w_2^*, p^{**})] - f'[\kappa(w_1^*, w_2^*, p^{**})](1 + w_1^*)]} = 0 \qquad (16)$$

Employing the implicit function rule and considering (12) it follows:

$$\frac{\partial p^{**}}{\partial \sigma_{h1}} = \frac{\lambda p^{**}(1 + \lambda p^{**})s}{(w_1^* - w_2^*)\sigma_{h1}^2} > 0,$$

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would overaccumulate human capital in autarky. However, this is the mirror image to regime I.

$$\frac{\partial p^{**}}{\partial \sigma_{h2}} = \frac{(1+\lambda p^{**})s}{(w_1^* - w_2^*)\sigma_{h2}^2} > 0.$$

Thus, p^{**} is the larger the larger the shares which countries 1 and 2 devote to human capital accumulation. p^{**} increases with σ_{h1} simply because country 1 accumulates more human capital. p^{**} increases with σ_{h2} because in regime II higher human capital accumulation in country 2 implies that it deviates more from the optimal accumulation structure and thus lowers its accumulation potential in the long run. This, in turn, reduces the productivity gap between both countries, *i.e.* raises p^{**}.

The equilibrium values w_1^* and w_2^* are the same under regime I and II. Furthermore, they are equal to the steady state capital intensities in autarky. This feature of the model makes the dynamic system and the comparative dynamics relatively easy to handle.

III. Long-Run Growth

This section will analyze the effects of free capital mobility on the long-run growth rate of per capita income (LGR) in both countries under both regimes. Since in the long run the intensities w_1, w_2, and the level of relative human productivity p are constant, per capita income of each country increases with domestic average human capital h_i, as can be seen from equations (7) and (8).

Considering equations (7) and (8), (10) to (12) and the steady state values of regime I, per capita income of country 1 and country 2 under regime I grow at the following rates:

$$g_1^I = \sigma_{h1}\left[f\left(\frac{s-\sigma_{h2}}{\sigma_{h2}}\right) + f'\left(\frac{s-\sigma_{h2}}{\sigma_{h2}}\right)\left(\frac{s-\sigma_{h1}}{\sigma_{h1}} - \frac{s-\sigma_{h2}}{\sigma_{h2}}\right)\right], \tag{17}$$

$$g_2^I = \sigma_{h2}\left(\frac{s-\sigma_{h2}}{\sigma_{h2}}\right). \tag{18}$$

Equations (17) and (18) show that the LGR of country 2, *i.e.* the growth rate of the country with the higher propensity to accumulate human capital, under regime I only depends on its own accumulation parameter. The LGR of country 1, on the other hand, depends on the accumulation parameters of both countries, though it should be noted that the production decision only depends on the accumulation parameter of country 2. Thus, country 1 becomes a small open economy under regime I and country 2 takes the position of an autarkic economy.

The following proposition highlights the effects of free international capital mobility on long-run growth under regime I:

Proposition 1: *Under regime I free international capital mobility leads to an increase of the* LGR *in country* 1, *while the* LGR *in country* 2 *does not change. However, the* LGR *in country* 1 *is lower than in country* 2.

Proof: See Appendix B.

Thus, under regime I only country 1 benefits from free capital mobility. Nevertheless, per capita income growth remains lower in country 1 than in country 2, even though both countries devote the same proportion of national income per period to accumulation. The different utilization of investment generates complete divergence of both countries in the long run.

Proposition 2 highlights the effects of national growth policies in an integrated world under regime I:

Proposition 2: *Under regime I an increase of* σ_{h2} *raises the* LGR *of both countries, while an increase of* σ_{h1} *raises only the* LGR *of country 1.*

Proof: Differentiation of equations (17) and (18) with respect to σ_{h1} and σ_{h2}, respectively, leads directly to proposition 2. ■

This property of the model shows an interesting prospect of international growth patterns. Growth enhancing policies of the economic leader positively influence economic growth in the rest of the world as well. Growth enhancing policies of small countries on the other hand only have domestic effects.

