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The Range and Determinants of the Real Exchange Rates

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Abstract

This paper examines the limit and determinants of the fluctuation of the real exchange rates between two countries. With a two-country and two-good model, where one country is a natural resource exporting country and the other is a manufacturing goods exporting country, this study employs a positive profit condition and proves the existence of the limit of the real exchange rates. The effects of determinants of the range of real exchange rates are investigated. It is found that the demand of non-wage earners is the only determinant which clearly widens or reduces the range while other determinants shift the range upward or downward. (JEL Classification: F3)

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I. Introduction

Traditional works on the fluctuation of the exchange rate had concentrated on the determination of the exchange rate in the foreign exchange market by demand and supply conditions. Little emphasis had been given to the mechanism by which the range of the exchange rate, in particular the real exchange rate, is determined. The real exchange rate, which is defined in terms of nominal exchange rates and price levels, is a convenient summary measure of the relative prices of domestic against foreign products and a basis for analyzing macroeconomic demand and supply conditions in open economies ⟨Krugman and Obstfeld [1994]⟩. Some studies explored the relation between changes in real sectors and the real exchange rate ⟨for example, see Saidi and Swoba [1983]⟩, however they did not show how the range of the exchange rate is bounded by real sectors.

Only recently did a group of economists focus on the range of an exchange rate, dealing with target zones. For example, Krugman [1991, 1992] considered the determination of the exchange rate when the authorities intervene to keep the exchange rate in a target zone. However, what they explored and developed is not the range of the real exchange rate, but issues on the dynamics of the exchange rate in a target zone regime, such as the behavior of the exchange rate inside the zone and the effects on the exchange rate of anticipation of (state-contingent or time-contingent) transition into a target zone ⟨Ichikawa, *et al.* [1990]⟩. They just mentioned that if there is no band (target zone) and money supply is held constant, the exchange rate moves together with real shocks.

A proof of the existence of the range of the exchange rate was endeavored by Ito [1990] from the view point of profitability which would be a very crucial condition for industries. Unfortunately, Itoh did not succeed in proving that the upper limit of the exchange rate is larger than the lower limit, which may undermine the validity of his proof of the existence of the range. His finding implies that if firms are pursuing non-negative profit, the upper and lower limits of the real exchange rate are determined by that profitability condition.

Dombusch [1987] investigated the adjustment of relative prices to exchange rate movement using industrial organization approach, however, his

major concern was not the range of the real exchange rate but the price adjustment.

In this paper, we set up a simple two-country two-good model. The existence of the natural range of the real exchange rate is proved and the effects of factors that determine the range are explored in section II where real wages are assumed to be given. In section III, the economy where real wages are not given but determined endogenously is considered.

II. The Range of Real Exchange Rates with Given Real Wages

A. Assumptions

In order to show the existence of the range of real exchange rates and the determinants of the range, several assumptions regarding production and consumption are required.

(i) There are two countries in the world. Country 1 produces and exports good 1 which are used as either consumption goods or capital goods. Country 2 produces and exports good 2 which are used as inputs for good 1 in country 1 or consumption goods in country 2.[1]

(ii) n units of labor and m units of good 2 are required to produce one unit of good 1, while n^* units of labor are required to produce one unit of good 2 (The superscript * stands for country 2).

(iii) The amount of employment does not adjust immediately. Therefore, while $N = nY$ units of labor are required to produce Y units of good 1, enterprises in country 1 actually employ $\bar{N} = n\bar{Y}$ units of labor, where Y stands for the amount actually produced and $\bar{Y}$ stands for the amount normally produced. In country 2, $\bar{N}^*(= n^*\bar{Y}^*)$ units of labor are employed while $N^* = n^*Y^*$ units are needed to produce Y units of good 2.

(iv) Workers spend all of their wage incomes for consumption. Workers in country 1 consume good 1 only. Workers in country 2 consume both good 1 and good 2 by the ratio of α and $(1 - \alpha)$ where α is less than or equal to one.

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1. Therefore, country 1 is a manufacturing country such as Japan or Korea which imports materials from country 2 and produces, consumes and exports manufactured goods while country 2 is a natural resource exporting country such as Australia.

(v) Non-wage earners in country 1 demand for good 1 only for consumption and investment. Those in country 2 consume both good 1 and good 2 by the ratio of β and $(1-\beta)$ where β is less than or equal to one.

