Journal of Economic Integration
12(3), September 1997; 344 – 358

Money Supply Growth and Exchange Rate Dynamics

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Abstract

The purpose of this paper is to re-examine the issue of exchange rate dynamics when the central bank undertakes a change in the growth rate of the money supply. The original analysis of exchange rate dynamics, the seminal model of Dornbusch, assumed that the growth rate of the money supply was zero and that the central bank permanently changed the money stock. In reality, however, central banks typically maintain money supply growth targets rather than money stock targets. Therefore, it seems appropriate to re-examine the issue of exchange rate dynamics using the money supply growth rate as the central bank's policy instrument. The paper analyzes the problem using a variable output version of the Dornbusch model. Perhaps the most significant finding in the paper is that money supply growth causes the exchange rate to either overshoot or undershoot. In addition, the real exchange rate depends inversely on the real interest rate during part of the adjustment process, in contrast to the real interest differential model. (JEL Classification: F41)

the money supply. The original analysis of exchange rate dynamics, the seminal model of Dornbusch [1976], assumed that the growth rate of the money supply was zero and that the central bank permanently changed the money stock. In reality, however, for several possible reasons central banks maintain money supply growth targets rather than money stock targets. One is that the long-run inflation rate depends on the growth rate of the money supply, and central banks are concerned with the long-term inflationary consequences of their actions. In addition, in a cyclically expanding economy the demand for real money balances is increasing, and the central bank therefore will have to undertake the appropriate amount of money supply expansion to achieve its short-term policy goals. Therefore, it seems appropriate to re-examine the issue of exchange rate dynamics using the money supply growth rate as the central bank's policy instrument.

The paper is organized in the following way. Section II develops a modified version of the Dornbusch model in which output is permitted to deviate temporarily from its natural level. In Section III the effects of money supply growth are examined in the model. Finally, Section IV summarizes the results. Perhaps the most significant finding in the paper is that money supply growth causes the exchange rate to either overshoot or undershoot. In addition, the real exchange rate depends inversely on the real interest rate during part of the adjustment process, in contrast to the real interest differential model.

II. The Model

Consider the following variable output version of the Dornbusch model, in which the money supply is growing at the constant rate, $\dot{m}$. Exchange rate expectations and inflationary expectations are assumed to be held with perfect foresight, and the inflation rate is determined by an expectations holds.[1] The model may be summarized as follows:

$$\dot{y} = \Omega \left[\quad + \frac{(e + p^{*} - p)}{D +} - \quad (r - \dot{p}) - (1 - \quad)y \right] \tag{1}$$

$$m - p = \quad y - \quad r \tag{2}$$

$$\dot{w} = \quad (y - \bar{y}) + \frac{\dot{p}^{e}}{D +} \tag{3}$$

and

$$r = r^{*} + \dot{e} \tag{4}$$

where y = log of real output; p = log of the price level of domestically produced goods; p^{*} = log of the foreign price level; e = log of the exchange rate on the foreign currency; r = domestic interest rate; $\underline{y}$ = log of the full-employment level of output; m = log of the money supply; r^{*} = foreign interest rate; w = log of the nominal wage rate; $\dot{p}^{e}$ = expected instantaneous rate of inflation; μ is a fiscal variable; and D is the differential operator.[2] Equation (1) is the output adjustment equation, according to which output gradually expands if aggregate demand exceeds the current level of output because of a production lag. Here aggregate demand depends on a fiscal variable, the real exchange rate, the real interest rate, and income. Observe that the real exchange rate affects aggregate demand with a lag, reflecting the well-known lag in the effect of the real exchange rate on trade flows.[3] This lag will be important in understanding the effect of money supply growth on interest rates.[4] Notice also that the real interest rate is defined as the nominal interest rate minus the actual rate of inflation on the assumption that

2. For a thorough discussion of the differential operator and its properties, see, for example, Baumol [1959, pp. 335-342].
3. By multiplying both sides of equation (1) by $D+\rho$, one can convert this equation to

expectations about the rate of inflation at the next instant in time turn out correctly to equal the actual rate of inflation. Thus, perfect foresight about very short-term inflation is assumed in the model. Equation (2) is the same money market equilibrium condition as in the Dornbusch model, but now output is variable. Equation (3) is an expectations augmented Phillips curve, according to which the rate of change of nominal wages depends on the gap between output and its natural level, $\bar{y}$, and on the expected rate of inflation. Notice that nominal wage increases would respond gradually to a sudden change in expected inflation, as a system of overlapping wage contracts would imply, although no sudden changes actually occur in the model.[5] Finally, equation (4) is again the interest parity condition.

