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Monopoly, Trade and Tariff Redundancy

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Abstract

The effects upon trade of protecting a domestic monopoly are analyzed formally, under conditions of both certainty and uncertainty in the exchange rate, by casting the firm's problem as one in the general format of constrained optimization. Existing studies assuming exchange rate certainty typically engage a diagrammatic approach, depicting a linear demand curve; the formal analysis presented here, involving non-differentiable methods of Lagrangean analysis, in particular identifies a scenario which has been "missed" in this previous work. In case of uncertainty in the exchange rate, under certain simplifying assumptions we can show that the probability of tariff redundancy is increasing with the level of protection, and also determine the comparative static effect of increased uncertainty on this probability and upon expected tariff receipts. The analytical structure developed here should lend itself in the future to such refinements as the introduction of a competitive domestic fringe. (JEL Classification: F13, L12)

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I. Introduction

The standard trade models explain the volume and direction of trade by comparative advantage: international differences in relative prices. In turn, comparative advantage is explained by supply factors such as differences between countries' technologies (the Ricardian model) or factor endowments (the Heckscher-Ohlin model); or by both supply and demand factors (the Neoclassical model). The general conclusion reached by these models is that trade based on comparative advantage results in welfare gains.

All these models are based on the assumption of perfect competition within and between countries. It has long been recognized, however, that the combination of a monopolistic producer and appropriately restrictive trade policies can affect a country's volume and direction of trade, and at the limit reverse trade flows by exporting a commodity in which the country under free trade conditions has comparative disadvantage. This possibility occurs from the ability of the monopolist to explore the restrictive power of trade barriers which prevent arbitrage, and thus facilitate market segmentation permitting price discrimination between foreign and domestic consumers. Trade restrictions allow the monopoly to exploit the opportunity of profit maximization by developing import substitution which at the margin it can carry over into export performance by 'dumping' – charging domestic consumers more than foreigners – which of course cannot arise under conditions of free trade: market integration disciplines domestic firms both by actual and potential competition.

These conclusions are entrenched in the international trade literature but have been derived disjointedly as special cases or by-products of the study of related issues, such as: the differences between tariffs, quotas and subsidies under imperfect competition ⟨Bhagwati [1965]⟩; the difference between monopoly and perfect competition in the use of tariffs ⟨Fishelson and Hillman [1979]⟩; and the potential for import substitution and exports by trade policy under different market structures, cost conditions and returns to scale ⟨Corden [1965]; Basevi [1970]; Frenkel [1971]; Pursell and Snape [1973]; White [1974]; Curtis [1983]⟩. Some of the welfare effects of protecting a domestic monopolist experiencing increasing costs have also been investigated but mostly diagrammatically ⟨Bhagwati [1988]; Rieber [1981];

White [1964]; Sodersten and Reed [1994]⟩.

In this paper, the effects upon trade of protecting a domestic profit maximizing monopoly are analyzed formally and systematically, under conditions of both certainty and uncertainty. This is done by posing the question, and exploring the issues, in the format of the general problem of constrained maximization. The paper is in four sections. Section I describes the basic model of a domestic monopolistic producer facing a world-wide competitive industry. Section II analyzes the production decisions of this monopolist under conditions of certainty in domestic demand, and also explores the effects on trade of different production-cost conditions and trade policies. Section III seeks to extend the basic analysis to the case of exchange rate uncertainty, a consideration which has been somewhat neglected in the literature to date.[1] Under the assumption that the domestic monopoly acts to maximize its expected profits denominated in world currency units (dollars), and with the help of a number of simplifying assumptions made both for realism and to ensure tractability, the features of the certainty solution are re-examined, and new insights obtained. The final section presents general conclusions and suggestions for further research.

II. Monopoly, Protection and Trade

The study employs an asymmetric monopoly model in partial equilibrium analysis which provides a rich variety of interesting cases. We assume a homogeneous commodity produced domestically by a single profit-maximizing firm facing a perfectly elastic supply in the world markets, so that it cannot affect import or export prices, and perfectly competitive input markets at home. There are no transport costs or other impediments to commodity arbitrage, but trade policy can potentially segment an otherwise integrated world market. In the domestic market, trade is the residual between domestic demand and domestic supply. So, when the price of the domestically pro-

1. The exceptions are the articles by White [1974], which deals exclusively with the comparison between the competitive industry and a quantity setting monopolist, and by Feldman [1993], which investigates tariff redundancy under foreign-price uncertainty. In each case uncertainty shows in the foreign price and expected profit is maximized in domestic currency units. We shall take issue with these assumptions.

duced and foreign commodities are equal, consumers buy first the domestic product in preference to the identical imported one. Therefore, trade occurs either as imports to meet excess home demand or as exports to relieve excess home supply. In the case of exchange rate uncertainty, to be considered later, the firm chooses output and price *before* the uncertainty is resolved, but sales take place afterwards; the direction and magnitude of trade is essentially stochastic.

Let $D_h(p)$ be the home demand curve, assumed downward sloping, and let $MR_h(Q)$ be the marginal revenue curve, also assumed to be downward sloping.[2] Let $MC(Q)$ be the firm's marginal cost curve, assumed for the present to be upwards sloping. Let p_w be the world price and let $MC(Q_0) = p_w$. In laying out the basic model here, we shall assume all prices to be denominated in the same currency – or that the exchange rate between the domestic currency and that of the world outside is fixed and equal to one. Later, we admit uncertainty in this exchange rate.

In free trade equilibrium, under conditions of certainty, the firm may be able to supply the home market entirely and also export, or there may have to be imports. This is the question of the firm's **cost advantage (CA)** or **cost disadvantage (CD)** in world trade:

$$\mathbf{CA} : D_h(p_w) < Q_0$$

$$\mathbf{CD} : D_h(p_w) > Q_0$$

Additionally (but not independently), the firm may either be able to achieve its overall profit maximum whilst engaging in the world market **(world market relevance, WMR)** or it may not **(world market irrelevance, WMI)**:

$$\mathbf{WMR} : MR_h(Q_0) < p_w$$

$$\mathbf{WMI} : MR_h(Q_0) > p_w$$

The relationships between the properties {**CA, CD**} and {**WMR, WMI**} will shortly become evident. Let $MC(Q)$ and $D_h(p)$ cross at p^0 and Q^0. Let p_1 be the price at which $MR_h = p_w$. The relationship between the prices p_w, p^0 and p_1 depends on which of the properties {**CA, CD**} and {**WMR, WMI**} hold.

2. There is a slight loss of generality here; MR_h would be increasing if the (inverse) demand curve were sufficiently convex to the origin; we return to this point later.

The following Lemma provides the key:

Lemma 1:

(1) **CA** implies $p^0 < p_w$ and **CD** implies $p^0 > p_w$

(2) **CA** implies **WMR**

(3) **WMR** implies $p^0 < p_1$

For the proof of this and all subsequent assertions, see the Appendix. It follows that there are at most four theoretically possible constellations for the properties {**CA, CD**} and {**WMR, WMI**} and the prices p_w, p^0 and p_1:

(a): **CA** and **WMR** and $p^0 < p_w < p_1$

(b): **CD** and **WMR** and $p_w < p^0 < p_1$

(c): **CD** and **WMI** and $p_w < p^0 < p_1$

(d): **CD** and **WMI** and $p_w < p_1 < p^0$

All of these constellations can occur: indeed, as Figure 1 shows, they can occur in the case of linear demand. Case (d) is typically overlooked in diagrammatic analyzes of the linear demand case ⟨see, for example, White [1964]; Fishelson and Hillman [1979]; Rieber [1981]; Bhagwati [1988]⟩.