B. The Model

We have two equilibrium equations for the commodity markets as follows.

$$pY = wN + pX + \alpha ew^*N^* + \beta ep^*X^* \tag{1}$$

$$p^*Y^* = p^*mY + (1-\alpha)w^*N^* + (1-\beta)p^*X^* \tag{2}$$

where p: price, Y: outputs, w: nominal wage rate, N: numbers employed, X: demand from non-wage earners, e: nominal exchange rate (the value of country 2's currency in terms of country 1's currency).

The total value of each country's product should be the same as the value of total demand from its domestic and foreign markets. (1) shows that good 1 is consumed by consumers in both countries while (2) shows that good 2 is consumed by consumers in country 2 only. Country 1 demands for good 2 in order to produce good 1.

Dividing equations (1) and (2) by p and p^* respectively gives equilibrium conditions in real terms as

$$Y = Rn\bar{Y} + X + \alpha R^*n^*\bar{Y}^* + \beta qX^* \tag{3}$$

$$Y^* = mY + (1-\alpha)(R^*n^*/q)\bar{Y}^* + (1-\beta)X^* \tag{4}$$

where $R = w/p$, $R^* = ew^*/p$, $q = ep^*/p$, $N = n\bar{Y}$, $N^* = n^*\bar{Y}^*$.

From equations (3) and (4), we have the real exchange rate q which is calculated from the nominal exchange rate and price levels.

The trade balance between the two countries respectively is

$$B = \alpha R^*n^*\bar{Y}^* + \beta qX^* - qmY \tag{5}$$

$$B^* = -B/q. \tag{6}$$

In order to sustain production and trade, enterprises in both countries must have positive profits. As enterprises in country 1 expend $Rn\bar{Y}$ for labor

costs and qmY for material costs when they produce Y units of output, country 1's profit in real terms is

$$\Pi = Y - Rn\bar{Y} - qmY. \tag{7}$$

Enterprises in country 2 do not require materials to produce good 2 but only need labor, as such, they pay R^*/q (per unit of labor) for $n^*\bar{Y}^*$ units of labor. Therefore, country 2's profit is

$$\Pi^* = Y^* - (R^*n^*/q)\bar{Y}^*. \tag{8}$$

Another expressions of profit can be derived from equations (3) to (6). For each country, producers' profits are

$$\Pi = X + B \tag{9}$$

$$\Pi^* = X^* + B^* \tag{10}$$

which indicate that profits are equal to the sum of demands from non-wage earners and the trade balance.

C. Profitability Conditions

In order to satisfy each country's profitability conditions, the following inequalities must be satisfied.

$$\Pi = Y - Rn\bar{Y} - qmY > 0 \tag{11}$$

$$\Pi^* = Y^* - (R^*n^*/q)\,\bar{Y}^* > 0 \tag{12}$$

To find the value of q which satisfies (11), we define $F(q)$ as follows, by replacing Y in (7) with (3):

$$F(q) = -m\beta X^*q^2 + \{\beta X^* - m(Rn\bar{Y} + X + \alpha R^*n^*\bar{Y}^*)\}q + X + \alpha R^*n^*\bar{Y}^*. \tag{13}$$

As a profit, $F(q)$ must always be equal to or greater than zero. In equation (13), F(0) $(= X + \alpha R^*n^*\bar{Y}^*)$ is positive. Moreover, the coefficient for q^2 in equation (13) is negative, which implies that the equation $[F(q) = 0]$ has an unique positive solution, q_u. When q is smaller than q_u (and, of course, greater than zero), country 1 enjoys positive profit. In brief, the profitability condition for country 1 is

$$0 < q < q_u. \tag{14}$$

Symmetrically, we can define $G(q)$ for country 2 as follows.

$$G(q) = m\beta X^* q^2 + \{m(Rn\bar{Y} + X + \alpha R^* n^* \bar{Y}^*) + (1-\beta)X^*\}q - \alpha R^* n^* \bar{Y}^*. \quad (15)$$

As $G(q)$ is the same as Π^*, $G(q)$ must always be positive. As $G(0)$ $(= -\alpha R^* n^* \bar{Y}^*)$ is negative and the coefficient for q^2 in (15) is positive, the equation $[G(q) = 0]$ has an unique positive solution q_l. When q is greater than q_l, country 2 enjoys positive profit. The profitability condition for country 2 is, therefore

$$0 < q_l < q. \quad (16)$$

In order for q to exist, satisfying both (14) and (16), q_u must be greater than q_1. From equations (13) and (15), $G(q)$ (the profit of country 2) can be expressed as