In order to close the model, the condition for perfect foresight for inflationary expectations is imposed

$$\dot{p}^e = \dot{p} \tag{5}$$

along with the assumption of markup pricing

$$\dot{p} = \dot{w}. \tag{6}$$

Substituting the solution for $\dot{p}^e$ from (5) and $\dot{w}$ from (6) into (3) then yields

$$\ddot{p} = \alpha\pi(y - \bar{y}) + \pi\dot{y}. \tag{7}$$

Thus, the rate of change of the inflation rate depends on the output gap as well as the rate of change of output. The first term in (7) shows that an output gap leads to accelerating inflation. An output gap produces wage increases and in turn price increases by way of markup pricing. However, inflationary expectations then rise, leading to even higher wage increases and price increases. The second term in (7) comes directly from the Phillips curve effect of an output gap on wage increases. Given this Phillips curve relationship, rising output must lead to rising wage increases and hence accelerating inflation.

It can be shown that system (1), (2), (7), and (4) has a characteristic equation of fifth degree because of the lags in the model. Furthermore, one can show that one root is positive because of the perfect foresight assumptions, that four negative roots are now required for the stability of the system, and that it is certainly possible for such roots to occur. On the assumption that the stability conditions are then satisfied, the four negative roots imply that the system will converge, perhaps with oscillations if two of the roots are complex, to a saddle point when the growth rate of the money supply increases.

Finally, consider the exchange rate expectations mechanism in the model. It can be shown, as proven in the Appendix, that the following scheme embodies perfect foresight about exchange rate expectations in the model:

$$\dot{e} = {}_1(\bar{e} - e) + {}_2\dot{p} + (1 - {}_2)\dot{m} + {}_3(p - \bar{p}) + {}_4\dot{y}. \tag{8}$$

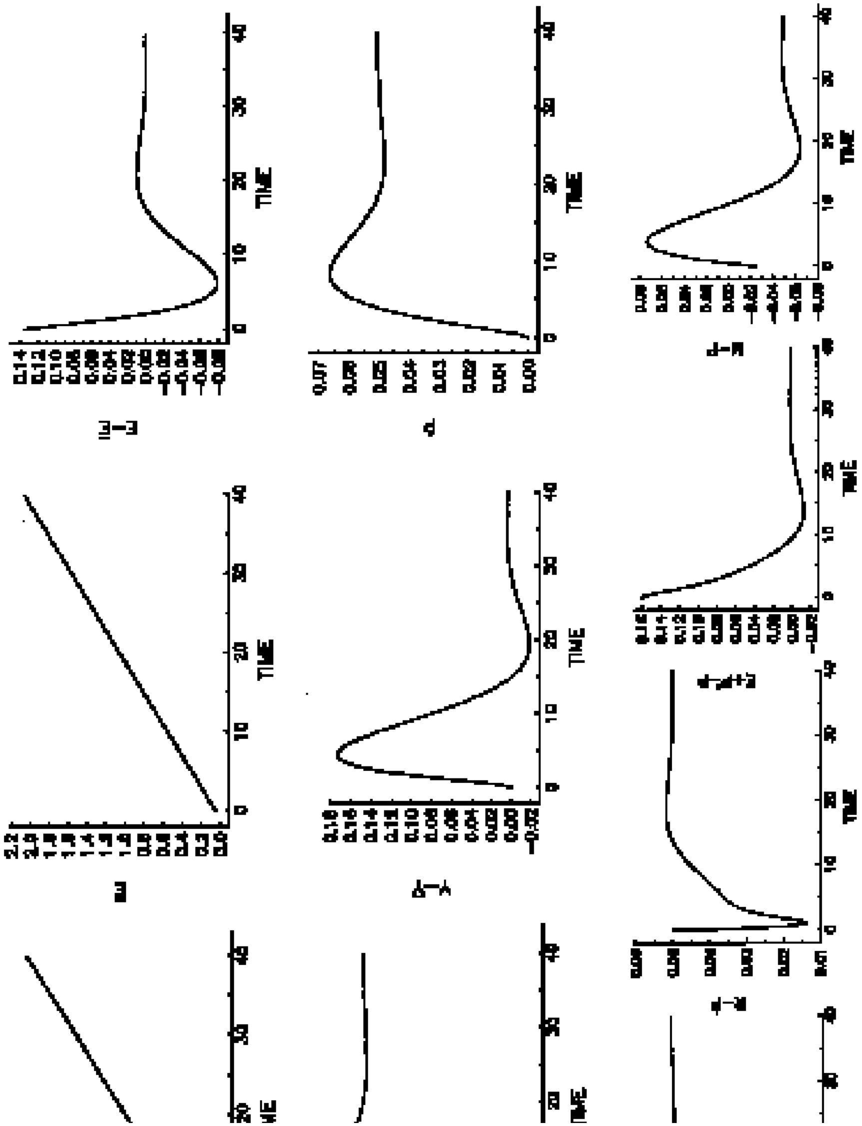
Observe that the expected rate of depreciation of the home currency depends regressively on the gap between the equilibrium exchange rate and its current level, the current rate of inflation, the rate of growth of the money supply, the price gap, and the growth rate of output. The rate of monetary growth enters the expectations mechanism because it determines the expected long-run inflation rate. The inflation rate, the price gap, and the growth rate of output also appear in the expectations scheme because, with three additional roots in the system, in comparison with the one negative root in the Dornbusch model, asset holders must have additional pieces of information about three endogenous variables to determine this path with perfect foresight. Combining equations (4) and (8) then yields the following equivalent version of the interest parity equation:

