Perfect Capital Mobility and Optimal Monetary Policy^{**}

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Abstract

This paper analyses the effects of perfect capital mobility on the choice of optimal monetary policy rules in a two country game-theoretic setting. The set of Nash equilibrium monetary policies is identified from the set of attainable outcomes. An optimal policy requires the central bank to manage both the exchange rate and, more vigorously, the interest rate. Management of both variables is more vigorous than would occur under an insulating monetary policy. Pursuing an optimal policy yields the unique Pareto optimal outcome as a Nash equilibrium. Explicit policy coordination is not necessary.

I . Introduction

Ever expanding opportunities for international trade and increasing worldwide financial integration appear to pose problems for the management of monetary policy. Increasing interdependence in commodity markets widens the direct channel through which foreign price disturbances are transmitted to domestic markets. At the same time, and perhaps more importantly, small disturbances in capital markets generate huge transfers in financial assets between countries. How foreign monetary authorities react to these disturbances directly alters the way by which the effects are transmitted to the domestic economy. Managing the domestic money supply, therefore, requires consideration of both how foreign disturbances exert influence on the domestic economy

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and how foreign policy makers react to the same.

With these issues in mind, Buiter and Eaton (1985) analyze optimal monetary policy in a two country world where goods markets and money markets are interdependent, but where there are no capital flows. Their analysis is explicitly game theoretic in view of the general recognition among policy makers of market interdependence. They find that optimal monetary policy requires some form of exchange rate management. The degree of intervention depends on the underlying structural parameters of the model and on the variability of the disturbances. In addition, Buiter and Eaton show that if central banks pursue just one policy target, say output stability, then with one instrument, the money supply, the Nash equilibrium is also the unique Pareto optimal outcome. Explicit coordination of monetary policies in this setting is not necessary.

Along different lines, Carlozzi and Taylor (1985) consider monetary policy in a world with perfect capital mobility and flexible exchange rates. Their analysis is motivated in part by the conjecture [Tobin (1978)] that capital mobility will hinder the effective performance of macroeconomic policies. In this context perfect capital mobility, per se, renders Nash equilibria suboptimal. Or, in the Buiter and Eaton framework, perfect capital mobility implies that, without explicit coordination of monetary policies, the Pareto optimal outcome is not achievable, even in the case where monetary authorities are pursuing just one policy target. Carlozzi and Taylor conclude that this is not necessarily true if exchange rates are flexible. However, their analysis does not consider the strategic elements of the problem and the monetary policies considered are not necessarily optimal. Indeed, the nature of their analysis, resting primarily on simulating certain type of policy, restricts the generality of their results.

This paper generalizes the Carlozzi and Taylor and Buiter and Eaton results, extending the analysis of optimal monetary policies to a two country world with perfect capital mobility. The objectives are, first, to characterize optimal money policies in an explicitly game theoretic setting. Second, to examine the conjecture of Tobin in an analytic framework, describing fully the optimality or otherwise of monetary policies in a framework with perfect capital mobility.

A monetary policy is interpreted as a choice of the degree to which the Fed manages the exchange rate and/or the interest rate around target values, subject to restrictions described in the model. One can think of an exchange rate policy as the choice of the degree to which the domestic monetary authority allows disturbances to be transmitted directly to domestic commodity markets by commodity arbitrage. Loosely speaking, an interest rate policy is a choice of just how much to insulate the domestic

capital market from disturbances in foreign capital markets, at some chosen nominal interest rate, while allowing perfect capital mobility.

Conversely, the degree to which both the exchange rate and interest rate are managed dictates, in part, the extent to which domestic disturbances are transmitted abroad. The domestic monetary authority has the ability to export domestic disturbances back through the commodity and capital markets by fixing either or both the exchange rate and the interest rate. The foreign monetary authority, of course, is similarly motivated.

To capture these aspects of monetary policy management, the model in this paper blends characteristics from both the Buiter and Eaton and the Carozzi and Taylor approaches. These models are themselves generalizations of approaches commonly found in the analysis of optimal monetary policy and exchange rate management in small open economies [Turnovsky (1984) is a representative example]. In this paper we model, along standard lines, the aggregate demand and aggregate supply sides of commodity markets in a two country world. Purchasing power parity provides the direct link between the two commodity markets. Financial integration is captured by the uncovered interest parity condition.

An optimal policy is a Nash equilibrium policy in the one shot game. As it turns out that there are many such Nash equilibria sharing the same qualitative properties, with one particular equilibrium being dominant. Since the properties of these equilibria are similar in nature to equilibria in Buiter and Eaton, it appears that the characterization of optimal monetary policy is quite robust with respect to different model specifications. The main result in the paper is that, in the presence of perfect capital mobility, the Pareto optimal outcome is achievable as the outcome of a noncooperative game only if the central bank manages the domestic interest rate as well as, and to a greater extent than, the exchange rate. Intuitively, while the central bank pursues just one target, output stability, with just one instrument, the money supply, there are two independent transmission channels for foreign disturbances. The central bank must independently use both components of the money supply, domestic credit and foreign reserves, to target each source of foreign disturbance. If the monetary authority is unwilling or unable to stabilize both the interest rate and the exchange rate then all noncooperative outcomes are Pareto suboptimal. Perfect capital mobility, therefore, need not hinder macroeconomic policy performance as monetary policy, properly chosen, can attain the best possible outcome without explicit policy coordination.

The paper proceeds as follows. Section 2 specifies the model. The solution, for given

monetary policies is given in section 3. Feasibility restrictions are discussed next. In section 5 insulating policies, which have been discussed extensively in the context of small country models, are analyzed. The payoff set and Nash noncooperative policies are derived in section 6. Brief conclusions follow.