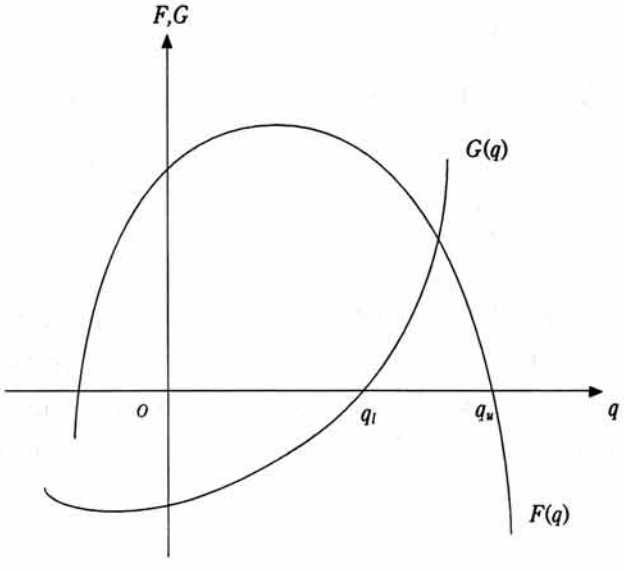
$$G(q) = -F(q) + X + qX^*. \quad (17)$$

As $F(q_u) = 0$, $G(q)$ is always positive when $q = q_u$, *i.e.*,

$$G(q_u) > 0. \quad (18)$$

Therefore, q_l is always smaller than q_u (see Figure 1).

Figure 1
Feasible Range of the Real Exchange Rate



$$q_l < q_u. \tag{19}$$

When q is lower than q_1, country 1 enjoys positive profit ($F(q) > 0$) while country 2 suffers a loss ($G(q) < 0$). When q is greater than q_u, only country 2 gains from production and trade. Only when q lies between q_l and q_u will both countries have positive profit. Therefore, $q = [q_l, q_u]$ is the feasible range of the real exchange rate for the two countries.

D. Determinants of the Range

As the range of the exchange rate is derived from the positive profit conditions for the two countries, the upper limit (q_u) and the lower limit (q_1) of the exchange rate are affected by changes in components constructing profitability.

From $F(q_u) = 0$, we can have

$$F_q dq + F_z dz = 0 \tag{20}$$

where z stands for the structural parameters, $F_q = \partial F/\partial q$ and $F_z = \partial F/\partial z$. From (13),

$$F(q) = qF_q + m\beta X^* q^2 + (X + \alpha R^* n^* \bar{Y}^*). \tag{21}$$

As $F(q_u) = 0$ and from (21), F_q evaluated at $q = q_u$ is negative.

$$F_q \mid_{q = q_u} < 0. \tag{22}$$

From equations (20) and (22), we can establish the effect of z on q where $q = q_u$ as follows.

$$\text{sign}(dq/dz) \mid_{q = q_u} = \text{sign}(F_z). \tag{23}$$

Similarly, the effect of the change in z on q where $q = q_1$ is

$$\text{sign}(dq/dz) \mid_{q = q_l} = \text{sign}(G_z).^2 \tag{24}$$

Based on (23) and (24), the effects of determinants on the range of the

2. Equation (24) is derived as follows:
Because $G(q)$ is expressed in terms of G_q as $G(q) = qG_q - m\beta X^* q^2 - \alpha R^* n^* \bar{Y}^*$ and $G(q_1) = 0$, $G_q \mid_{q = q_l}$ is positive. Therefore, sign $(dq/dz) \mid_{q = q_l} = -\text{sign}(G_z)$.

Table 1

The Effects of Structural Parameters on the Range of Exchange Rates

	n	m	n^*	R	R^*	X	X^*	α	β	$\bar{Y}$	$\bar{Y}^*$
q_u	−	−	+	−	+	+	+	+	+	−	+
q_1	−	−	+	−	+	−	−	+	+	−	+

exchange rate are summarized in Table 1. It is shown that when structural parameters except X and X^* increase or decrease, both the upper and lower limits of q move in the same direction. If the productivity of country 1 increases (*i.e.*, n and m decrease), demands for good 1 in country 2 increase (*i.e.*, α and β increase), and real wage rates in country 1 decrease (those in country 2 increase) or the amount of normal production of good 1 decreases (that of good 2 increases), then the range of the real exchange rate between country 1 and country 2 moves upwards. It needs more investigation to find whether the range widens or shrinks as moving upwards or downwards. Only the increase in non-wage earners' demands (X and X^*) in both countries increases the upper limit and decreases the lower limit, which clearly widens the range of the real exchange rate between the two countries.