can now be imposed. Suppose for convenience that the growth rate of the money supply is initially zero. Then since real output is constant, the inflation rate is also initially zero. In addition, the system can be further normalized by setting $\bar{y}$ to zero, the value of m prior to the monetary shock to zero, r^* to 5%, and p^* to zero. Finally, μ is set equal to the value of σr prior to the monetary shock. This guarantees from equation (1) that the real exchange rate, p^*+e-p, prior to the monetary shock, is normalized at zero. It follows that the values of e and p prior to the monetary shock are equal. From equation (2) these pre-shock values must equal βr, where the pre-shock value of r equals 5% because interest parity holds. Notice that the pre-shock value of the real interest rate is also 5% since the inflation rate is initially zero.

Equations (1), (2), (7), and (4′) constitute a simultaneous differential equation system in the variables e, p, r, and y, and the solution equations for these variables involve the system's initial conditions and the four negative characteristic roots. A computer program of the model was then developed and simulated for a 5% growth rate of the money supply after normalizing the system.[6] Figure 1 shows the response of the system to this monetary shock. Observe first the impact effects. Notice in particular that the domestic currency depreciates (e rises from .025 to .185) and overshoots its new long-run equilibrium level ($\bar{e}$ rises from .025 to .05).[7] Overshooting is due not to a decline in interest rates, as in the Dornbusch model, but to the development of expected long-term inflation. The long-run inflation rate is now positive, and the domestic currency is therefore expected to begin depreciating to maintain long-run purchasing power parity. Consequently,

6. The following values of the parameters, which are based on a number of empirical studies, were used in this simulation: $\pi=.2$; $\beta=.5$; $\delta=1.0$; $\sigma=1.0$; $\gamma=.5$; and $\phi=.68$. In addition, α is given the value of .2, Ω is set equal to 10.0, and ρ is set to .5. These values guarantee that the system is dynamically stable.

