

## Foreign Capital Inflow with Public Input Production

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### Abstract

*This paper develops a general equilibrium trade model of a less developed country, facing imperfect international capital mobility, and producing a public input. Within this framework, the paper examines the welfare effects of an inflow of foreign capital when the government finances the provision of the public input either (i) by taxing the return to foreign capital, or (ii) by imposing a tariff on the imported good. Using the gross domestic product (GDP) function with public input production, the paper shows that (i) in the presence of a tariff, the inflow of foreign capital may increase the country's welfare, even if the imported good is capital intensive, and (ii) in the presence of capital taxes, the inflow of foreign capital may decrease the country's welfare. The paper examines also within the two-good, two-factor model the effect of a capital inflow on factors rewards. (JEL: F13, F20)*

### I. Introduction

Most trade theorists assume that governments distribute tariff or factor tax revenue as a lump sum to consumers. But, governments, especially in less developed countries (LDCs), primarily use such revenue to finance the

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provision of public goods or public inputs such as infrastructure and technical training. Some trade studies introduce into their analysis this more realistic assumption for the use of tariff revenue by the governments of LDCs. For example, Feehan [1988] examines the implications of using tariff revenue to finance the provision of a public good, and Abe [1992] considers the welfare effects of tariff reform when tariff revenue finances the provision of a public good. When the government provides a public input, Abe [1990] shows how differences in the level of the public input affect the trade patterns between countries, and Feehan [1992] derives efficiency rules for public input provision when the tariff revenue is the sole revenue source.

These studies, however, ignore international capital mobility, and thus the option to a government to tax foreign capital, and use capital tax revenue to finance the provision of public goods or inputs. LDCs are net capital importers, but, in most cases, the mobility of capital between them and the capital-exporting countries is not perfect. This paper examines the welfare effects of foreign capital inflows when the government finances the provision of the public input either (i) by taxing capital, or (ii) by imposing a tariff on the imported good. In a world without public goods or distortions (*e.g.*, tariffs, factor taxes), an exogenous foreign capital inflow improves the country's welfare, if it decreases its domestic rate of return, and does not affect welfare, if it does not affect its rate of return (*e.g.*, two-good, two-factor Heckscher-Ohlin model). In the presence of a tariff, however, an exogenous inflow of foreign capital reduces the country's welfare, if the imported good is capital intensive (Brecher and Diaz Alejandro [1977]), or if the endogenous capital is specific in the production of the imported good (Brecher and Findlay [1983]), and tariff revenue is distributed to consumers as a lump-sum. In the absence of trade restrictions, and in models where an increase in the supply of capital either reduces or does not affect its domestic rate of return, an inflow of foreign capital always improves welfare, if its rate of return is taxed and the tax revenue is distributed to consumers as a lump-sum. The present analysis shows that when the government uses the revenue to finance the provision of a public input then an inflow of foreign capital changes capital tax or tariff revenue, and changes the supply of the public input that causes factor rewards to change. The change in factor rewards due to the change in the supply of the public input alters some traditional

results. Thus, within this framework, the paper concludes that (i) in the presence of a tariff, the inflow of foreign capital may increase the host-country's welfare, even if the imported good is capital intensive, and (ii) in the presence of capital taxes, the inflow of foreign capital may decrease the host-country's welfare.

In section II, I develop the model and the GDP function with public input production. In sections III and IV, I examine the welfare effects of a foreign capital inflow in the presence of capital taxes and in the presence of a tariff, respectively. In section V, using a two-good, two-factor model, I consider the effects of a foreign capital inflow on welfare and on the rate of return to capital and labor. The final section provides some concluding remarks.

## II. The Model

Consider a small open economy with  $n$  private goods,  $m$  factors of production, and one public good that is used as an input in the production of private goods. All private goods and one factor, capital, are internationally traded. The production function of the  $i$ -th traded good is given by

$$x_i = h^i(g) f^i(v_i^p), \quad (1)$$

where  $g$  is the amount of the public input that the government provides free of charge to the private sector and  $v_i^p$  is the vector of factors available for the production of the  $i$ -th private good. It is assumed that  $f^i$  is homogeneous of degree one and quasi-concave in  $v_i^p$ , so that the public input is of the 'atmosphere' or 'factor-augmenting' variety.<sup>1,2</sup>

The production function for the public input is given by

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1. One complication that arises with these production functions is that the production possibility frontier (PPF) may not be everywhere concave to the origin, and thus has multiple equilibrium or corner solutions. See the work of Tawada and Abe [1984], Okamoto [1985], and Altenburg [1987]. In general, the PPF remains concave to the origin as long as the degree of factor substitutability is sufficient to overcome the tendency towards convexity due to the increasing returns to scale effects from the public input. It is assumed, at least in the neighborhood of equilibrium, that the PPF is strictly concave to the origin.
  2. The gains generated by the provision of factor-augmenting public inputs are cap-

$$g = g(v^g) \quad (2)$$

where  $v^g$  is the vector of factors used in the production of the public input. It is assumed that  $g$  is homogeneous of degree one and quasi-concave in  $v^g$ . All markets of the private goods and factors of production are perfectly competitive.