Now define p^m and Q^m as the domestic profit-maximizing price and quantity, and let p_0 be the price at which quantity Q_0 is demanded domestically, so that $D_h(p_0) = Q_0$. A further relationship between prices is contained in the following Lemma:

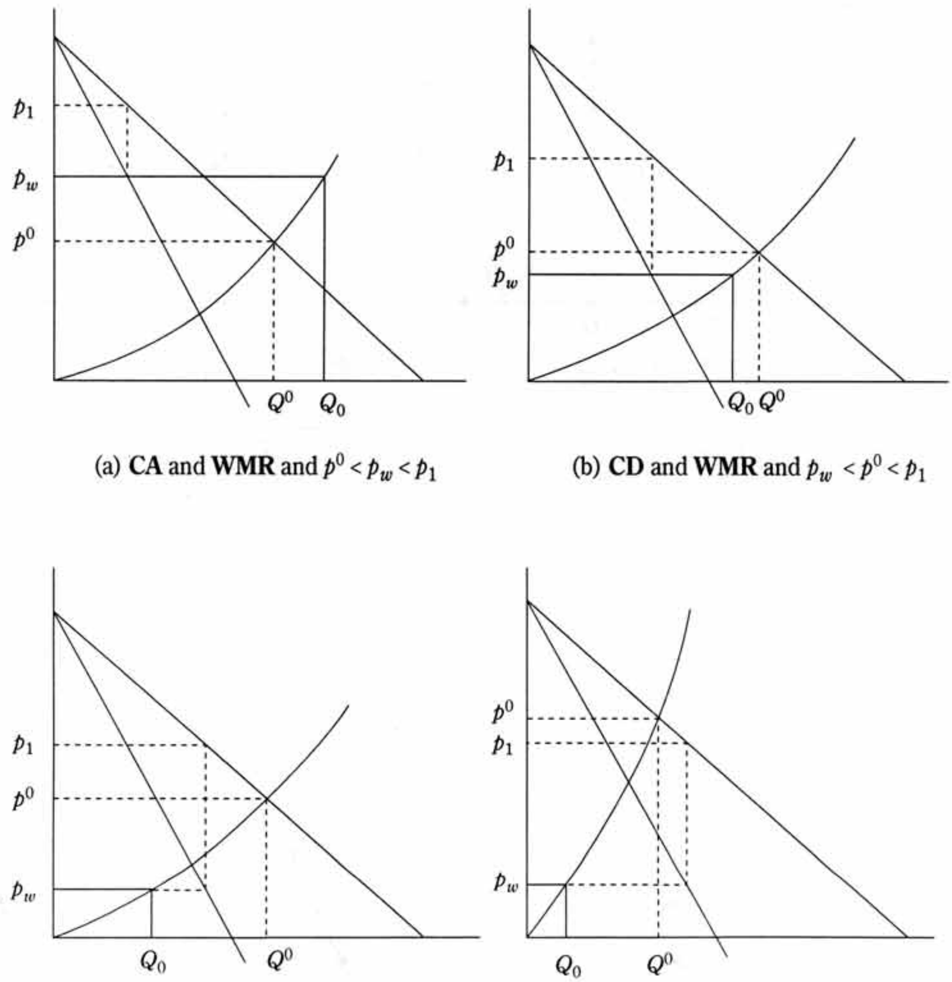
Lemma 2:

$p^0 < p^m$, and if both **CD** and **WMR** then $p^0 < p_0 < p_1$

III. The Firm's Decision Problem: The Certainty Case

Our concern is with tariff protection: we assume that an *ad valorem* tariff is imposed on imports, raising their price from p_w to $p_T = p_w(1+\tau)$. The firm can set a lower domestic price p_h, but not a higher one, and can sell any amount of its output abroad at price p_w. The firm must therefore choose its domestic price $p_h \in [p_w, p_T]$ and output Q to maximize its total profit $\pi(p_h, Q)$ which is given by:

Figure 1
The Four Constellations



(a) **CA** and **WMR** and $p^0 < p_w < p_1$

(b) **CD** and **WMR** and $p_w < p^0 < p_1$

(c) **CD** and **WMI** and $p_w < p^0 < p_1$

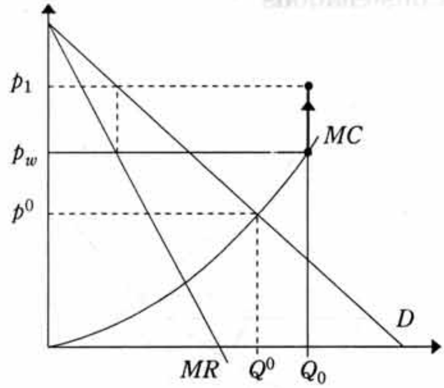
(d) **CD** and **WMI** and $p_w < p_1 < p^0$

$$\pi(p_h, Q) = p_w Q - C(Q) + (p_h - p_w).\ min\ \{D_h(p_h), Q\} \tag{1}$$

where $C(Q)$ is the total cost function (*viz.* $C'(Q) = MC(Q)$).[3] For the linear demand cases illustrated in Figure 1, the domestic price p_h^* and output Q^*

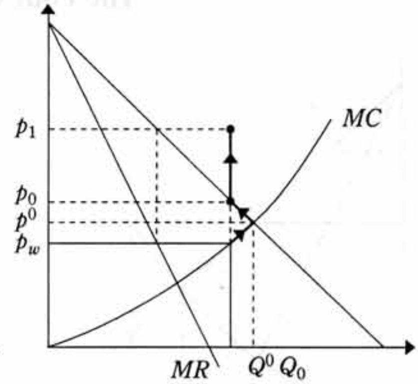
3. In the case of monopsonistic input conditions, the firm would of course choose factor input levels to maximize its profits, and not output.

Figure 2
The Solution Path for p_h^*, Q^* (arrowed as τ increases)



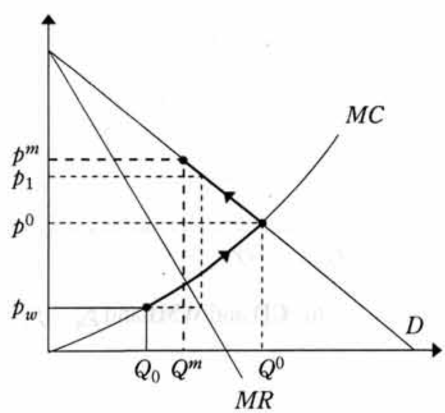
(a)

$p_T < p_1 \rightarrow$	$p_h^* = p_T$ $Q^* = Q_0$ *exports*
$p_T < p_1 \rightarrow$	$p_h^* = p_t$ $Q^* = Q_0$ *exports* *water in tariff*



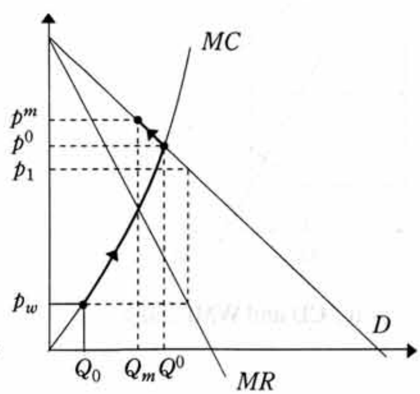
(b)

$p_T < p^0 \rightarrow$	$p_h^* = p_T$ $MC(Q^*) = p_T$ *imports*
$p^0 < p_T < p_0 \rightarrow$	$p_h^* = p_T$ $Q^* = D_h(p_T)$ *market closed*
$p_0 < p_T < p_T \rightarrow$	$p_h^* = p_T$ $Q^* = Q_0$ *exports*
$p_T > p_1 \rightarrow$	$p_h^* = p_1$ $Q^* = Q_0$ *exports*