A completely different situation arises under regime II. Considering equations (7) and (8), (10) to (12) and the steady state values of regime II, the LGRs are given by:

$$g_1^{II} = \sigma_{h1}\left[f[\kappa(w_1^*, w_2^*, p^{**})] + \frac{1}{1+\lambda p^{**}} f'[\kappa(w_1^*, w_2^*, p^{**})]\,(w_1^* - w_2^*) \right], \quad (19)$$

$$g_2^{II} = \sigma_{h2}\left[f[\kappa(w_1^*, w_2^*, p^{**})] - \frac{\lambda p^{**}}{1+\lambda p^{**}} f'[\kappa(w_1^*, w_2^*, p^{**})]\,(w_1^* - w_2^*) \right]. \quad (20)$$

Equations (19) and (20) show that bilateral dependencies between both countries are still valid in the long run. A comparison of the LGRs between autarky and free international capital mobility under regime II yields the following proposition:

Proposition 3: *Under regime II free international capital mobility will equalize the* LGRs *of both countries and will raise them, for a given investment rate s, to its maximum level.*

Proof: Appendix A shows that the following equality holds under regime II:

$$\frac{f[\kappa(w_1^*, w_2^*, p^{**})]}{f'[\kappa(w_1^*, w_2^*, p^{**})]} = 1 + \kappa(w_1^*, w_2^*, p^{**}). \quad (21)$$

Rearranging (21) yields:

$$\begin{aligned} f[\kappa(w_1^*, w_2^*, p^{**})] - \kappa(w_1^*, w_2^*, p^{**}) f'[\kappa(w_1^*, w_2^*, p^{**})] \\ = f'[\kappa(w_1^*, w_2^*, p^{**})]. \end{aligned} \quad (22)$$

This equality shows that free capital mobility in regime II not only equalizes the returns of physical capital between both countries but also the returns of physical capital and labor in efficiency units in each country. An equivalent condition is already known from section II.a. Again, no other allocation of investment yields a higher return of one asset, without lowering the return of the other asset to a greater extent. Henceforth, growth of per capita income cannot be augmented further, if (22) holds. ■

Under regime II free capital mobility leads to international convergence of per capita income growth, though both countries employ different relative shares of their investment per period for human capital accumulation. Free capital mobility thus not only leads to an efficient international allocation of physical capital but also to an efficient accumulation structure. Though each country chooses a suboptimal accumulation structure, efficient international investment of physical and human capital accumulation emerges. It is noteworthy that this result holds for a wide range of values of σ_{h1} and σ_{h2}.[10]

IV. Summary and Conclusion

This paper has developed a two-country model of growth and international investment. The objective was to show that international divergence as well as international convergence patterns of economic growth can be explained without relating to any externalities. Depending on the accumulation propensities of both countries, two regimes of long-run growth were identified. Under the first regime, the model predicts a strict divergence of per capita income growth, though only the country with the lower long-run growth rate of per capita income benefits from free capital mobility. Under the second regime, the model predicts convergence of per capita income growth. Both countries benefit from free capital mobility. Moreover, capital mobility leads to an international accumulation structure such that the long-run growth rate of per capita income takes on a maximum in each country.

Though the paper has not considered welfare but only growth effects of international capital movements, some conclusions about the desirability of capital market liberalization may be drawn. In regime I only the country with the lower rate of growth can augment its income by international investment. However, only capital owners gain from capital market liberalization by investing in the country with the higher steady state rate of growth. The position of individuals supplying only labor respectively human capital as a factor of production, on the other hand, deteriorates. Thus, capital market liberalizations may have adverse distributional effects within the poor country. In the second regime both countries gain. International investments open up growth potentials which exist because households do not arbitrage domestic differences in the return to physical and human capital.

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10. Assume for example a Cobb-Douglas function $f = \kappa^{1-\alpha}$, this result holds for $\sigma_{h1}/s \in (0, \alpha)$ and $\sigma_{h2}/s \in (\alpha, 1)$.

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Appendix A

The purpose of this appendix is to derive the results summarized in Table 1.

Proposition A1: *The system (13) to (15) contains a unique, globally stable point* $(w_1^*, w_1^*, p^*) = ((s - \sigma_{h1})/\sigma_{h1}, (s - \sigma_{h2})/\sigma_{h2}, 0)$ *if the propensities to accumulate human capital fulfill the condition* $\sigma_{h1}/s < \sigma_{h2}/s < \varepsilon_2$.