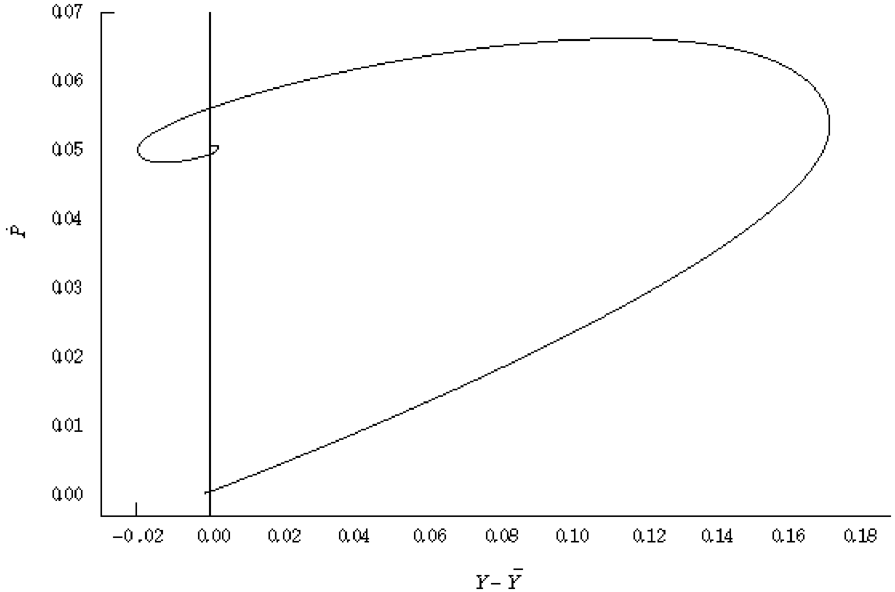
Figure 1
The Dynamic Effects of Money Supply Growth



overshooting must occur to re-establish the expectation that the exchange rate will not change and restore interest parity. The depreciation of the home currency, however, has no initial effect on real output because of the exchange rate lag on aggregate demand and the production lag in the goods sector. Consequently, the inflation rate remains at zero, the nominal interest rate remains constant (due to unchanged y and p), and the real interest rate remains unchanged. In addition, the real exchange rate on the home currency initially depreciates because of the depreciation of the nominal exchange rate and the initially unchanged domestic price level. Finally, recall that the home currency initially depreciates by 16% in response to the 5% monetary expansion. Thus, this model suggests that monetary growth can have a very large impact effect on the exchange rate. Moreover, note that the exchange rate initially overshoots by 13.5%, and therefore it is possible for much of the depreciation to reflect the overshooting factor.

Now consider the dynamic behavior of the system over time. Observe that after the impact effect output begins to rise and then gradually declines and converges with oscillations around the natural level. There are several observations to make here. First, the behavior of output reflects movements in the real exchange rate and the real interest rate. The expansion in output is caused by the real depreciation of the home currency that occurs on impact and by the effect of the gradual temporary decline in the real interest rate following the monetary shock. However, since the real exchange rate on the home currency subsequently appreciates back toward its original level, and the real interest rate eventually rises to its initial level, output eventually returns to its natural level. Second, although oscillations in output need not occur, they arise here because of the output adjustment lag, the exchange rate lag on aggregate demand, and the lagged response of wages to expected inflation in the model.[8] Finally, the inflation rate rises

Figure 2
The Output Gap and the Inflation Rate



new equilibrium level of 5%.

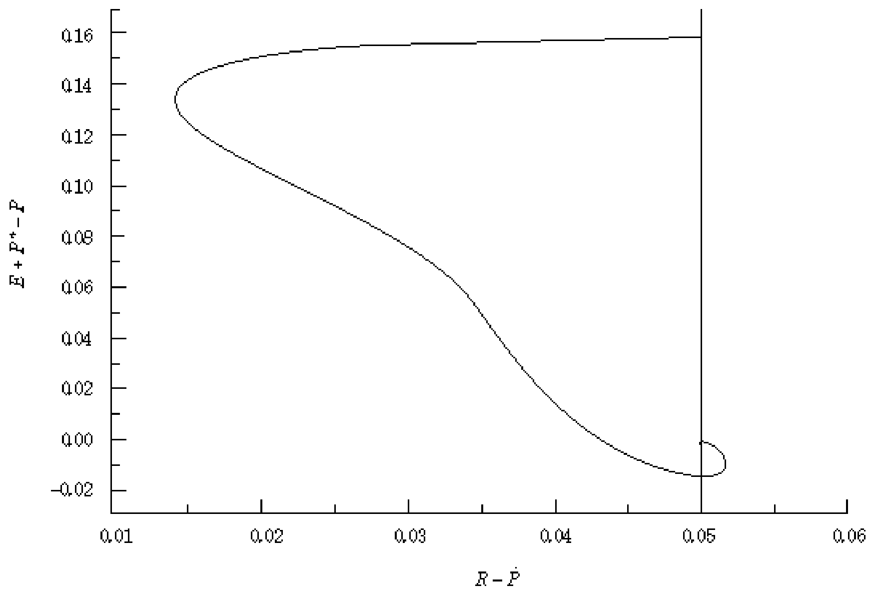
The relationship between the inflation rate and the output gap is shown in Figure 2. Notice that as output rises, the inflation rate rises. Eventually output begins to fall, but inflation continues to rise because of the buildup of inflationary expectations. As output continues to fall, however, the inflation rate eventually begins to decline as the Phillips curve effect begins to dominate the inflationary expectations effect. Eventually, both output and the inflation rate converge in a cyclical spiral to the new steady state.