III. Extension – Mark-up Pricing

In section II, it was assumed that real wage rates are given. In this section, we relax that assumption and examine the determination of the range of the real exchange rate. Throughout the model, mark-up pricing is assumed, where the price is determined by the sum of production costs (costs for both wages and materials) and profit margin per unit. The wage costs are the nominal wage rate (w) multiplied by the amount of labor required to produce one unit of a good (n). When actual outputs are not the same as normal outputs, the wage costs the enterprise actually expends per unit of product ($= wn\,\bar{Y}/Y$) are different from the wage costs considered in the mark-up price ($= wn$). When actual outputs Y are greater than normal outputs $\bar{Y}$ (in a boom period), the wage costs, which are higher than actually expended, are included in the price. When actual outputs are less than normal outputs (in a recession period), the wage costs, which are lower than

actually expended, are included in the price. Material costs for unit of good 1 are, by considering the amount of material required and the exchange rate, obtained as ep^*m.

The mark-up price of good 1 is

$$p = (wn + ep^*m)\mu \quad \mu > 1 \tag{25}$$

where μ stands for (1 + mark-up ratio). Dividing (25) by p gives

$$1 = (Rn + qm)\mu. \tag{26}$$

From the mark-up price of good 2 and by applying the same method as for good 1, we can obtain another equation for the mark-up price of good 2.

$$1 = (R^*n^*/q)\mu^*. \tag{27}$$

From (3), (4), (26) and (27),

$$\begin{aligned} Y &= (\alpha\bar{Y}^*/\mu^* + \beta X^* - m\bar{Y})q + \bar{Y}/\mu + X \\ &= (A - m\bar{Y})q + C \end{aligned} \tag{28}$$

$$Y^* = m(A - m\bar{Y})q + mC - A + \bar{Y}^*/\mu^* + X^* \tag{29}$$

where $A = \alpha\bar{Y}^*/\mu^* + \beta X^*$ and $C = \bar{Y}/\mu + X$.

Using (7), (8), (26), (27), (28) and (29), the respective profit for each country is obtained as:

$$\Pi = -m(A - m\bar{Y})q^2 + (A - mC)q + X \tag{30}$$

$$\Pi^* = m(A - m\bar{Y})q + mC - A + X^*. \tag{31}$$

Define $\Pi = H(q)$ and $\Pi^* = J(q)$. If there exists some range of q which guarantees both $H(q)$ and $J(q)$ to be positive (*i.e.*, both countries have positive profits), that range is the range of real exchange rates between the two countries. In order to discover this range, the signs of $(A - m\bar{Y})$ and $(mC - A + X^*)$ from (30) and (31) should be investigated. We assume that $(A - m\bar{Y})$ is positive based on at least two reasons:[3] first, when the ex-

3. Some may regard this assumption too strict. If this assumption is released (*i.e.*, if $(A - m\bar{Y})$ is negative), it means that outputs decrease as the country's currency is depreciated. In this case, the results are dependent on the sign and size of $(A - mC)$. It

change rate depreciates (q increases), outputs would increase as the increase of exports owing to the depreciation of country 1's currency would be larger than the decrease of consumption owing to the decrease of real wage rates caused by depreciation. Second, $-(A - m\bar{Y})$ is the trade balance of country 2 under normal operation of production. It may be assumed that the manufacturing country (country 1) records a trade account surplus.

As $H(0) = X > 0$, the profit equation $H(q) = 0$ has one positive solution q_u when $(A - m\bar{Y})$ is positive. Therefore q should be less than q_u. From (31) it is found that the profit of country 2 increases as q increases because $(A - m\bar{Y})$ is assumed to be positive. If the intercept $(mC - A + X^*)$ is positive, country 2 will enjoy profit for any $q > 0$. If $(mC - A + X^*)$ is negative, country 2 will have positive profit when $q > q_l$, where q_l satisfies $J(q_1) = 0$. From (30) and (31), $H(q) = -qJ(q) + qX^* + X$. Therefore $H(q_l) = q_l X + X > 0$, which implies that q_u is larger than q_l.