II . The Model

Consider two countries that produce and consume the same perfectly substitutable fixed basket of goods. Short run production occurs with a variable labor input and a fixed capital input in each country. We suppose that the deviation of actual output from its natural level is proportional to the unanticipated increase in the price level:

$$Y_t^s = Y_n + \theta(p_t - E_{t-1}p_t) + \epsilon_t \quad (1)$$

$$Y_t^{s*} = Y_n^* + \theta^*(p_t^* - E_{t-1}p_t^*) + \epsilon_t^* \quad (2)$$

Y_t^s and Y_t^{s*} are the logs of output in each country. p_t and p_t^* are the logs of the price level. ϵ_t^* are gaussian white noise processes with zero mean and variance σ_ϵ^2 and $\sigma_{\epsilon^*}^2$, respectively. $E_t(\cdot)$ denotes the expectation of the variable $(\cdot)$ conditioned on time t information.

As in Barro (1976, 1980), commodity demand in each country is assumed to depend on the real interest rate. For simplicity we abstract from real balance effects. If the two countries are of equal size and relative price elasticities in each country are equal (for convenience set equal to one) then the worldwide commodity market clearing condition can be written

$$Y_t^s + Y_t^{s*} = A_n - r_t - r_t^*. \quad (3)$$

A_n is the natural level of world consumption. The real interest rate is defined by the Fisher equation:

$$i_t = r_t + (E_t p_{t+1} - p_t) \quad (4)$$

$$i_t^* = r_t^* + (E_t p_{t+1}^* - p_t^*). \quad (5)$$

In the absence on transactions costs and other impediments to trade commodity arbitrage ensures that purchasing power parity (PPP) always holds:

$$p_t = p_t^* + e_t. \quad (6)$$

e_t is the log of the nominal exchange rate, or the domestic price of foreign currency. In light of recent experience, this assumption is somewhat restrictive and is maintained here to highlight the role played by commodity markets in transmitting real disturbances between countries.

The demand for money in each country is positively related to domestic prices and real income, and negatively related to the nominal interest rate:

$$M_t^d = p_t + Y_t - \beta i_t \quad (7)$$

$$M_t^{d*} = p_t^* + Y_t^* - \beta^* i_t^* \quad (8)$$

There is no currency substitution as in Buiter and Eaton. In contrast to their model, however, there is one kind of perfectly mobile bond so that uncovered interest parity (UIP) always holds:

$$i_t = i_t^* + (E_t e_{t+1} - e_t). \quad (9)$$

An immediate, and convenient, implication of this framework is that real interest rates are equalized, $r_t = r_t^*$. This can be verified by subtracting (5) from (4) and solving for $r_t - r_t^* = (i_t - i_t^*) - (E_t p_{t+1} - E_t p_{t+1}^*) + (p_t - p_t^*)$.

UIP and PPP imply that the RHS is identically zero.

It remains to specify the money supply process. Abstracting from a commercial banking sector, the money stock in each country is equal to its money base which is composed of holdings of domestic credit and international reserves. Each central bank may want to manage either or both the domestic interest and the exchange rate. A deviation of the domestic interest rate from its target level induces an offsetting intervention, open market operation, in the domestic credit market. Similarly, a deviation of the exchange rate from its target level induces an intervention in the foreign exchange market. Both these activities directly effect the domestic monetary base. Thus the monetary reaction functions are given by:

$$M_t = \rho_t + m_e(\bar{e}_t - e_t) + m_i(i_t - \bar{i}_t) + \mu_t \quad (10)$$

$$M_t^* = \rho^* t - m_e^*(\bar{e}_t^* - e_t) + m_i^*(i_t^* - \bar{i}_t^*) + \mu_t^* \quad (11)$$

$\bar{e}_t$ is the domestic target exchange rate; $\bar{i}_t$ the target interest rate; and ρ the systematic rate of growth of the domestic money supply. μ_t is a gaussian white noise process with variance σ_μ^2 . The foreign variables are similarly defined. The parameters m_e and m_i are

the domestic bank's policy parameters, determining the degree to which the bank supports a particular exchange rate or interest rate. (10) and (11) are generalizations of the money supply rules in both Buiter and Eaton and Carlozzi and Taylor. Here the central banks are able to contemporaneously observe and track the two indicators or prices relevant for international arbitrage operations.

Let the domestic target exchange rate be

$$\bar{e}_t = e_0 + \eta t, \quad (12)$$

where η is a chosen rate of devaluation. η nonzero corresponds to a crawling peg or sliding parity exchange regime. Similarly for the foreign country we have

$$\bar{e}_t^* = e_0^* + \eta^* t. \quad (13)$$

In choosing a fixed target for the nominal interest we require that the target be achievable by the appropriate selection of m_t . This is discussed in greater detail below.*