Proof: Because of the properties of f, $f(\kappa) - ra$ and $f(\kappa) + \lambda pra$ are always positive and it follows:

$$\dot{w}_1 = 0 \Leftrightarrow w_1^* = \frac{s - \sigma_{h1}}{\sigma_{h1}} \text{ and } \dot{w}_2 = 0 \Leftrightarrow w_2^* = \frac{s - \sigma_{h2}}{\sigma_{h2}}. \tag{A.1}$$

Therefore, a sufficient condition for uniqueness of $(w_1^*, w_2^*, 0)$ is given by:

$$\begin{aligned}&\sigma_{h1}[f[\kappa(w_1^*, w_2^*, p)] + r(w_1^*, w_2^*, p)a(w_1^*, w_2^*, p) - \sigma_{h2}[f[\kappa(w_1^*, w_2^*, p)] \\ &\quad - \lambda pr(w_1^*, w_2^*, p)a(w_1^*, w_2^*, p) < 0\end{aligned} \tag{A.2}$$

for all $p \in [0, \infty]$. Considering equations (10) to (12) and (A.1), it follows that (A.2) is equivalent to:

$$\frac{f[\kappa(w_1^*, w_2^*, p)]}{f'[\kappa(w_1^*, w_2^*, p)]} > 1 + \kappa(w_1^*, w_2^*, p) \tag{A.3}$$

for $\sigma_{h1} < \sigma_{h2}$. Considering (12), it follows: $\partial\kappa(w_1^*, w_1^*, p)/\partial p > 0$. Then, both sides of inequality (A.3) are strictly increasing in p. Because of the properties of f, the left hand side increases faster in p than the right hand side. Therefore, inequality (A.3) must hold, if it holds for $p = 0$. It is easy to show that for $p = 0$ inequality (A.3) holds if:

$$\frac{\sigma_{h2}}{s} < \varepsilon_2. \tag{A.4}$$

To show global stability of (w_1^*, w_1^*, p^*) the differential equation (15) will be written as:

$$\dot{p} = pf'\Bigg[\underbrace{\left(\sigma_{h1} - \sigma_{h2}\right)}_{<0}\underbrace{\left(\frac{f[\kappa(w_1, w_2, p)]}{f'[\kappa(w_1, w_2, p)]} - [1 + \kappa(w_1, w_2, p)]\right)}_{\substack{>0 \\ \text{for } w_1 \to w_1^* \\ \text{and } w_2 \to w_2^* \\ \text{and } \sigma_{h1}/s < \sigma_{h2}/s < \varepsilon_2}}$$

$$+ \underbrace{\sigma_{h1}(w_1 - w_1^*)}_{\substack{=0 \\ \text{for } w_1 \to w_1^*}} - \underbrace{\sigma_{h2}(w_2 - w_2^*)}_{\substack{=0 \\ w_2 \to w_2^*}}\Bigg].$$

and global stability follows, since w_1 and w_2 converge to its equilibrium values independently of p, and $\dot{p}$ becomes negative in course of time. ■

Proposition A2: *The system (13) to (15) contains a saddle point* (w_1^*, w_2^*, p^*) $= ((s - \sigma_{h1})/\sigma_{h1}, (s - \sigma_{h2})/\sigma_{h2}, 0)$, *if the propensities to accumulate human capital fulfill the condition* $\sigma_{h1} < \sigma_{h2}$ *and* $\sigma_{h2}/s > \varepsilon_2$.

Proof: Linear approximation of system (13) to (15) in the neighborhood of the equilibrium (w_1^*, w_2^*, p^*) yields the following Jacobian:

$$\begin{pmatrix} \frac{\partial \phi_1}{\partial w_1}\Big|_{(w_1^*, w_2^*, p^*)} & 0 & 0 \\ 0 & \frac{\partial \phi_2}{\partial w_2}\Big|_{(w_1^*, w_2^*, p^*)} & 0 \\ 0 & 0 & \frac{\partial \phi_p}{\partial p}\Big|_{(w_1^*, w_2^*, p^*)} \end{pmatrix} \tag{A.6}$$

where

$$\frac{\partial \phi_1}{w_1}\Big|_{(w_1^*, w_2^*, p^*)} = -\sigma_{h1}[f(w_2^*) + f'(w_2^*)(w_1^* - w_2^*)] < 0,$$

$$\frac{\partial \phi_2}{w_2}\Big|_{(w_1^*, w_2^*, p^*)} = -\sigma_{h2} f(w_2^*) < 0,$$

$$\frac{\partial \phi_p}{\partial p}\Big|_{(w_1^*, w_2^*, p^*)} = \sigma_{h1}[f(w_2^*) + f'(w_2^*)(w_1^* - w_2^*)] - \sigma_{h2} f(w_2^*) > 0, \quad \text{if} \quad \sigma_{h1} < \sigma_{h2} \quad \text{and} \quad \sigma_{h2}/s > \varepsilon_2.$$