Therefore we have two ranges depending on the signs of $(mC - A + X^*)$:

CASE I: If $(mC - A + X^*) < 0$, then $q_l < q < q_u$. (Figure 2)

CASE II: If $(mC - A + X^*) > 0$, then $0 < q < q_u$. (Figure 3)

The effects of structural parameters on the range of exchange rates are

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can be summarized as follows:

(i) If $(A - mC)$ is negative, $H(q)$ has two distinct positive solutions q_{u1} and q_{u2} $(q_{u1} < q_{u2})$. Therefore, country 1 enjoys positive profit if $0 < q < q_{u1}$ or $q_{u2} < q$.

Country 2 has positive profit if $0 < q < q_1 = (A - mC - X^*[m(A - m\bar{Y})]$ where $J(q_1) = 0$. As $J(q) = -H(q)/q + X/q + X^*$ and $H(q_u) = 0$, hence $J(q_u) = X/q_u + X^* > 0$. Therefore $q_u < q_l$. As country 2's profit is negatively linear with respect to q, its profit is maximized when $q = 0$. With q_l, country 1's profit $H(q_1)$ is smaller than $H(0) = X$. It means both countries 1 and 2 can maximize their profit by keeping $q = 0$, which is a very unrealistic result.

(ii) If $(A - mC)$ is positive and greater than X^* (*i.e.*, $mC - A + X^* < 0$), the unique solution of $J(q) = 0$ is negative. With any $q > 0$, country 2 never realizes profit, which is unrealistic, too.

(iii) If $(A - mC)$ is positive and smaller than X^*, $J(q) = 0$ has a positive solution $q_l = (A - mC - X^*)/[m(A - m\bar{Y})]$. As $H(q) > 0$ and $H_q > 0$ for any $q > 0$, $q = [0, q_l]$ is the fluctuation range of exchange rates. In this case, the lower limit is fixed at zero and not affected by any changes in structural parameters. Only q_l is variable by parameters, which implies that only country 2's profitability binds the range.

Figure 2
Feasible Range of the Real Exchange Rate – Mark-up Pricing (Case I)

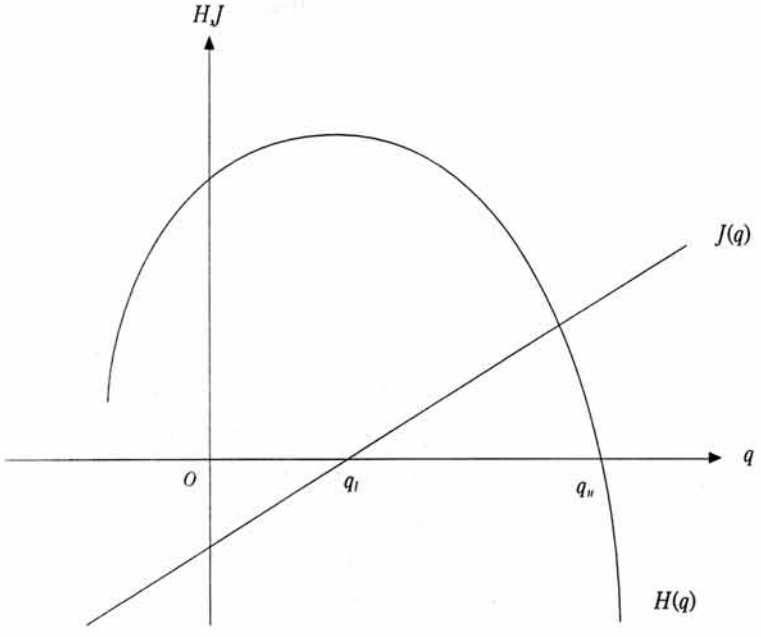

Figure 3
Feasible Range of the Real Exchange Rate – Mark-up Pricing (Case II)