III . The Solution

The strategy is to solve for p_t and p_t^* given m_e , m_i , m_e^* and m_i^* . The solution method is the familiar method of undetermined coefficients. Using the fact that $r_t = r_t^*$, substitute (1) and (2) into (3) and solve for

$$r_t = r_n - \theta(p_t - E_{t-1} p_t) - \theta^*(p_t^* - E_{t-1} p_t^*) - (\epsilon_t + \epsilon_t^*), \quad (14)$$

where $r_n = A_n - Y_n - Y_n^*$ is the natural real interest rate that ex ante clears the goods market. Substitute (4) – (6), (9) and (14) into each of the money market clearing conditions

$$M_t^d = M_t^s \quad (15)$$

and

$$M_t^{d*} = M_t^{s*} \quad (16)$$

to derive the quasi-reduced form solutions:

$$\bar{R}P_t = R_0 + R_1 t + R_2 T + R_3 E_{t-1} P_t + R_4 V_t. \quad (17)$$

$P_t = (p_t \ p_t^*)'$ is the vector of endogenous prices, $T = (e_0 \ \bar{i} \ e_0^* \ \bar{i}^*)'$ is the vector of targets values and $V_t = (\epsilon_t \ \mu_t \ \epsilon_t^* \ \mu_t^*)'$ is the vector of disturbances. The elements of the matrices $\bar{R}$ and R_t are nonlinear functions of the underlying structural parameters and are defined in the Appendix. Guessing a solution

$$P_t = K_0 + K_1 t + K_2 T + K_3 V_t, \quad (18)$$

it is a simple matter to solve for the K_i :

$$K_0 = (\bar{R} - R_3)^{-1} R_0, \quad K_1 = (\bar{R} - R_3)^{-1} R_1, \quad K_2 = (\bar{R} - R_3)^{-1} R_2, \quad K_3 = \bar{R}^{-1} R_4.$$

Defining $\pi = E_{t-1}(p_{t+1} - p_t)$ to be the systematic or fully anticipated rate of inflation, then the K_i in terms of the policy parameters of the model are given by:

$$\begin{aligned} K_0 &= \begin{pmatrix} \Pi_1^* [(\beta + m_i)(r_n + \pi) - Y_n] + (1 - \Pi_1^*)[(\beta^* + m_i^*)(r_n + \pi^*) - Y_n^*] \\ (1 - \Pi_1)[(\beta + m_i)(r_n + \pi) - Y_n] + \Pi_1[(\beta^* + m_i^*)(r_n + \pi^*) - Y_n^*] \end{pmatrix} \\ K_1 &= \begin{pmatrix} \Pi_1^* (\rho + m_e \eta) + (1 - \Pi_1^*) (\rho^* - m_e^* \eta^*) \\ (1 - \Pi_1) (\rho + m_e \eta) + \Pi_1 (\rho^* - m_e^* \eta^*) \end{pmatrix} \\ K_2 &= \begin{pmatrix} \Pi_1^* m_e & -\Pi_1^* m_i & -(1 - \Pi_1^*) m_e^* & -(1 - \Pi_1^*) m_i^* \\ (1 - \Pi_1) m_e & -(1 - \Pi_1) m_i & -\Pi_1 m_e^* & -\Pi_1 m_i^* \end{pmatrix} \\ K_3 &= \Pi_4 \begin{pmatrix} -\Pi_2^* (1 + \beta + m_i) - \Pi_3 (\beta^* + m_i^*) & \Pi_2^* & -\Pi_2^* (\beta + m_i) - \Pi_3 (1 + \beta^* + m_i^*) & \Pi_3 \\ -\Pi_3^* (1 + \beta + m_i) - \Pi_2 (\beta^* + m_i^*) & \Pi_3^* & -\Pi_3 (\beta + m_i) - \Pi_2 (1 + \beta^* + m_i^*) & \Pi_2 \end{pmatrix} \end{aligned}$$

where

$$\begin{aligned} \Pi_1 &= (1 + m_e) / (1 + m_e + m_e^*) & \Pi_1^* &= (1 + m_e^*) / (1 + m_e + m_e^*) \\ \Pi_2 &= (1 + \theta)(1 + \beta + m_i) + m_e & \Pi_2^* &= (1 + \theta^*)(1 + \beta^* + m_i^*) + m_e^* \\ \Pi_3 &= m_e - \theta^*(\beta + m_i) & \Pi_3^* &= m_e^* - \theta(\beta^* + m_i^*) \\ \Pi_4 &= (\Pi_2 \Pi_2^* - \Pi_3 \Pi_3^*)^{-1}. \end{aligned}$$

The interdependence of policy choices is clear from this solution. Notice that the pairs (Π_2, Π_3) and (Π_2^*, Π_3^*) are linear transformations of the two policy pairs (m_e, m_i) and (m_e^*, m_i^*) . This observation facilitates the derivation of equilibrium policies below.

IV . Feasibility Restrictions

Before considering optimal and equilibrium policies we need to verify the feasibility of

certain policies. Of concern is the existence of well defined price levels, interest rates and exchange rates when either or both central banks attempt to fix things in any arbitrary way. In particular, a policy combination with both central banks fixing both the exchange rate and/or the interest rate may cause problems.

From (18) we have that the systematic rates of inflation are constant for all t :

$$\begin{aligned}\pi &= \Pi_1^* (\rho + m_e \eta) + (1 - \Pi_1^*) (\rho^* - m_e^* \eta^*) \\ \pi^* &= (1 - \Pi_1) (\rho + m_e \eta) + \Pi_1 (\rho^* - m_e^* \eta^*).\end{aligned}$$

From the definition of Π_1 and Π_1^* it is clear that both π and π^* have finite limits if either m_e or m_e^* approach infinity. Solving for the domestic interest rate,

$$i_t = (r_n + \pi) + \alpha_1 \epsilon_t + \alpha_2 \mu_t + \alpha_3 \epsilon_t^* + \alpha_4 \mu_t^*, \quad (19)$$

where

$$\begin{aligned}\alpha_1 &= -[(1 + \theta)K_3^{11} + \theta^* K_3^{21} + 1] & \alpha_2 &= -[(1 + \theta)K_3^{12} + \theta^* K_3^{22}] \\ \alpha_3 &= -[(1 + \theta)K_3^{13} + \theta^* K_3^{23} + 1] & \alpha_4 &= -[(1 + \theta)K_3^{14} + \theta^* K_3^{24}].\end{aligned}$$