(c)

$p_T < p^0 \rightarrow$	$p_h^* = p_T$ $MC(Q^*) = p_T$ *imports*
$p^0 < p_T < p^m \rightarrow$	$p_h^* = p_T$ $Q^* = D_h(p_T)$ *market closed*
$p_T > p^m \rightarrow$	$p_h^* = p^m$ $Q^* = Q^m$ *market closed* *water in tariff*



(d)

$p_T < p^0 \rightarrow$	$p_h^* = p_T$ $MC(Q^*) = p_T$ *imports*
$p^0 < p_T < p^m \rightarrow$	$p_h^* = p_T$ $Q^* = D_h(p_T)$ *market closed*
$p_T > p^m \rightarrow$	$p_h^* = p^m$ $Q^* = Q^m$ *market closed* *water in tariff*

which maximize the firm's total profit are shown in Figure 2 as functions of the tariff rate τ. In fact, as the following Theorem reveals, the solutions indicated by Figure 2 apply more generally, indeed they exhaust all possibilities:

Theorem 1:

(a) If **CA** and **WMR** and

a1: $p_T < p_1$, then $p_h^* = p_T$ and $Q^* = Q_0$; the firm exports

a2: $p_T > p_1$, then $p_h^* = p_1$ and $Q^* = Q_0$; the firm exports and there is 'water in the tariff' ⟨White [1974]; Fishelson and Hillman [1979]⟩.

(b) If **CD** and **WMR** and

b1: $p_T < p^0$, then $p_h^* = p_T$ and $MC(Q^*) = p_T$; there are imports

b2: $p^0 < p_T < p_0$, then $p_h^* = p_T$ and $Q^* = D_h(p_T)$; the market is closed

b3: $p_0 < p_T < p_1$, then $p_h = p_T$ and $Q^* = Q_0$; the firm exports

b4: $p_T > p_1$, then $p_h^* = p_1$ and $Q^* = Q_0$; the firm exports and there is 'water in the tariff ⟨White [1974]; Fishelson and Hillman [1979]; Rieber [1981]; Bhagwati [1988]⟩

(cd) If **CD** and **WMI** and

cd1: $p_T < p^0$, then $p_h^* = p_T$ and $MC(Q^*) = p_T$; there are imports

cd2: $p^0 < p_T < p^m$, then $p_h^* = p_T$ and $Q^* = D_h(p_T)$; the market is closed

cd3: $p_T > p^m$, then $p_h^* = p^m$ and $Q^* = Q^m$; the market is closed and there is 'water in the tariff' ⟨Fishelson and Hillman [1979]; Bhagwati [1988]⟩.

As this theorem confirms, there are essentially no new cases to consider beyond those thrown up by a diagrammatic study using a linear demand schedule.[4] (However, the linear demand case (d), typically overlooked in diagrammatic analyzes, is revealed by our analytics). Case (a) of the Theorem confirms that a firm with cost advantage relative to its foreign competitors will supply the entire domestic market and export part of its output under free trade; with gradually rising protection, domestic sales are contracted and exports expanded maintaining the same volume of output. In

4. Recall that we have assumed an increasing marginal cost curve and that we ruled out *ab initio* the pathological case of an increasing marginal revenue curve MR_h; the analysis upon which Theorem 1 is based is otherwise quite general.

cases (b) and (cd) of the Theorem, the firm has cost disadvantage relative to its foreign competitors and so does not supply the entire domestic market under free trade. Then, there is a range of tariff protection under which the market is closed; the domestic monopoly exactly serves the domestic market: there are neither imports nor exports.[5] The introduction of protection, from the free-trade position, first causes the volume of the domestic monopoly's output to rise whilst domestic sales and imports fall, and then causes a fall in domestic output, leading to autarky or even to a reversal from imports to exports.

Redundant protection, *i.e.* unused tariff or "water in the tariff" in Corden's [1974, p. 206] terms, can occur in all combinations of the market conditions {**CA, CD**} and {**WMR, WMI**}. Our condition for this in cases (a) and (b) (*viz.* $p_T > p_1$) is equivalent to Fishelson and Hillman's [1979] general necessary condition (that $MR_h(D_h(p_T)) > p_w$), but is stronger than their necessary condition in cases (cd) (when we require $p_T > p^m > p_1$).

IV. The Firm's Decision Problem: The Uncertainty Case

We have assumed heretofore that the monopolist knows the world price p_w with certainty. With a fixed exchange rate of 1, it did not matter whether we denominated values in the domestic currency (call it the pound sterling) or in the world currency (call it the dollar). But with uncertainty in the exchange rate between the currencies, the sterling equivalent of the known world price $\$p_w$ becomes stochastic. The monopolist clearly faces a complex problem in this scenario. In particular, it makes a difference whether he acts to maximizes *expected sterling profits* or *expected dollar profits*.

Consider specifically, as motivation for the modeling, the case in which a state monopoly has been set up, with protection, in order to foster domestic output of a tradeable good; or the parallel case of a branch of a multinational company, licensed and protected by the domestic government for similar reasons.[6] It seems quite natural to model this firm as facing production costs

5. In this range the firm's optimization problem must be solved using non-differentiable techniques: see on.

6. As examples, consider: the production primarily for exports of petrochemicals by petrol exporting countries (*e.g.* Saudi Arabia); or of manufactures by the ex-"social-

denominated in dollars rather than pounds,[7] and as maximizing dollar profits.[8]

Let the domestic demand curve be $D_h(p_h)$ where p_h is price in pounds, and let the world price be p_w in dollars. An *ad valorem* tariff is imposed at rate τ on imports, as before raising their dollar price from p_w to $p_T = p_w(1+\tau)$; we assume $0 \leq \tau < 1$. Let the exchange rate be £1 = \$$\theta$ where θ is uncertain. The firm must choose an output level Q and set a domestic price p_h before the exchange rate uncertainty is realized; sales occur only *after* θ becomes known. If $p_h \leq p_T/\theta$ the firm sells $min\ \{D_h(p_h), Q\}$ domestically and if $p_h > p_T/\theta$ it sells nothing domestically. It can sell any amount of its output abroad at a dollar price of p_w. If the probability density function for θ is $f(\theta)$ and the distribution function and first moment function are $F(\theta) = \int_0^\theta f(t)dt$ and $F_1(\theta) = \int_0^\theta tf(t)dt$ respectively, then expected dollar profits II are given by:

$$\text{II} = p_w Q - C(Q) + ([p_T F_1(q)/q] - p_w F(q)).min\ \{D_h(p_T/q), Q\} \qquad (2)$$

where $p_h = p_T/q$ and, now, $C(Q)$ is the total production cost *in dollars*. The firm's decision problem is to choose q and Q to maximize (2). The objective function in the certainty case, given by (1), was similar in form to this, but in (2) there is new complexity due to the introduction of uncertainty, through the terms $F_1(q)$ and $F(q)$.

In order to achieve explicit results given this increased complexity, we must necessarily make simplifying assumptions. Retaining generality in the distribution of the exchange rate θ for the moment, we make two simplifica-

ist" countries (*e.g.* steel from Hungary); or the assembly of cars in the developing world, initially for domestic consumption only, by the licensing and protection of a branch of a multinational company (*e.g.* of Peugeot in Nigeria in the middle 1970s).