All the partial derivatives are evaluated at the steady state (w_1^*, w_2^*, p^*). It is obvious that the eigenvalues of the system have different signs. □

Proposition A3: *The system (13) to (15) contains a locally stable equilibrium $(w_1^*, w_2^*, p^{**}) = ((s - \sigma_{h1})/\sigma_{h1}, (s - \sigma_{h2})/\sigma_{h2}, p^{**}(\sigma_{h2}, \sigma_{h2}))$, if the propensities to accumulate human capital fulfill the condition $\sigma_{h1}/s < \varepsilon_1$ and $\sigma_{h2}/s > \varepsilon_2$.*

Proof: For $w_1 = w_1^*$ and $w_2 = w_2^*$ the differential equation (15) contains another equilibrium besides p^*, if:

$$\frac{f[\kappa(w_1^*, w_2^*, p)]}{f'[\kappa(w_1^*, w_2^*, p)]} = 1 + \kappa(w_1^*, w_2^*, p). \tag{A.7}$$

Assume p^{**} is a realization of p so that (A.7) holds, then, considering (12), it follows:

$$p^{**} = -\frac{f[\kappa(w_1^*, w_2^*, p^{**})] - f'[\kappa(w_1^*, w_2^*, p)](1 + w_2^*)}{\lambda[f[\kappa(w_1^*, w_2^*, p^{**})] - f'[\kappa(w_1^*, w_2^*, p^{**})](1 + w_1^*)]}. \tag{A.8}$$

Define the following function:

$$\varphi(p) = -\frac{z(p)}{n(p)}, \tag{A.9}$$

where

$$z(p) = f[\kappa(w_1^*, w_2^*, p)] - f'[\kappa(w_1^*, w_2^*, p)](1 + w_2^*),$$

$$n(p) = \lambda[f[\kappa(w_1^*, w_2^*, p)] - f'[\kappa(w_1^*, w_2^*, p)](1 + w_1^*)]$$

are both at least once continuously differentiable for all $p \in [0,\infty)$. If $\varphi(p)$ contains a fixed point, at least one p^{**} exists. The following inequalities hold:

i) $z(0) < 0$ and $n(0) < 0$, so that $\varphi(0) < 0$, if $\sigma_{h2}/s > \varepsilon_2$,

ii) $n > 0$ for $p \to \infty$, if $\sigma_{h1}/s < \varepsilon_1$,

iii) $z' > 0$ and $n' > 0$, for all $p \in [0, \infty)$.

Thus, there exists a $p = \bar{p}$ with $n(\bar{p}) = 0$. Because of $w_2^* < w_1^*$, $z(\bar{p}) > n(\bar{p})=0$ and it follows $\lim_{p\to\bar{p}^-} \varphi(p) = \infty$. Since $\varphi(0) < 0$, at least one p^{**} with $0 < p^{**} < \bar{p}$ exists, so that $p^{**} = \varphi(p^{**})$.

To proof uniqueness of p^{**} it suffices to show that sign $[d(p - \varphi(p))/dp] < 0$ for all $p = p^{**}$. Considering (12), (A.7) and (A.8) it follows:

$$\left.\frac{d(p - \varphi(p))}{dp}\right|_{p = p^{**}}$$

$$= \frac{f[\kappa(w_1^*, w_2^*, p^{**})] f''[\kappa(w_1^*, w_2^*, p^{**})](w_1^* - w_2^*)^2}{(1 + \lambda p^{**})^2 [f[\kappa(w_1^*, w_2^*, p^{**})] - f'[\kappa(w_1^*, w_2^*, p^{**})](1 + w_1^*)]^2} < 0. \tag{A.10}$$

The Jacobian of the system in the neighborhood of (w_1^*, w_2^*, w^{**}) is given by:

$$\begin{pmatrix} \left.\frac{\partial \phi_1}{\partial w_1}\right|_{(w_1^*, w_2^*, p^{**})} & 0 & 0 \\ 0 & \left.\frac{\partial \phi_2}{\partial w_2}\right|_{(w_1^*, w_2^*, p^{**})} & 0 \\ \left.\frac{\partial \phi_p}{\partial w_1}\right|_{(w_1^*, w_2^*, p^{**})} & \left.\frac{\partial \phi_p}{\partial w_2}\right|_{(w_1^*, w_2^*, p^{**})} & \left.\frac{\partial \phi_p}{\partial p}\right|_{(w_1^*, w_2^*, p^{**})} \end{pmatrix}. \tag{A.11}$$