The K_3^{ij} are elements of the matrix K_3 . From (19)

$$\lim_{m_i \rightarrow \infty} i_t = r_n + \pi.$$

If $\bar{i}_t = r_n + \pi$ then, since r_n is exogenous to the actions of the central bank, any targeted nominal interest rate is achievable with the appropriate choice of π . Given the behavior of the other central bank (m_e^* constant and less than ∞) any π can be attained by the appropriate choice of ρ or η . Thus a policy combination of $(m_e \rightarrow \infty, m_i \rightarrow \infty)$ is feasible so long as m_e^* is finite.

Consider the implications of both central banks fixing both the exchange rate and the interest rate. Since $e_t = p_t - p_t^*$, we have from (18), dropping the disturbance terms,

$$\begin{aligned}e_t &= \Pi_5 [(\beta + m_i)(r_n + \pi) - (\beta^* + m_i^*)(r_n + \pi^*) - (Y_n - Y_n^*) + (\rho - \rho^*) \\ &\quad + m_e(\eta + e_0) + m_e^*(\eta^* + e_0^*) - m_i \bar{i} + m_i^* \bar{i}^*],\end{aligned}$$

where $\Pi_5 = (1 + m_e + m_e^*)^{-1}$. Let $m_e = m_i = m_e^* = m_i^* = m$. Then

$$\lim_{m \rightarrow \infty} e_t = \bar{e} = (1/2)[\pi + \pi^* + \eta + \eta^* + e_0 - \bar{i} + e_0^* - \bar{i}^*].$$

And

$$\lim_{m \rightarrow \infty} p_t = (1/2)[\beta(r_n + \pi) + \beta^*(r_n + \pi^*) - (Y_n + Y_n^*) + (\rho + \rho^*)],$$

if

$$e_0 = e_0^*$$

$$\eta = \eta^* = 0$$

$$\bar{i} = \bar{i}^* = r_n + \hat{\pi}$$

where

$$\hat{\pi} = (1/2)(\rho + \rho^*).$$

With these restrictions $\bar{e} = e_0 = e_0^*$. In other words, each central bank cannot independently fix both its nominal interest rate and the exchange rate. In order to do so they must coordinate their policies and select a common exchange rate and a common nominal interest rate.

V . Insulating Policies

Policies that have received much attention are so-called insulating policies: forms of exchange rate and interest rate management that eliminate the international transmission of disturbances [see Saidi (1982), Turnovsky (1979, 1984), Flood and Marion (1982), amongst many others]. The insulating properties of different policies became important prior to and following the general move to floating exchange rates as such regimes were presumed to provide more insulation from foreign inflationary pressures and to allow domestic monetary authorities free pursuit of independent monetary policies. For the small open economy framework, a large literature has developed with the general conclusion that flexible exchange rates (an insulating policy) seems to be preferable when the major source of disturbances is foreign and fixed rates are preferable when the disturbances are domestically sourced. These results can be modified when distinctions between nominal and real disturbances and their respective transmission channels are made. A good discussion of these results is found in Turnovsky (1984).

In this model there are two channels of transmission of disturbances: the commodity market via PPP and the bond market via UIP. It should not be surprising then that,

with an indicator as to each source of disturbance, the central banks can effectively isolate the domestic economy from the rest of the world. This is achieved by floating the exchange rate ($m_e=0$) and managing the interest rate ($m_i=-\beta$). This insulating policy requires the central bank to increase the money supply by an amount equal to the increase in the demand for money whenever the interest rate falls below its target level. This is a policy of leaning with the wind for interest rates and may appropriately be called an accommodative policy. Significantly, the foreign central bank can independently pursue the same policy so that mutual complete insulation is a jointly feasible policy.

Given that complete insulation from international disturbances is possible, one may ask is it desirable under any circumstances, as in Turnovsky (1985), for example. This question is answered in the next section where we derive closed form solutions for equilibrium policies for the two country setting.

VI . Nash Equilibrium Policies

The economy described so far is one with two large agents, possibly with different objectives, independently choosing policy responses that impinge upon each other. An appropriate equilibrium concept to consider is the one shot Nash noncooperative equilibrium, where the action of the two players are the money supply policy parameters: $\{\rho, \eta, m_e, m_i\}$ and $\{\rho^*, \eta^*, m_e^*, m_i^*\}$. The corresponding strategy set for each player is the Euclidean space R^4 . The question of payoff functions is a difficult one. Ordinarily we would think of the central bank as being concerned with a range of characteristics of the economy. But in the interest of simplicity, and in accordance with the literature, we will assume that each central bank desires to minimize the variation of actual output around its ex ante expected value. Thus, the central bank is merely trying to minimize the effects of unanticipated disturbances on real income. In this framework, this is the same as minimizing the variance of the forecast error on output and in the absence of supply disturbances is the same as minimizing the variance of inflation. From (1) and (18) we have

$$Y_t = Y_n + \theta[(K_3^{11} + 1/\theta)\epsilon_t + K_3^{12}\mu_t + K_3^{13}\epsilon_t^* + K_3^{14}\mu_t^*]. \quad (20)$$