7. The major costs would be in \$ – *e.g.* as for a "completely knocked down (CKD) car industry" = "screwdriver operations" of cars assembled from imported parts – which typically has domestic value added content of labour input less than 0.30, paid in local currency.

8. The alternative is to maximize pound profits. The distinction seems not to have been discussed in the literature to date. We go for dollars, which is consonant with the scenario in which the operator is a branch of a foreign-owned multinational firm in a small country which has no influence on the production and cost decisions of the firm; it also accords with the case of a developing country protecting an infant state monopoly and wishing to use the profit generated as a source of foreign exchange for development purchases. Krugman [1989] discusses a range of problems raised for firms as decision-makers by the presence of exchange rate volatility.

tions in order to ensure tractability and also to permit interesting comparative statics later on. The first simplification, which is at variance with what has gone before, is that ***marginal cost is constant and equal to the world price:***

$$MC = C'(Q) = p_w \tag{3}$$

The full significance of this assumption will emerge shortly.[9] The second simplifying assumption is that the firm faces a constant price elasticity of demand, denoted $e > 1$. Then the optimum is to locate on the domestic demand curve,[10] and to choose:

$$q_1 = argmax\ [(1+\tau)F_1(q) - qF(q)]\ .\ q^{e-1} \tag{4}$$

The optimal domestic price is $p_h = p_T/q_1$ and the probability of tariff redundancy, call it Φ, is then given simply by:

$$\Phi = Prob\ \{p_h = p_T/q_1 < p_T/\theta\} = F(q_1) \tag{5}$$

It is difficult to predict the properties of p_h and Φ independently of the probability distribution of the exchange rate θ. However, if $min_£\, p_T$ and $max_£\, p_T$ are the bounds for the uncertain domestic price of imports, p_T/θ,[11] then the following general results do obtain:

Theorem 2:

Suppose *w.l.o.g* that the mean exchange rate is $\bar{\theta} = 1$. In the absence of a tariff the firm sets $p_h = max_£ p_w$. In the presence of a tariff $min\ \{min_£\, p_T,\ ep_w/(e-1)\} \le p_h < max_£\, p_T$ and the probability Φ of tariff redundancy is positive.

First, if the domestic monopoly does not enjoy tariff protection, it will set its price p_h equal to the highest possible price of imported goods – and thereby ensure neither sales nor profits. This apparently perverse result can be understood as follows. Under the assumption of a constant marginal cost \$$p_w$, which is also the dollar equivalent of the price at which imports can be

9. This assumption can be justified, *e.g.* if the domestic industry is a branch of a multinational firm: see note 8.

10. Given q, it is maximized by any $Q \ge D_h(p_T/q)$. But if the firm chose to locate above the domestic demand curve and to export, this would have no effect on expected profits because the constant unit production cost equals the world price.

11. If $f(\theta)$ has support $[a, b]$, then $min_£\, p_T = p_T/b$ and $max_£\, p_T = p_T/a$.

bought by domestic consumers, it is as if the firm buys units of the good *from the importer*, at $\$p_w$ each, and can only sell them in a market *in which the importer also trades*. In order to make a profit, the firm would have to sell the good at a price exceeding the sterling equivalent of $\$p_w$ – but then no-one would buy, as imports would be cheaper – whilst, if the firm chose a price which, after exchange rate uncertainty were resolved, undercut the sterling price of imports, it would perforce make a dollar loss. Regarding this as a 'dollar market', in which the firm has to set an uncertain price, but the importer does not, the firm's strategy must be to ensure that its price is undercut with probability one in order to avoid expected losses: hence it must set $p_h = max_{£}\, p_w$. It is precisely our simplifying assumption $MC = \$p_w$ in (3) which ensures that tariff protection is necessary in order to foster domestic output of a tradeable good; shortly we shall vary this assumption.

In the presence of a tariff the firm will set a price below the highest possible import price – thereby entering the market with a non-zero probability of both sales and profits. It is not possible to vouch that the firm will set a price within the range of the uncertain price of imports (see on to Theorem 3). But this will happen if the tariff rate is low, or if it is high and the price elasticity is low: the condition $min_{£}\, p_T \leq ep_w(e-1)$ is met if either $1+\tau \leq \theta^{\max}$ or, with $1+\tau > \theta^{\max}$, if $e < (1+\tau)/[1+\tau-\theta^{\max}]$ where $\theta^{\max}$ is the highest possible exchange rate. If these conditions both fail we know in general only that $p_h \geq ep_w/(e-1)$, which can be written $MR_h \geq p_w$; marginal revenue (in pounds) wi11 not fall below expected marginal cost.

Further general results appear inaccessible, but a full solution to the firm's problem can be obtained in the case that the exchange rate θ is uniformly distributed:

Theorem 3:

Suppose that θ is uniformly distributed on the interval $[1-c, 1+c]$, and let $\tau_0 = 2c(ec+1)/(c^2+2ec+1)$ and $\tau_1 = (ec+1)/(e-1) > \tau_0$. Then:

$$0 < \tau < \tau_0 \;\rightarrow\; min_{£}\, p_T < p_h < max_{£}\, p_T \;\;\&\;\; 0 < \Phi < 1$$
$$\tau_0 \leq \tau \leq \tau_1 \;\rightarrow\; p_h = min_{£}\, p_T \;\;\&\;\; \Phi = 1$$
$$\tau > \tau_1 \;\rightarrow\; p_h = ep_w/(e-1) < min_{£}\, p_T \;\;\&\;\; \Phi = 1$$

The comparative statics for the case $0 < \tau < \tau_0$ are as follows:

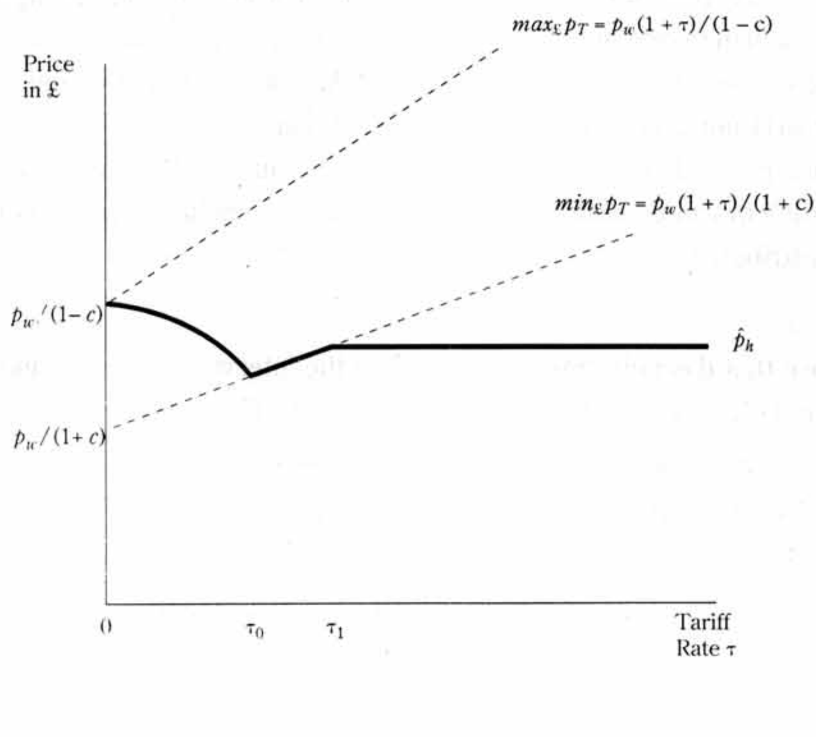
$$\partial p_h/\partial \tau < 0,\ \partial p_h/\partial e < 0,\ \partial p_h/\partial c > 0$$
$$\partial \Phi/\partial \tau > 0,\ \partial \Phi/\partial e > 0,\ \partial \Phi/\partial c < 0$$

These results show *inter alia* that, as protection is increased, then – at least in the case of a uniformly distributed exchange rate – the price p_h initially falls; for $\tau < \tau_0$ it lies strictly between the minimum and maximum possible prices of imports; and the probability of tariff redundancy is positive and rising. When the tariff passes the critical level τ_0, the firm sets p_h equal to the lowest possible sterling price of imports, and tariff redundancy is certain. This price $p_h = min_£\, p_T$ of course now *rises* as τ continues to increase beyond τ_0, finally becoming constant at $p_h = ep_w/(e-1)$ for all $\tau \geq \tau_1$. For $\tau > \tau_1$, this chosen price lies outside the range of possible prices of imported goods, and in fact is where $MR_h = p_w$. See Figure 3.