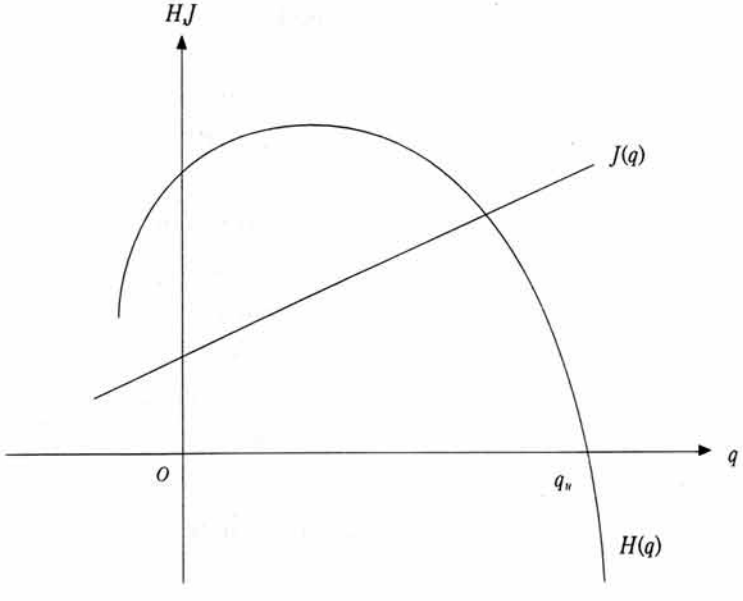

Table 2
The Effects of Structural Parameters on the Range of Exchange Rates (Mark-up Pricing)

		m	μ	μ^*	X	X^*	α	β	$\bar{Y}$	$\bar{Y}^*$
q_u	(Case I & II)	?	+	−	+	+	+	+	−	+
q_l	(Case I)	?	+	−	−	−	+	+	−	+
	(Case II)	0	0	0	0	0	0	0	0	0

summarized in Table 2. For case I, the results are fundamentally the same as Table 1. The lower limit is bounded by the material goods exporting country's (country 2) profitability and the upper limit is bounded by the industrial goods exporting country's (country 1) profitability. Only X and X^* clearly widen the range. While the increase of the mark-up ratio in country 1 shifts the range upwards, that in country 2 shifts the range downwards. For case II, however, only the parameters affecting country l's profitability can change the range of real exchange rates because q_l from the profitability condition of country 2, is always zero. Any attempt to control the range through the profitability of country 2 will not be effective.

IV. Discussion

When an economy experiences overshooting of exchange rates, sometimes it is followed by a temporary departure of the real exchange rate from its long run equilibrium level (as suggested by PPP). This departure of real exchange rates from equilibrium, which is referred to as "misalignment", has received a wide range of attention, in particular, concerning its effects on industry and trade. For example, Marston [1987] investigated the influence of an appreciation in real exchange rates on the British economy, and Baldwin and Krugman [1986] examined the US dollar misalignment. They argued that a large appreciation or depreciation of the currency has significant effects on the industrial structure, an outcome known as "hysteresis".

Our research provides critical findings about an endurable criterion, which should be considered by economic agents when they observe and

analyze the fluctuation of the real exchange rate. The endurable criterion is given from this research in the form of the band between the upper limits of the appreciation and the lower limits of the depreciation.

As discussed above, misalignment, by nature, happens in the short run, as the exchange rate will return to the long run equilibrium level after a while. As such, the focus of this paper is on the determination of the real exchange rate in the short run. In the short run, some variables are not able to be adjusted immediately after a misalignment occurs. As a result, firms will suffer a loss until they complete all of their adjustment. In the long run, all variables will be adjusted, firms may enter or exit, and zero profit will be prevailing in the economy.

The findings of this study have additional practical implications. For example, the upper and lower limits of exchange rates represent the border out of which firms suffer a loss. Therefore, by providing the relevant information, it gives policy implications including the target zone for government. In Japan, for example, the bound for the exchange rate has been investigated by *the Economic Planning Agency* with special attention given to the estimation of the exchange rate where the profit from exporting becomes zero. The low bound of real exchange rates which guarantees a positive profit (the profitable rate) and the low bound of real exchange rates which can cover at least a part of variable costs are calculated and compared with the realized actual exchange rate (between Yen and US Dollar) in *Kezaihakusho*. However, their low bound is not calculated by the two-country model we suggested, but derived from survey questionnaire distributed to Japanese exporters.

V. Conclusion

With a two-country two-good model, where one country is a natural resource exporting country and the other is a manufacturing goods exporting country, this paper proved the existence of the range of the exchange rate between the two countries. Profitability conditions were used throughout the analysis to prove the existence. Factors such as technical conditions, distribution conditions and demands of both countries for goods, appear to affect the width of this range. It was found that a change in any parameter

except X and X^* (demand from non-wage earners in each country) makes both the upper and lower limits of the real exchange rate moved in the same direction. An increase in X and X^* results in a widening of the range (*i.e.*, the upper limit increases and the lower limit decreases). In a special case, only the upper limit is bounded by the manufacturing goods exporting country's profitability while the lower limit is zero. In that case, the resource exporting country's profitability does not bound because it has positive profit if only the real exchange rate is greater than zero.

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