Given our distributional assumptions, the variance of Y_t is thus

$$\sigma_Y^2 = \theta^2[(K_3^{11} + 1/\theta)^2\sigma_\epsilon^2 + (K_3^{12})^2\sigma_\mu^2 + (K_3^{13})^2\sigma_{\epsilon^*}^2 + (K_3^{14})^2\sigma_{\mu^*}^2]. \quad (21)$$

A similar expression holds for the variance of foreign output,

$$\sigma_{Y^*}^2 = \theta^{*2} [(K_3^{21})^2 \sigma_\epsilon^2 + (K_3^{22})^2 \sigma_\mu^2 + (K_3^{23} + 1/\theta^*)^2 \sigma_{\epsilon^*}^2 + (K_3^{24})^2 \sigma_{\mu^*}^2]. \quad (22)$$

Since the K_3^{ij} are functions only of the pairs (m, m_i) and (m_e^*, m_i^*) , each central bank need only choose an interest rate and an exchange rate policy in order to minimize output variance. However, the K_3^{ij} are relatively complicated functions of the policy parameters and the requisite algebra becomes messy and uninformative very quickly. We can simplify the problem by exploiting the linear transformations implicit in the definitions of Π_2 , Π_3 , Π_2^* and Π_3^* . Making the appropriate substitutions yields the transformed payoff pairs:

$$\Sigma = -a + \frac{b\Pi_2^* - c\Pi_3^*}{\Pi_2\Pi_2^* - \Pi_3\Pi_3^*} - \frac{e\Pi_2^{*2} - d\Pi_3\Pi_2^* + f\Pi_3^2}{(\Pi_2\Pi_2^* - \Pi_3\Pi_3^*)^2} \quad (23)$$

and

$$\Sigma^* = -a^* + \frac{b^*\Pi_2 - c^*\Pi_3^*}{\Pi_2\Pi_2^* - \Pi_3\Pi_3^*} - \frac{e\Pi_3^{*2} - d\Pi_2\Pi_3^* + f\Pi_2^2}{(\Pi_2\Pi_2^* - \Pi_3\Pi_3^*)^2} \quad (24)$$

where $\Sigma = -(1/\theta)^2 \sigma_Y^2$ and $\Sigma^* = -(1/\theta^*)^2 \sigma_{Y^*}^2$. The coefficients are defined in terms of the underlying parameters, given in the Appendix.

The payoff possibility frontier in this game, the set of possible equilibrium outcomes, includes those outcomes on and below the boundary ABC in Figure 1. It was derived as follows. Consider choosing $(\Pi_2, \Pi_3, \Pi_2^*, \Pi_3^*)$ to maximize Σ . Solving the four first order conditions yields the following restrictions:

$$\Pi_3 = E\Pi_2^* \quad (25)$$

and

$$\Pi_2 = E\Pi_3^* + F, \quad (26)$$

where $E = (bd - 2ce)/(2fb - cd)$ and $F = (4fe - d^2)/(2fb - cd)$. Substituting back into Σ yields:

$$\Sigma = -a + (fb^2 - bcd + c^2e)/(4fe - d^2).$$

Thus the maximal value of Σ , which we denote Σ_0 , is independent of Π_2^* and Π_3^* . In terms of the underlying parameters

$$\Sigma_0^* = - \frac{\sigma_\mu^2 \{ (1+\theta^*)^2 \sigma_\epsilon^2 \sigma_\mu^2 + \theta^2 \sigma_\epsilon^2 \sigma_\mu^2 \sigma_\epsilon^2 \}}{\theta \{ \sigma_\mu^2 \sigma_\mu^2 + \sigma_\epsilon^2 [(1+\theta^*)^2 \sigma_\mu^2 + \theta^2 \sigma_\epsilon^2] \} + \sigma_\epsilon^2 \{ \theta^2 \sigma_\mu^2 + (1+\theta)^2 \sigma_\mu^2 \} + \sigma_\epsilon^2 \sigma_\epsilon^2} \quad (27)$$

Substituting (25) and (26) back into (24) however, indicates that Σ^* varies with the choices of Π_2^* and Π_3^* . This means that part of the payoff frontier is contained in the set of points $S = \{ \Sigma_0, \Sigma^* : \Sigma^* \in \mathbb{R} \}$.

Similarly, minimizing Σ^* and solving the first order conditions yields the restrictions:

$$\Pi_2^* = E^* \Pi_3 + F^* \quad (28)$$

and

$$\Pi_3^* = E^* \Pi_2, \quad (29)$$

where $E^* = (b^*d - 2fc)/(2eb^* - dc^*)$ and $F^* = (4fe - d^2)/(2eb^* - dc^*)$. Substituting (28) and (29) back into (24) yields

$$\Sigma^* = -a^* + (fc^{*2} - b^*c^*d + b^{*2})/(4fe - d^2).$$

The symmetrical nature of the problems reveals that the maximal value of noted Σ_0^* , is independent of Π_2 and Π_3 . In terms of the underlying parameters

$$\Sigma_0^* = - \frac{\sigma_\mu^2 \{ \theta^{*2} \sigma_\epsilon^2 \sigma_\mu^2 + (1+\theta)^2 \sigma_\epsilon^2 \sigma_\mu^2 + \sigma_\epsilon^2 \sigma_\epsilon^2 \}}{\theta^{*2} \{ \sigma_\mu^2 \sigma_\mu^2 + \sigma_\epsilon^2 [(1+\theta^*)^2 \sigma_\mu^2 + \theta^{*2} \sigma_\epsilon^2] \} + \sigma_\epsilon^2 \{ \theta^2 \sigma_\mu^2 + (1+\theta)^2 \sigma_\mu^2 \} + \sigma_\epsilon^2 \sigma_\epsilon^2} \quad (30)$$

Thus more of the payoff frontier is contained in $S^* = \{ \Sigma, \Sigma_0^* : \Sigma \in \mathbb{R} \}$.