The more elastic is domestic demand, the lower the price p_h for any given tariff and the higher the probability of tariff redundancy; and the more uncertain is the exchange rate θ, the higher the price p_h for any given tariff and the less probable is tariff redundancy.

Figure 3

The Optimal Price as a Function of the Tariff Rate τ



These new insights are of course particular to the model assumptions we have made – principal among which are that the exchange rate is uniformly distributed and that the firm's marginal production cost is constant and equal to the world price $\$p_w$. We relax this latter assumption shortly, but first we take the present case a little further, and analyze the comparative statics of the government's expected revenue from the tariff, call this T:

$$T = \int_0^\infty T(\tau|\theta)f(\theta)d\theta \tag{6}$$

where $T(\tau \mid \theta)$ is actual revenue conditional upon the realization of θ:

$$T(\tau|\theta) = \begin{cases} p_w \cdot \tau \cdot D_h(p_T/\theta) \cdot & p_h > p_T/\theta \\ 0 & p_h \leq p_T/\theta \end{cases} \tag{7}$$

It is clear that $T > 0$ for $0 < \tau < \tau_0$ and $T = 0$ when $\tau = \tau_0$. The following comparative static effects can now be stated:

Theorem 4:

For tariff values τ such that $1/(e-1) < \tau < \tau_0$, $\partial T/\partial\tau < 0$. For uncertainty parameter values c such that $c > 1/e$, $\partial T/\partial c > 0$.

These effects cannot be signed unambiguously; they depend in general on the configuration of the triple (τ, e, c). The straightforward results cited here, however, are the more encompassing the more elastic is domestic demand. Then, for high demand elasticity, raising the tariff rate for sure brings in less revenue;[12] and the more uncertain is the exchange rate in these circumstances, the higher is expected tariff revenue.

Finally, we consider the effect upon our results of varying the marginal cost assumption $MC = p_w$, which *inter alia* implies that the domestic firm will not export (since exporting cannot be profitable). Suppose instead that:

$$MC = \mu = p_w/(1+v) \tag{8}$$

where $v>0$. In this case, exports become profitable. In fact the domestic firm could make unlimited profits (of $v\mu$ per unit) by producing solely for export. But this is implausible, at least in the cited case of an infant industry set up as a branch of a multinational company. If the goods are effectively

12. This property is not implied by an increasing probability of tariff redundancy *per se*.

supplied by the parent company at cost, and this cost is less than p_w per unit, the parent would clearly forbid the infant from entering the world market;[13] and in that case, very little changes in our analysis. Expected dollar profit for the domestic firm becomes:

$$\Pi = ([p_T F_1(q)/q] - \mu F(q)) \cdot D_h(p_T/q) \tag{9}$$

(compare (2)), and this is maximized by the value:

$$q_1 = argmax\ [(1+\sigma)F_1(q) - qF(q)] \cdot q^{e-1} \tag{10}$$

where $(1+\sigma) = (1+\tau)(1+v)$ (compare (4)). In particular, Theorem 3 can be reformulated for the new scenario, with only minor change, as follows:

Theorem 5:

Let $MC = \mu = p_w/(1+v)$ and $1+\sigma = (1+\tau)(1+v)$, with the other conditions of Theorem 3 unchanged. Then:

$$\begin{array}{ll} 1 < \sigma < \tau_0 \rightarrow & min_\varepsilon p_T < p_h < max_\varepsilon p_T \ \ \&\ \ 0 < \Phi < 1 \\ \tau_0 \le \sigma < \tau_1 \rightarrow & p_h = min_\varepsilon p_T \ \ \&\ \ \Phi = 1 \\ \sigma > \tau_1 \quad \rightarrow & p_h = e\mu/(e-1) < min_\varepsilon p_T \ \ \&\ \ \Phi = 1 \end{array}$$

The comparative statics for the case $1 < \tau < \sigma_0$ are exactly as in Theorem 3; additionally, we now have $\partial p_h/\partial v < 0$ and $\partial \Phi/\partial v > 0$.

A small modification is needed in Figure 3. The origin on the horizontal σ-axis corresponds to no tariff and no cost advantage ($\tau = v = 0$) and the point $\sigma = v$ corresponds to no tariff. Hence, if the firm has a cost advantage ($v > 0$), it will enter the market with non=zero probability; and if the cost advantage is sufficient ($v > \tau_0$), tariff redundancy becomes certain ($\Phi = 1 \forall \tau$). The very low unit cost means there is no need for tariff protection in this case. Quite generally, an increasing cost advantage lowers the firm's chosen price and increases the probability of tariff redundancy. The comparative statics of expected tariff revenue are complex in the presence of a cost advantage, and are omitted.

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13. This is the usual case with franchising by a multinational company which restricts the franchisee's sales of the product to an agreed geographical area in which no other franchisee will be appointed.

In order to model the domestic firm as a price and quantity setter, with access to the world market, it would be natural to depart from our chosen scenario, in which the multinational parent company licenses the setting up under protection of an infant domestic industry, and to allow trade flows. For this, we should posit in particular a convex (rather than linear) total cost function $C(Q)$, just as we did in the case of exchange rate certainty. This refinement, which would substantially complicate the mathematics of the uncertainty case, is left to future work.

IV. Concluding Summary

In this paper we have investigated the relationship running from domestic market structure – monopoly – to trade flows under conditions of increasing tariff protection. Our main conclusions for certainty in the exchange rate may be summarized in the following propositions, corresponding to the cases our analysis has thrown up:

(*a*) If the firm has cost advantage relative to its foreign competitors and therefore under free trade it exports part of its output, gradually rising protection contracts domestic sales and expands exports under the same volume of output. This process ceases, and "water" appears, after the tariff protection reaches the critical level at which domestic marginal revenue would equal the world price (and the domestic marginal cost).

(*b*) If the firm has cost disadvantage relative to its foreign competitors, rising tariff protection causes the volume of its output to rise, while domestic sales and imports fall, and later causes the level of output to fall, leading to autarky. Further increases in the tariff cause a reversal from imports to exports. This process ceases, and "water" appears, as in case (*a*).

These two scenarios have been identified by Fishelson and Hillman (1979, page 54), but a new case, revealed here, is:

(*cd*) If, as in (*b*), the firm has cost disadvantage relative to its foreign competitors, and furthermore such high cost relative to the world price, that it could not achieve its overall profit maximum without high protection,[14] then rising tariff protection causes the volume of its output to rise, while domes-

14. Recall the condition **WMR**: $MR(Q_0) < p_w$ identifying the case (*cd*) of Theorem 1.

tic sales and imports fall, and later causes the level of output to fall, leading to autarky. Further increases in the tariff cause domestic sales to fall further, but *do not lead to a reversal from imports to exports*; the critical level for tariff protection in this case is higher, and permits the domestic firm to achieve its overall profit maximum.