The behavior of interest rates reflects movements in the real money stock and in real output. Interest rates initially fall because the money supply of the money supply. The real money stock subsequently begins to rise as the inflation rate falls below the growth rate of the money supply, and the demand for money falls as output eventually contracts. Thus, the interest rate declines and converges with oscillations to its new long-run equilibrium level of 10%.[9] Moreover, as the inflation rate, the interest rate, and output converge to their long-run equilibrium levels, then from the interest parity equation (4′) the gap between e and e_declines over time and is eventually eliminated. Of course, both e and e_rise over time as the money stock expands. In fact, the home currency gradually depreciates at an increasingly faster rate and then converges with oscillations to a 5% rate of depreciation equal to the growth rate of the money supply.[10] At that point long-run purchasing power parity is achieved.

Next, consider the behavior of the real interest rate and the real exchange rate. Initially, the real interest rate does not respond to the monetary shock because the nominal interest rate remains at 5% and the inflation rate is initially zero. However, over time the real interest rate declines because the nominal interest rate declines and the inflation rate rises. Subsequently, as the nominal interest rate eventually rises, it soon rises faster than the inflation rate, and the real interest rate begins to rise. Ultimately, the inflation rate reverses itself, and the real interest rate continues to rise and converges cyclically to its original level. Finally, the real exchange rate on the home currency, which initially depreciates because of the depreciation of the nominal exchange rate, gradually appreciates and converges with oscillations to its original level. Because of its initial depreciation, the real exchange rate overshoots in this model. However, in contrast to the real interest differential model developed by Frankel [1979], the real exchange rate depends inversely on the real interest rate during part of the adjustment process. Indeed, when the real interest rate is first declining, the real

Figure 3

The Real Interest Rate and the Real Exchange Rate



although the nominal exchange rate on the home currency eventually begins to depreciate, prices are rising sufficiently fast to cause real appreciation. This is shown in Figure 3, where the relationship between the real exchange rate and the real interest rate is presented. Thus, the real interest differential model is not supported here.[11]

Finally, the overshooting result reported here for the nominal exchange rate is derived from a specific set of parameters. However, the finding of overshooting of the nominal exchange rate is sensitive to the choice of parameters. When the model was simulated for low, medium, and high values of the nine parameters, undershooting occurred in 1,128 of the 11,021 dynamidiate price gap, $p-\underline{p}$. Since $\underline{p}$ initially rises, the price gap is negative, and this can lead to the expectation that interest rates will rise and the home currency appreciate.[12] Therefore, the home currency will immediately depreciate but undershoot its new long-run equilibrium level to produce an offsetting expected depreciation. Overall, then, asset holders will expect the domestic currency to remain unchanged, thereby restoring interest parity. Consequently, while the real exchange rate always overshoots in the model, the nominal exchange rate can either overshoot or undershoot in response to a monetary growth shock.[13]

IV. Conclusions

This paper has analyzed the effects of money supply growth on the exchange rate and other variables in a variable output version of the Dornbusch model. In this model the domestic currency depreciates on impact in response to money supply growth but may overshoot or undershoot its new long-run equilibrium level. Overshooting can occur because money supply growth has no immediate effect on the nominal interest rate, and therefore overshooting may be required to offset expected depreciation due to the effect of money supply growth on expected long-run inflation. This would keep the expected exchange rate movement unchanged, thereby restoring interest parity. However, since money supply growth also raises the long-run equilibrium price level, the expectation could develop that interest rates would rise and the home currency appreciate. Hence, undershooting might be necessary to keep the expected exchange rate movement unchanged. In addition, because the nominal exchange rate initially depreciates, the real

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12. See Levin [1995] for an analysis of the effect of the price level gap on exchange rate

exchange rate on the home currency initially depreciates and overshoots its long-run equilibrium level.