The complete payoff frontier is the set $\{ S \cap S^* : \Sigma \leq \Sigma_0, \Sigma^* \leq \Sigma_0^* \}$. This is the locus ABC in Figure 1, where point B = (Σ_0, Σ_0^*) is the unique outcome that would be attained in a cooperative game. Notice that perfect output stabilization is possible for each country only if the variance of the unanticipated monetary disturbance in the other country is zero.

The derivation of the set of Nash equilibrium outcomes is mechanical and relegated to the Appendix. The upshot of the exercise is that, because there are many different possible sets of beliefs that could be held by each central bank as to the behavior of the other, there are many Nash equilibria. The complete set comprises the points {B, RS, TU, VWX} in Figure 1. However, not all these outcomes are equally plausible. Clearly, the behavior associated with point B dominates any other action. The corresponding

Figure 1

strategies that yield this equilibrium outcome are:

$$m_e = \frac{\theta^*(1+\theta^*)\sigma_\mu^2}{\sigma_\epsilon^2} \quad m_i = -\frac{[\theta^*(1+\theta^*)\sigma_\epsilon^2 + \theta^2\sigma_{\epsilon^*}^2]\sigma_\mu^2}{\sigma_\epsilon^2\sigma_{\epsilon^*}^2} - (1+\beta) \quad (31)$$

and

$$m_e^* = \frac{\theta(1+\theta)\sigma_\mu^2}{\sigma_\epsilon^2} \quad m_i^* = -\frac{[\theta^2\sigma_\epsilon^2 + \theta(1+\theta)\sigma_{\epsilon^*}^2]\sigma_\mu^2}{\sigma_\epsilon^2\sigma_{\epsilon^*}^2} - (1+\beta^*). \quad (32)$$

The payoffs associated with these actions are given in (27) and (30).

This dominating equilibrium should be easily achievable with minimal pre-play communication. As long as each country somehow indicates that it will play the strategy associated with this equilibrium the best response of the other country is to play that strategy also. Any announcement by a central bank to that effect is credible because B is the best attainable outcome for that central bank. However, if for some reason the two countries are unable to agree to play to B then, given prevailing beliefs, some of the other Nash equilibria might emerge. For what follows it does not matter, however, as all the Nash equilibria are qualitatively similar.

Note first that it is never an optimal policy to simultaneously float the exchange rate and the interest rate. A purely floating exchange rate is an optimal policy only in

the unlikely event that there are no domestic monetary disturbances. A purely floating interest rate is never an optimal policy. Thus, intervention will occur in both the foreign exchange and the domestic capital markets. At the other end of the scale, a decision by the domestic central bank to fix the exchange rate would only emerge if there were no foreign real disturbances. This enables domestic real disturbances to be directly exported to the foreign country. In the presence of foreign real disturbances, some exchange rate flexibility is required to buffer their effects.

From (31) and (32), $m_i = -m_e - (\theta^2 \sigma_\mu^2) / \sigma_\epsilon^2 - (1 + \beta)$ which implies $|m_i| > |m_e|$. Similarly $|m_i^*| > |m_e^*|$. Thus, a central bank following an optimal policy will "more vigorously" manage the interest rate than it will manage the exchange rate. It has long been observed, sometimes disparagingly, that central banks often give the impression of being preoccupied with managing interest rates. As this analysis shows, this merely reflects optimal behavior under general circumstances. Further, management of both the interest rate and exchange rate is more vigorous under the optimal policy than it would be under a policy designed to insulate the domestic economy from foreign disturbances, where the exchange rate is floated and only the interest rate is managed.

As in Buiter and Eaton, the variability of the foreign money supply has no effect on optimal domestic monetary policy. This follows because optimal foreign monetary policy requires the foreign central bank to minimize the effects of such disturbances, effectively allowing the domestic monetary to ignore them. Moreover, optimal monetary policy remains accommodative, both with respect to interest rate and exchange rate deviations from target values. The domestic money supply increases whenever the value of the domestic currency rises too much or the price of domestic assets falls too low. As already noted, monetary policy is more accommodative in equilibrium, both with respect to exchange rate and interest rate deviations, than would a purely insulation policy be.

An increase in the variability of foreign real disturbances reduces the degree of intervention in both the foreign exchange market and the capital market. In this instance, as foreign disturbances are transmitted through both commodity markets and bond markets, the exchange rate and the domestic interest rate are allowed to absorb more of the underlying variation.

VII. Conclusions

This paper employs a simple stylized model to demonstrate that perfect capital mobility need not create additional problems for optimal monetary management. In this

setting, optimal monetary policy not only requires the central bank to intervene in the foreign exchange market, as in Buitier and Eaton, but also to intervene (more heavily) in the domestic credit market. Optimally pursuing both a nominal interest rate target and an exchange rate target results in the best attainable outcome emerging as a Nash noncooperative equilibrium. This policy requires active management of both the exchange rate and the domestic nominal interest rate because foreign disturbances impinge on the domestic economy through both the commodity market, via PPP, and the capital market, via UIP. While the central bank has just one instrument, the money supply, to use in pursuit of its goal of output stability, that instrument has two components: domestic assets and foreign assets. Each component of the money supply is thus directed at one of the two channels through which foreign disturbances impact on the domestic economy.