The rigorous mathematics supporting all of this, which is contingent only upon increasing marginal cost and decreasing marginal revenue curves, confirms that there are no other cases to consider, beyond these three – all of which can be illustrated by diagrammatics using a linear demand schedule, as in Figures 1 and 2 – but one of which, arising from the scenario shown in panel (*d*) in Figure 1, has apparently been overlooked in the literature to date. This case was *revealed* by the rigorous approach adopted here, necessitating the application of both differentiable and non-differentiable methods of Lagrangean analysis. The non-differentiable analytics, required precisely to deal with the autarky case, may be of independent interest.

In the case of exchange rate uncertainty which we have considered, involving some additional model restrictions, the characteristics of the domestic firm's pricing policy have been ascertained, and it has been shown too that "water in the tariff" can occur (and does occur with increasing probability as the tariff is increased): this essential feature of the certainty case is maintained. Moreover, we have been able to analyze the effects of increasing exchange rate uncertainty, both upon the pricing policy and upon the government's expected tariff revenue, and these insights are new.

Further work is clearly needed to weaken the simplifying assumptions driving these results. Most importantly, the assumption of (short-run) constant returns to scale in domestic production needs to be relaxed; and the question of internal economies of scale and dynamics could be addressed. But the assumptions we have maintained seem quite natural for at least one politically and economically relevant scenario, that in which the multinational parent company licenses the setting up under protection of an infant domestic industry, and our specific results should be of interest on this account.

Perhaps the most important future step would be to generalize the analysis to admit a competitive fringe in the domestic country: such a model could well describe the situation faced by a major producer in the European

Union *vis-a-vis* the rest of the world. Other concerns for the future, rather more ambitious, could be to replace the single price-setting monopolist of our model by two firms in an oligopoly, each dumping in the other's market, and to introduce monopsonistic or oligopsonistic power in factor markets, all under conditions of exchange rate uncertainty.

Appendix

From the definitions, $MC(Q^0) = p^0$, $D_h(p^0) = Q^0$ and $MR_h(D_h(p_1)) = p_w$. Four simple results which will be useful for the analysis are as follows:

P1: for all p, $MR_h(D_h(p)) = p + D_h(p)/Dh'(p) < p$
P2: $p > \{<\}\ p_1 \Leftrightarrow MR_h(D_h(p)) > \{<\}\ p_w$
P3: $p > \{<\}\ p^0 \Leftrightarrow MC(D_h(p)) < \{>\}\ p$
P4: $p_w < p_1$.

These derive readily from the model assumptions (including upward sloping MC curve and downward sloping MR curve).

(a) The Proof of Lemma 1

Since MC is increasing, **CA** implies $MC(D_h(p_w)) < p_w$ and **CD** implies $MC(D_h(p_w)) > p_w$. (1) follows from P3. Since MR_h is decreasing, **CA** implies $MR_h(Q_0) < MR_h(D_h(p_w))$. By P1, $MR_h(D_h(p_w)) < p_w$. Combining these, **CA** implies **WMR**, which is (2). For (3), note that $MR_h(D_h(p_1)) = p_w$ and that **WMR** implies $MR_h(Q_0) < p_w$. Thus **WMR** implies $D_h(p_1) < Q_0$, or $MC(D_h(p_1)) < MC(Q_0) = p_w$. Now $p_w < p_1$ from P4. Hence **WMR** implies $MC(D_h(p_1)) < p_1$. (3) follows from P3.

(b) The Proof of Lemma 2

We have $MC(Q^m) = MR_h(Q^m)$ and $Q^m = D_h(p^m)$. Now $MC(Q^m) < p^m$ by P1, and so $p^m > p^0$ by P3. If **CD** then $D_h(p_w) > Q_0 = D_h(p_0)$ *i.e.* $p_0 > p_w = MC(Q_0)$, whence $p_0 > p^0$ by P3. If **WMR** then $p_0 < p_1$ by P2.

(c) The Proof of Theorem 1

Let $m(p, Q) = min\{D_h(p), Q\}$ and let $Q_h = m(p_h, Q)$. Total revenue is $TR_h + TR_w$ where $TR_h = 0$ and $TR_w = p_wQ$ if $p_h > p_T$ whilst $TR_h = p_hQ_h$ and $TR_w =$

$p_w(Q - Q_h)$ if $p_h \leq p_T$. It follows that total profit $\pi(p_h, Q) = TR_h + TR_w - C(Q)$ is maximized by choice of p_h in the range $p_w \leq p_h \leq p_T$. The Lagrangean function for the firm's decision problem is:

$$L(p_h, Q, \underline{\lambda}) = p_w Q - C(Q) + (p_h - p_w)\cdot m(P_h, Q) - \lambda_1\cdot[p_h - p_T] + \lambda_2\cdot[p_h - p_w]$$

The function $m(p, Q)$, which is not differentiable at $D_h(p) = Q$, has partial derivatives if $D_h(p) \neq Q$ as follows:

$$m_1 = 0 \text{ if } D_h(p) > Q, = D_h'(p) \text{ if } D_h(p) < Q$$
$$m_2 = 1 \text{ if } D_h(p) > Q, = 0 \text{ if } D_h(p) < Q$$

With appropriate concavity assumptions, there exist $\lambda_1^* \geq 0$ and $\lambda_2^* \geq 0$ such that the solution values $p_h = p_h^*$ and $Q = Q^*$ for the problem satisfy:

(i) $L(p_h, Q^*, \underline{\lambda}^*) \geq L(p_h, Q, \underline{\lambda}^*)$ for all $p_h \leq p_T$ and all Q

(ii) $\lambda_1^*\cdot[p_h^* - p_T] = \lambda_2^*\cdot[p_h^* - p_w] = 0$

⟨see Lambert [1985], pages 117-119⟩. Two cases must be distinguished:

(I) $D_h(p_h^*) \neq Q^*$ (II) $D_h(p_h^*) = Q^*$

In case (I), the FOCs $\partial L/\partial p_h = \partial L/\partial Q = 0$ hold at the optimum. In case (II), the optimum occurs at the point of non-differentiability of the Lagrangean function, and different solution methods must be engaged.

<u>Case (I)</u>: The conditions to be satisfied at the optimum include:

$$\partial L/\partial p_h = 0: m + (p_h - p_w)\cdot m_1 = \lambda_1 - \lambda_2 \quad \text{(A1)}$$
$$\partial L/\partial Q = 0: p_w - MC(Q) + (p_h - p_w)\cdot m_2 = 0 \quad \text{(A2)}$$
$$\lambda_1\cdot[p_h - p_T] = \lambda_2\cdot[p_h - p_w] = 0 \quad \text{(A3)}$$

Note first that $\lambda_2^* = 0$: if $\lambda_2^* > 0$ then, by (A3), $p_h^* = p_w$ and $\lambda_1^* = 0$, in which case $m < 0$ from (A1), a contradiction. We consider three sub-cases:

$$\lambda_1^* = 0; \quad \lambda_1^* > 0 \ \& \ Q^* > D_h(p_h^*); \quad \lambda_1^* > 0 \ \& \ Q^* < D_h(p_h^*)$$

in turn (recalling that by assumption $D_h(p_h^*) \neq Q^*$).