Since only the elements in the lower triangle of the Jacobian are different from zero, $\partial\phi_p/\partial w_1$ and $\partial\phi_p/\partial w_2$ do not influence the sign of the real parts of the eigenvalues. Because

$$\left.\frac{\partial\phi_1}{\partial w_1}\right|_{(w_1^*,w_2^*,p^{**})} = -\sigma_{h1}[f[\kappa(w_1^*,w_2^*,p^{**})] + r(w_1^*,w_2^*,p^{**})a(w_1^*,w_2^*,p^{**})] < 0,$$

$$\left.\frac{\partial\phi_2}{\partial w_2}\right|_{(w_1^*,w_2^*,p^{**})} = -\sigma_{h2}[f[\kappa(w_1^*,w_2^*,p^{**})] - \lambda p^{**} r(w_1^*,w_2^*,p^{**})a(w_1^*,w_2^*,p^{**})] < 0,$$

$$\left.\frac{\partial\phi_p}{\partial p}\right|_{(w_1^*,w_2^*,p^{**})} = \frac{\lambda p^{**}(\sigma_{h1}+\lambda p^{**}\sigma_{h2})(w_1^*-w_2^*)^2}{(1+\lambda p^{**})^3} f''[\kappa(w_1^*,w_2^*,p^{**})] < 0,$$

the Routh-Hurwitz conditions for local stability are satisfied. All three eigenvalues of the Jacobian have negative real parts. □

Appendix B
Proof of Proposition 1

The LGRs of country 1 in autarky, g_1^{aut}, and in regime I under free capital mobility, g_1^I, respectively, are given by:

$$g_1^{aut} = \sigma_{h1} f\left(\frac{s-\sigma_{h1}}{\sigma_{h1}}\right), \tag{B.1}$$

$$g_1^I = \sigma_{h1}\left[f\left(\frac{s-\sigma_{h2}}{\sigma_{h2}}\right) + f'\left(\frac{s-\sigma_{h2}}{\sigma_{h2}}\right)\left(\frac{s-\sigma_{h1}}{\sigma_{h1}} - \frac{s-\sigma_{h2}}{\sigma_{h2}}\right)\right]. \tag{B.2}$$

(B.1) and (B.2) include:

i) $\lim_{\sigma_{h1}\to 0} g_1^{aut} = sf'[(s-\sigma_{h1})/\sigma_{h1}] < \lim_{\sigma_{h1}\to 0} g_1^I = sf'[(s-\sigma_{h2})/\sigma_{h2}]$, since $\sigma_{h1} < \sigma_{h2}$ and $f'' < 0$.

ii) $\lim_{\sigma_{h1} \to \sigma_{h2}} g_1^{aut} = g_2^I$,

iii) $\lim_{\sigma_{h1} \to \sigma_{h2}} g_1^I = g_2^I$,

iv) $dg_1^I / d\sigma_{h1} > 0$ for all $\sigma_{h1} \in (0, \sigma_{h2})$, if $\sigma_{h2} / s < \varepsilon_2$.

From iii) and iv) it follows $g_1^I < g_2^I$ for all $\sigma_{h1} \in (0, \sigma_{h2})$, if $\sigma_{h2}/s < \varepsilon_2$. Further, from i) to iv) it follows $g_1^I > g_1^{aut}$ for all $\sigma_{h1} \in (0, \sigma_{h2})$, if $\mathrm{sign}[d(g_1^I - g_1^{aut})/d\sigma_{h1}]$ is unique. Considering (B.1) and (B.2) it follows:

$$d(g_1^I - g_1^{aut}) / d\sigma_{h1} < 0$$
$$\Leftrightarrow f(w_2^*) - f'(w_2^*)w_2^* - f'(w_2^*) < f(w_1^*) - f'(w_1^*)w_1^* - f'(w_1^*).$$

The left hand side represents the difference between the return of labor in efficiency units and physical capital in the autarky steady state in country 2. The right hand side represents the respective difference in autarky in country 1. This inequality must hold for every $\sigma_{h1} < \sigma_{h2}$. □