It appears that the basic structure of optimal monetary policy in these frameworks is quite robust to different model specifications. Generally speaking, monetary policy should be accommodative, or "leaning with the wind". The degree of intervention depends on the underlying structural parameters and variances in quite straightforward ways. Optimal intervention is more active than intervention designed to insulate the domestic economy from all foreign shocks, both real and monetary.

Finally a remark concerning coordination and cooperation. We have shown that there exists a need for some minimal communication in the game described, even in the absence of explicit coordination of monetary policies. This communication relates to chosen exchange rate and interest rate targets. Following agreement on these, each central bank merely has to announce that it will (noncooperatively) play the action associated with the best possible Nash equilibrium (point B). Since there is no reason for it not to be believed the Pareto optimal outcome then emerges without the need for further coordination.

<Appendix>

Coefficients in equation (17):

$$\bar{R} = \begin{pmatrix} m_e + (1 + \theta)(1 + \theta + m_i) & \theta^*(\theta + m_i) - m_e \\ \theta(\beta^* + m_i^*) - m_e^* & m_e^* + (1 + \theta^*)(1 + \beta^* + m_i^*) \end{pmatrix}$$

$$R_0 = \begin{pmatrix} (\beta + m_i)(r_n + \pi) - Y_n \\ (\beta^* + m_i^*)(r_n + \pi^*) - Y_n^* \end{pmatrix}$$

$$R_1 = \begin{pmatrix} \rho + m_e \eta \\ \rho^* - m_e^* \eta^* \end{pmatrix}$$

$$R_2 = \begin{pmatrix} m_e & -m_i & 0 & 0 \\ 0 & 0 & -m_e^* & -m_i^* \end{pmatrix}$$

$$R_3 = \begin{pmatrix} \theta + (1 + \theta)(\beta + m_i) & \theta^*(\beta + m_i) \\ \theta(\beta^* + m_i^*) & \theta^* + (1 + \theta^*)(\beta^* + m_i^*) \end{pmatrix}$$

$$R_4 = \begin{pmatrix} -(1 + \beta + m_i) & 1 & -(\beta + m_i) & 0 \\ -(\beta^* + m_i) & 0 & -(1 + \beta^* + m_i^*) & 1 \end{pmatrix}$$

Coefficients in the payoff functions :

$$a = [(1 + \theta^*)^2 \sigma_\epsilon^2 + \theta^2 \sigma_{\epsilon^*}^2] / \theta^2 (1 + \theta + \theta^*)^2$$

$$b = 2[\theta^*(1 + \theta^*) \sigma_\epsilon^2 + \theta(1 + \theta) \sigma_{\epsilon^*}^2] / \theta(1 + \theta + \theta^*)^2$$

$$c = 2[(1 + \theta^*)^2 \sigma_\epsilon^2 + \theta^2 \sigma_{\epsilon^*}^2] / \theta(1 + \theta + \theta^*)^2$$

$$d = 2[\theta^*(1 + \theta^*) \sigma_\epsilon^2 + \theta(1 + \theta) \sigma_{\epsilon^*}^2] / (1 + \theta + \theta^*)^2$$

$$e = [\theta^{2*} \sigma_\epsilon^2 + (1 + \theta^*)^2 \sigma_{\epsilon^*}^2 + (1 + \theta + \theta^*)^2 \sigma_\mu^2] / (1 + \theta + \theta^*)^2$$

$$f = [(1 + \theta^*)^2 \sigma_\epsilon^2 + \theta^2 \sigma_{\epsilon^*}^2 + (1 + \theta + \theta^*)^2 \sigma_\mu^2] / (1 + \theta + \theta^*)^2$$

$$a^* = [\theta^{*2} \sigma_\epsilon^2 + (1 + \theta)^2 \sigma_{\epsilon^*}^2] / \theta^{*2} (1 + \theta + \theta^*)^2$$

$$b^* = 2[\theta^*(1 + \theta^*) \sigma_\epsilon^2 + \theta(1 + \theta) \sigma_{\epsilon^*}^2] / \theta^*(1 + \theta + \theta^*)^2$$

$$c^* = 2[\theta^{*2} \sigma_\epsilon^2 + (1 + \theta)^2 \sigma_{\epsilon^*}^2] / \theta^*(1 + \theta + \theta^*)^2$$

Derivation of the set of Nash equilibria.

The equilibrium set is easiest derived by considering a number of possible cases.

Case 1. $\Pi_2 = \Pi_2^* = 0$.

In this case payoff functions simplify to

$$\Sigma = -a + \frac{c}{\Pi_3^*} - \frac{f}{\Pi_3^{*2}} \quad (A1)$$

and

$$\Sigma^* = -a^* + \frac{c^*}{\Pi_3} - \frac{e}{\Pi_3^2}. \quad (A2)$$

The payoff to each country depends only on the action of the other country and not on its own actions. Thus, for $\Pi_2^* = 0$, any (Π_2, Π_3) is a best reply. Similarly, for $\Pi_2 = 0$, any (Π_2^*, Π_3^*) is a best reply. This yields large set of Nash equilibria.

$$\{(\bar{\Pi}_2, \bar{\Pi}_3), (\bar{\Pi}_2^*, \bar{\Pi}_3^*)\} = \{(0, \Pi_3), (0, \Pi_3^*) : \Pi_3 \in \mathbb{R}, \Pi_3^* \in \mathbb{R}\}.$$