Suppose first $\lambda_1^* = 0$. As $m > 0$, by (A1) $p_h^* > p_w$ and $m_1(p_h^*, Q^*) < 0$. Thus $D_h(p_h^*) < Q^*$ and $m_2(p_h^*, Q^*) = 0$, so by (A2), $\mathbf{Q}^* = \mathbf{Q}_0$. From (A1) and P1, $p_w = p_h^* + D_h(p_h^*)/D_h'(p_h^*) = MR_h(D_h(p_h^*))$. Thus $p_h^* = p_1$ by P2, yielding ***a2*** and ***b4***.

Now suppose $\lambda_1^* > 0$ and $Q^* > D_h(p_h^*)$. From (A3), $\boldsymbol{p_h^*} = \boldsymbol{p_T}$. Now substitute $m_1 = D_h'(p_T) < 0$ and $m_2 = 0$ in (A2) and (A1): the respective conclusions are $\boldsymbol{Q^*} = \boldsymbol{Q}_0$ and $p_w = p_h^* + m/m_1 - \lambda_1^*/m_1 > p_h^* + m/m_1 = MR_h(D_h(p_h^*))$ (using P1). Hence by P2, $\boldsymbol{p_h^*} < \boldsymbol{p}_1$. Under the initial assumptions for this case, $D_h(p_0) = Q_0 = Q^* > D_h(p_h^*)$, whence $\boldsymbol{p_h^*} > \boldsymbol{p}_0$, yielding ***al*** and ***b3***.

Finally, suppose $\lambda_1^* > 0$ and $Q^* < D_h(p_h^*)$. From(A3), $\boldsymbol{p_h^*} = \boldsymbol{p_T}$. Now substitute $m_2 = 1$ in (A2), to obtain $\boldsymbol{MC(Q^*)} = \boldsymbol{p_T}$. Two inequalities follow: $D_h(p_w) > D_h(p_T) > Q^* > Q_0$, so that the solution pertains to the **CD** case; and $MC(D_h(p_h^*)) > MC(Q^*) = p_h^*$, whence by P3 $\boldsymbol{p_h^*} < \boldsymbol{p}^0$, yielding ***b1*** and ***cd1***.

<u>Case (II)</u>: In this case $D_h(p_h^*) = Q^*$ and the Lagrangean is not differentiable at this point. We know, however, that:

$$p_h^* Q^* - C(Q^*) \geq p_w Q - C(Q) + (p - p_w)\cdot m(p, Q) - \lambda_1^* \cdot [p - p_T] + \lambda_2^* \cdot [p_h - p_w] \quad \text{(A4)}$$

for all Q and for all $p \in [p_w, p_T]$, and also that:

$$\lambda_1^*[p_h^* - p_T] = \lambda_2^*[p_h^* - p_w] = 0. \qquad \text{(A5)}$$

It is immediate that $p_h^* > p_w$; otherwise, since $m > 0$, by (A4) $p_w Q^* - C(Q^*) > p_w Q - C(Q)$ for all Q, a contradiction. Let $\pi(Q)$ be the profit function:

$$\pi(Q) = D_h^{-1}(Q)\cdot Q - C(Q) \qquad \text{(A6)}$$

Since $\lambda_1^* \geq 0$, $i = 1, 2$, (A4) implies the weaker condition:

$$\pi(Q^*) \geq H(p, Q) \text{ for all } Q \text{ and all } p \in [p_w, p_T] \qquad \text{(A4a)}$$

where:

$$H(p, Q) = \begin{cases} p_w Q - C(Q) + (p - p_w)\cdot D_h(p) & D_h(p) < Q \\ pQ - C(Q) & D_h(p) \geq Q \end{cases} \qquad \text{(A7)}$$

Set $p = D_h^{-1}(Q)$ in (A4a), using (A7):

$$\pi(Q^*) \geq \pi(Q) \text{ for all } Q \in [D_h(p_T), D_h(p_w)] \qquad \text{(A8)}$$

Suppose first that $\boldsymbol{p^m} \in [\boldsymbol{p_w}, \boldsymbol{p_T}]$. By (A8), $\pi(Q^*) \geq \pi(Q^m) = \pi_{\max}$, so $\boldsymbol{Q^*} = \boldsymbol{Q^m}$ and $\boldsymbol{p_h^*} = \boldsymbol{p^m}$. Now set $p = p^m$ and $Q > Q^m$ in (4a) and use (A7):

$$\pi(Q^m) \geq p_w Q - C(Q) + (p^m - p_w).\ Q^m \text{ for all } Q > Q^m \tag{A9}$$

Substituting $\pi(Q^m) = p^m Q^m - C(Q^m)$ and rearranging, this yields:

$$C(Q) - C(Q^m) \geq p_w.\ (Q - Q^m) \text{ for all } Q > Q^m \tag{A10}$$

which, letting $Q \to Q^m$ and using P1 implies $p^m > MR_h(Q^m) = MC(Q^m) \geq p_w$. It follows that $Q_0 \leq Q^m = D_h(p^m) < D_h(p_w)$ and that $MR_h(Q_0) \geq MR_h(Q^m) \geq p_w$, *i.e.* that **CD** and **WMI** both hold: this is case ***cd3***.

Next suppose $p^m < p_w$. Then $D_h(p_w) < D_h(p_m) = Q^m$, so the (concave) function $\pi(Q)$ is increasing for $Q \in [D_h(p_T), D_h(p_w)]$. Thus $Q^* \geq D_h(p_w)$ and $p_h^* \leq p_w$ from (A4a). But already $p_h > p_w$, so this case cannot arise.

Finally suppose $p^m > p_T$. Then $D_h(p_T) > D_h(p^m) = Q^m$, so $\pi(Q)$ is decreasing for $Q \geq D_h(p_T)$. From (A8), $Q^* \leq D_h(p_T)$ and since $Q^* = D_h(p_h^*)$ and $p_h^* \leq p_T$, $\boldsymbol{p_h^*} = \boldsymbol{P_T}$ and $\boldsymbol{Q^*} = \boldsymbol{D_h(p_T)}$

Now put $p = p_T$ and $Q < Q^* = D_h(p_T)$ in (4a):

$$p_T Q^* - C(Q^*) \geq p_T Q - C(Q) \text{ for all } Q < Q^* \tag{A11}$$

It follows that the function $p_T Q - C(Q)$ is increasing for $Q < Q^*$ and in particular that $MC(Q^*) \leq p_T$. By P3, $p_T \geq p^0$. Put $p = p_T$ and $Q = Q_0$ in (4):

$$p_T Q^* - C(Q^*) \geq p_w Q_0 - C(Q_0) + p_w \tau \cdot min\ \{Q^*, Q_0\} \tag{A12}$$

If $Q^* < Q_0$ this says $p_w Q^* - C(Q^*) \geq p_w Q_0 - C(Q_0)$, a contradiction since the function $p_w Q - C(Q)$ takes its maximum value at $Q = Q_0$. Hence $Q^* \geq Q_0$. Now $D_h(p_w) > D_h(p_T) = Q^* \geq Q_0$, whence **CD** holds and $p_T \leq p_0$. In view of what has gone before, the feasible range for p_T in this case is $p_T \in [p^0, min\{p^m, p_0\}]$. But:

$$p^m > \{<\}\ p_0 \Leftrightarrow Q^m > \{<\}\ Q_0 \Leftrightarrow MR_h(Q^m) = MC(Q^m) > \{<\}\ MC(Q_0) = p_w.$$

Hence $\boldsymbol{p_T} \in [\boldsymbol{p^0}, \boldsymbol{p_0}]$ if **WMR** (case ***b2***) and $\boldsymbol{p_T} \in [\boldsymbol{p^0}, \boldsymbol{p^m}]$ if **WMI** (case ***cd2***).