With respect to the dynamic effects in the model, interest rates initially begin to fall because monetary growth is accompanied by a constant demand for money due to the exchange rate lag on aggregate demand. Eventually, however, interest rates rise to their new long-run equilibrium level.[14] Similarly, the real interest rate declines substantially before returning to its original level. Also, the inflation rate rises over time before declining to its new long-run equilibrium level. Finally, the real interest differential model is not supported here because during part of the adjustment process the real exchange rate depend inversely on the real interest rate.

Finally, consider an overall evaluation of the model. It seems reasonable to believe that inflation will gradually respond to monetary growth and that initially interest rates will decline after a monetary growth shock. In addition, a variable output model is a plausible description of the economy. Therefore, the behavior of the exchange rate and other variables in the model appears to be a sensible characterization of the system.

Appendix
The Exchange Rate Expectations Scheme

Consider the interest parity condition (4′):

$$r = r^{*} + \theta_1(\bar{e} - e) + \theta_2\dot{p} + (1 - \theta_2)\dot{m} + \theta_3(p - \bar{p}) + \theta_4\dot{y}. \qquad (4')$$

Differentiating (4′), noting that $\dot{e}$ equals $\dot{m}$ and $\dot{m}$ is a constant, and solving for $\dot{e}$ yields

$$\dot{e} = \frac{(\theta_1 - \theta_3)\dot{m} + \theta_3\dot{p} + \theta_2\ddot{p} - \dot{r} + \theta_4\ddot{y}}{\theta_1}. \qquad (A1)$$

Linearizing equation (1) around the long-run equilibrium and using the differential operator D yields

$$\ddot{y} = \ \Omega\ddot{p} - [\Omega(1-\ \ ) + \ \ ]\dot{y} - \ \Omega\dot{r} + \ \Omega\ \ (\dot{p} - \dot{m}) - \ \Omega\ \ (1-\ \ )(y-\bar{y}) - \ \Omega\ \ (r-\bar{r}) - \ \Omega\ \ (p-\bar{p}) - \ \Omega\ \ (\bar{e}-e). \qquad \text{(A2)}$$

Differentiating equation (2) yields

$$\dot{m} - \dot{p} = \ \ \dot{y} - \ \ \dot{r}. \qquad \text{(A3)}$$

Linearizing equation (2) around the long-run equilibrium yields

$$p - \bar{p} = \ \ (r-\bar{r}) - \ \ (y-\bar{y}). \qquad \text{(A4)}$$

Next, note that

$$r-\bar{r} = r - r^* - (\bar{r} - r^*) \qquad \text{(A5)}$$

or equivalently, using (4) and the steady state condition $r - r^* = \dot{m}$ yields

$$r - \bar{r} = \dot{e} - \dot{m}. \qquad \text{(A6)}$$

Then using (A6) in (A2) and (A4) and substituting the solution for $(y-\bar{y})$ in (A4) into (A2) and then the solution for $\dot{r}$ from (A3) into (A2) and finally the solution for $\ddot{y}$ from (A2), $\ddot{p}$ from (7), using (A4) again, and $\dot{r}$ from (A3) into (A1) yields

$$\dot{e} = \frac{\Omega\ \ {}_4(\bar{e}-e)}{\Delta} + \frac{[-\ \ (\Omega\ \ {}_4 + \ {}_3) + \ \Omega\ {}_4 + 1]\dot{p}}{\Delta} + \frac{[\ \ ^2\ (\Omega\ \ {}_4 + \ {}_2) - \ ^2\Omega\ \ (1-\ \ )\ {}_4 + \ \ (\ {}_3 - \ {}_1) - \ (\Omega\ \ {}_4 + 1)]\dot{m}}{\Delta} + \frac{\{\ \ (\Omega\ \ {}_4 + \ {}_2) + \ \Omega\ {}_4[\ \ -(1-\ \ )]\}(p-\bar{p})}{\Delta}$$

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