This set of equilibria corresponds to the points UVW in Figure 1, where V is the dominant equilibrium within this class. Point V found by choosing Π_3^* to maximize Σ , and choosing Π_3 to maximize Σ^* . This yields equilibrium strategies:

$$m_e = \frac{\theta^*(1+\theta)(1+\theta+\theta^*)\sigma_\mu^2}{\theta^2\sigma_\epsilon^2 + (1+\theta)^2\sigma_{\epsilon^*}^2} \quad m_i = -\frac{\theta^*(1+\theta+\theta^*)\sigma_\mu^2}{\theta^2\sigma_\epsilon^2 + (1+\theta)^2\sigma_{\epsilon^*}^2} - (1+\beta)$$

and

$$m_e^* = \frac{\theta(1+\theta^*)(1+\theta+\theta^*)\sigma_\epsilon^2}{(1+\theta^*)^2\sigma_\epsilon^2 + \theta^2\sigma_{\epsilon^*}^2} \quad m_i^* = -\frac{\theta(1+\theta+\theta^*)\sigma_\mu^2}{(1+\theta)^2\sigma_\epsilon^2 + \theta^2\sigma_{\epsilon^*}^2} - (1+\beta^*).$$

The associated payoffs are:

$$\Sigma_1 = -a + \frac{c^2}{4f} = -\frac{\sigma_\mu^2 [(1+\theta^*)^2\sigma_\epsilon^2 + \theta^2\sigma_{\epsilon^*}^2]}{\theta^2[(1+\theta)^2\sigma_\epsilon^2 + \theta^2\sigma_{\epsilon^*}^2 + (1+\theta+\theta^*)^2\sigma_\mu^2]} < \Sigma_0$$

and

$$\Sigma_1^* = -a^* + \frac{c^{*2}}{4e} = -\frac{\sigma_\mu^2 [\theta^*\sigma_\epsilon^2 + (1+\theta)^2\sigma_{\epsilon^*}^2]}{\theta^{*2}[\theta^2\sigma_\epsilon^2 + (1+\theta)^2\sigma_{\epsilon^*}^2 + (1+\theta+\theta^*)^2\sigma_\mu^2]} < \Sigma_0^*.$$

Case 2. $\Pi_2 = 0$ and $\Pi_2^* \neq 0$ or $\Pi_2 \neq 0$ and $\Pi_2^* = 0$.

Consider the case where $\Pi_2 = 0$ and $\Pi_2^* \neq 0$. Since $\Pi_2 = 0$ the payoff to the foreign country, Σ^* , depends only on Π_3 . So, as in Case 1, any (Π_2^*, Π_3^*) is a best reply. But for the domestic country,

$$\Sigma = -a - \frac{b\Pi_2^*}{\Pi_3\Pi_3^*} - \frac{e\Pi_2^{*2} - d\Pi_3\Pi_2^* + f\Pi_3^2}{(\Pi_3\Pi_3^*)^2} \quad (A3)$$

Maximizing Σ over Π_3 yields the best reply for the domestic country

$$(\Pi_2, \Pi_3) = (0, \frac{2e\Pi_2^*}{d-b\Pi_3^*}), \quad (A4)$$

for any arbitrary (Π_2^*, Π_3^*) . Once again there is a large set of equilibria of this kind. Substituting (Π_2, Π_3) back into Σ yields the optimal payoff as a function of Π_3^* only. Maximizing Σ over Π_3^* (to derive point R in Figure 1) yields,

$$\Pi_3^* = (4fe - d^2)/(2ce - db). \quad (A5)$$

Substituting this back into Σ gives the maximum payoff, for this class of equilibria, to the domestic country, denoted Σ_2 . It is a simple matter to show that $\Sigma_2 = \Sigma_0$. That is, the domestic country can attain the maximal payoff if Π_3^* is chosen to satisfy (A5). Given (A5), we have from (A4)

$$\Pi_3 = E\Pi_3^*. \quad (A6)$$

Substituting this value into Σ^* yields:

$$\Sigma_2^* = -a^* + \frac{c^{*2}}{4e} = \Sigma_1^*. \quad (A7)$$

Moreover, one can show that, in this case, Σ_2^* is the maximal payoff the foreign country could attain given freedom of choice over Π_2^* and Π_3^* . For any choice of Π_3^* different from that given in (A5) the payoffs to both the domestic and the foreign country are lower. Thus a set of equilibria indexed by Π_3^* is mapped out. This set corresponds to the curve RS in Figure 1.

Correspondingly, setting $\Pi_2 = 0$ and letting $\Pi_2^* \approx 0$ maps out a symmetrical set of equilibria, given by the curve TU in Figure 1.

Case 3: $\Pi_2 \approx 0$ and $\Pi_2^* \approx 0$.

Consider deriving the best reply mappings for the domestic central bank, given Π_2^* and Π_3^* . A simple way to proceed is to maximize Σ over Π_2 , holding Π_3 fixed to derive the optimal Π_2 as a function of Π_3 . Substitute this value back into Σ and then maximize over Π_3 . This yields:

$$\Pi_2 = F + E \Pi_3^* \quad (\text{A8})$$

and

$$\Pi_3 = E \Pi_2^* \quad (\text{A9})$$

Exploiting the symmetrical nature of the problem yields the best replies for the foreign country:

$$\Pi_2^* = F^* + E^* \Pi_3 \quad (\text{A10})$$

and

$$\Pi_3^* = E^* \Pi_2 \quad (\text{A11})$$

Solving (A8)–(A11) yields a unique equilibrium. However, notice that these equations are identical to (25)–(28). Thus the best attainable equilibrium, point B in Figure 1, corresponds to the Nash equilibrium in this case. The corresponding strategies are (31) and (32). The corresponding payoffs are given in (27) and (30).

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