(d) The Proof of Theorems 2-5

Letting:

$$S = S(p_h, Q|\theta) = \begin{cases} min\ \{D_h(p_h), Q\} & p_h \leq p_T/\theta \\ 0 & p_h > p_T/\theta, \end{cases}$$

the firm's profit in dollars is $\pi(p_h, Q|\theta) = \theta p_h \cdot S + p_w \cdot (Q-S) - C(Q)$, whence expected dollar profit is:

$$\Pi = E_\theta\, \pi(p_h, Q|\theta) = \int_0^\infty (\theta p_h - p_w) \cdot S(p_h, Q|\theta) \cdot f(\theta) d\theta + p_w Q - C(Q)$$

$$= \int_0^q ([\theta p_T/q] - p_w) \cdot \min\{D_h(p_T/q), Q\}, f(\theta) d\theta + p_w Q - C(Q)$$

Given $MC = p_w$, the solution is to set $q_1 = argmax\ ([p_T F_1(q)/q] - p_w F(q))$. $D_h(p_T/q)$, which simplifies to (D) in the text assuming constant elasticity of demand e. Now define:

$$H(q) = [(1+\tau)F_1(q) - qF(q)] \cdot q^{e-1} \tag{A13}$$

so that q_1 maximizes $H(q)$. Now suppose θ has support $[a, b]$ and mean 1. Note first that since $F_1(q) \le qF(q)\ \forall q$, $\tau = 0 \Rightarrow q_1 = a\ \&\ F(q_1) = 0$. If $\tau > 0$ then using $aF(q) \le F_1(q) \le bF(q)\ \forall q$, we have $[(1+\tau)a - q] \cdot F(q) \le H(q)/q^{e-1} \le [(1+\tau)b - q] \cdot F(q)\ \forall q$, whence $\exists x$: $a < x \le (1+\tau)a\ \&\ H'(q) > 0\ \forall q \in (a, x]$. Now let $\kappa = (1+\tau)(e-1)$, so that:

$$H'(q) = [\kappa F_1(q) - eqF(q) + \tau q^2 f(q)] \cdot q^{e-2} \tag{A14}$$

Since $f(q) = 0\ \forall q > b\ \&\ F(q) = F_1(q) = 1\ \forall q \ge b$, we have $H'(q) = [\kappa - eq] \cdot q^{e-2}\ \forall q > b$, whence if $q > b$ and also $q > \kappa/e$, $H'(q) < 0$. Since $H'(q_1) = 0$, we can conclude that $x < q_1 \le \max\{b, \kappa/e\}$. Further general results are hard to come by,[15] but if θ is uniformly distributed on the interval $[1-c, 1+c]$ then, for $1-c \le q \le 1+c$, we have $f(\theta) = 1/2c$, $F(\theta) = [\theta + c - 1]/2c$ and $F_1(\theta) = [\theta^2 - (1-c)^2]/4c$, and so from (A14):

$$4c \cdot H'(q)/q^{e-2} = -(1-\tau)(e+1)q^2 + 2e(1-c)q - (1+\tau)(e-1)(1-c)^2 \tag{A15}$$

Denoting the quadratic on the *r.h.s.* of (A15) by $J(q)$, $J(1-c) = 2\tau(1-c)^2 \ge 0$ and $J(1+c) = 2(c^2+2ec+1) \cdot (\tau - \tau_0)$ where τ_0 is as defined in Theorem 3. If $\tau \ge \tau_0$, $H(q)$ is strictly increasing on $[1-c, 1+c]$ whilst if $0 < \tau < \tau_0$, $H(q)$ has a unique maximum on $(1-c, 1+c)$. The condition $\kappa/e > b$ above translates in this scenario to $\tau > \tau_1$, with $\tau_1 > \tau_0$ as in Theorem 3, and ensures $q_1 = \kappa/e > b = 1+c$ (and $F(q_1) = 1$). If $\kappa/e \le b$, so that $\tau \le \tau_1$ and on general grounds $q_1 \le b$,

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15. Note, in particular, that since $H'(b) = \lim_{q \downarrow b} H'(q) + \tau f(b) \cdot b^e$, the function $H(q)$, though continuous, has discontinuous derivative at $q = b$ if $f(b) \ne 0$.

it is clear that if $\tau \geq \tau_0$ then $q_1 = b = 1+c$ (and $F(q_1) = 1$), whilst if $0 < \tau < \tau_0$ then q_1 is the unique root of the quadratic equation $J(q) = 0$ which lies in $(1-c, 1+c)$, *viz*:

$$q_1 = (1-c)(e+R)/[(1-\tau)(e+1)] \tag{A16}$$

where $R = (\tau^2e^2 + 1 - \tau^2)^{1/2} > 1$, and $0 < F(q_1) < 1$. The comparative statics for this case are straightforward: for changes in τ and e, q_1 and $\Phi = F(q_1)$ move in the same direction, whilst $\partial\Phi/\partial c < 0$ because $F(q_1) = [q_1 - (1-c)]/2c$ can be written in the form $[(1-c)/2c]$. Δ where $\Delta > 0$ is independent of c; and, from (A16):

$$p_h = p_T/q_1 = p_w(e+1)(1-\tau^2)/[(1-c)(e+R)] \tag{A17}$$

from which the signs of $\partial p_h/\partial\tau$, $\partial p_h/\partial e$ and $\partial p_h/\partial c$ follow, using $\partial R/\partial e = \tau(e^2-1)/R$ and $\partial R/\partial e = e\tau^2/R$.

Letting $D_h(p) = Ap^{-e}$, expected tariff revenue is $T = Ap_w\tau\cdot\int_{q1}^{1+c} \theta^e d\theta/[2cp_T^e] = [p_w^{1-e}A/2c(e+1)]\cdot[\tau/(1+\tau)^e]\cdot[(1+c)^{e+1} - q_1^{e+1}]$ which can be written as:

$$2(e+1)T/[p_w^{1-e}A] = [(1+c)^{e+1}\,\tau/(1+\tau)^e - \tau(1+\tau)\cdot(p_w/p_h)^{e+1}]/c \tag{A18}$$

As already ascertained, the second term on the *r.h.s.* increases with τ; the derivative *w.r.t.* τ of the first term is $(1+c)([1-(e-1)\tau]/(1+\tau)$; hence $\partial_T/\partial\tau < 0$ if $\tau > 1/(e-1)$. Rewriting (A18) as:

$$2(e+1)(1+\tau)^eT/[p_w^{1-e}A\tau] = [(1+c)^{e+1} - (1-c)^{e+1}\cdot G(\tau,e)]/c \tag{A19}$$

where $G(\tau, e) = [q1/(1-c)]^{e+1}$ is independent of c, from (A16), and differentiating with respect to c, we find:

$$sign\ \partial T/\partial c = sign\ [(ce-1)(1+c)^e + (ce+1)(1-c)^e.G(\tau, e)] \tag{A20}$$

so that $\partial T/\partial c > 0$ if $c > 1/e$.

When $MC = p_w/(1+v) < p_w$, the effect is to replace τ in (A13) and all subsequent mathematics by σ, up to *but excluding* (A17), since $p_h \neq p_w(1+\sigma)/q_1$. However, we can write

$$p_h = [p_w(1+\sigma)/q_1]/(1+v) \tag{A21}$$

and so the *r.h.s.* in (A17), after replacing τ by σ, should be divided by $(1+v)$. Thus the comparative statics of p_h *w.r.t.* e and c are not affected, whilst the previous arguments now show that $\partial[(1+v)p_h]/\partial\sigma < 0$. It is immediate from this that both $\partial p_h/\partial\tau < 0$ and $\partial p_h/\partial v < 0$.

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