Let  $R^*(p, g, v^p)$  denote the maximum private production for the vector of goods prices  $p$ , for given level of the public input, and for the vector of factors of production available to the private sector. The partial derivative of the private revenue function with respect to  $p$  (*i.e.*,  $R_p^*$ ) gives the production vector of the private goods, with respect to  $g$  gives the increase in private production due to the increase in the public input (*i.e.*, the value of the private marginal product of the public input), and with respect to  $v^p$  gives the value of marginal products of factors. In equilibrium,

$$\omega = R_v^*(p, g, v^p), \quad (3)$$

where  $\omega$  is the vector of factors prices, and the subscript denotes a partial derivative (*i.e.*,  $R_v^* = (\partial R^* / \partial v^p)$ ).

Let  $E(p, u)$  denote the minimum expenditure needed to achieve utility level  $u$  at prices  $p$ . The derivatives of the expenditure function with respect to prices give the compensated demand functions for the private goods (*i.e.*,  $E_p$ ).

Let  $C^g(\omega)$  denote the unit cost function of public production, which is homogeneous of degree one and concave in  $\omega$ . Using the properties of the unit cost function, the government demand vector for factors of production is given by

$$v^g = C_\omega^g(\omega)g. \quad (4)$$

It is assumed that factors are either employed in the private or public sectors. That is, full employment requires that

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tured entirely by the private factors. Such public inputs include education, training workers and in general anything that improves the productivity of private factors. Other varieties of public inputs are firm-augmenting and semi-public inputs. See Feehan [1989] for a description of the conditions required to achieve economy-wide efficiency with each of these varieties of public inputs.

$$v^p + v^g = v, \quad (5)$$

where  $v$  is the vector of factor supplies in the economy.

To derive the gross domestic product (GDP) function with the production of the public input that contains information on the private and public production, substitute equation (4) into (5), and use (3) to get

$$v^p + C_{\omega}^g(R_v^*(p, g, v^p))g = v. \quad (6)$$

Equation (6) gives  $v^p$  as a function of  $p$ ,  $g$ , and  $v$ . But since by assumption only the supply of capital can change, define  $v^p$  to satisfy (6) as

$$v^p = v^p(p, g, K). \quad (7)$$

Substituting equation (7) into  $R^*(p, g, v^p)$  results in the GDP function with public input production as follows:

$$R(p, g, K) = R^*(p, g, v^p(p, g, K)). \quad (8)$$

The following lemma gives the properties of the GDP function with public input production as follows:

**Lemma 1:** *If  $R_{vv}^* = 0$ , then the  $R(p, g, K)$  function has the following properties:*

$$(i) R_g = R_g^* - C^g(\omega); \quad (ii) R_p = R_p^*; \quad (iii) R_K = R_K^*; \quad (iv) R_{Kg} = R_{Kg}^*; \quad \text{and} \quad (v) R_{pK} = R_{pK}^*.$$

**Proof:** See Appendix A.

The condition that  $R_{vv}^* = 0$  means that factor prices are not affected by a change in factor supplies available to the private sector. For this to occur, the number of goods should be equal to or higher than the number of factors (e.g., Ethier [1984]). Condition (i) states that when  $R_g > 0$ , GDP increases with the increase in  $g$ . Thus, if the provision of the public input could be financed with nondistortionary taxes, then the optimum level of the public input is the one that makes  $R_g = 0$ . That is, the provision of the public input should increase until its unit cost equals the value of its private marginal product (i.e.,  $C^g(\omega) = R_g^*$ ).

### III. Capital Taxes and Foreign Capital Inflow

The capital mobility between most LDCs and the capital-exporting countries is not perfect. Thus, the domestic rate of return to capital differs from its world rate of return. Consider the case of a small exogenous inflow of foreign capital into a LDC. The government of the host-country taxes the rate of return to capital and uses the proceeds to finance the provision of the public input. It is assumed, for simplicity, that all domestically located capital is owned by foreigners.<sup>3</sup> Thus, the government budget constraint requires that tax revenue equals the cost of producing the public good. That is,

$$B_1 = \tau rK - gC^g(\omega) = 0, \quad (9)$$

where  $B_1$  is the net government revenue (*i.e.*, tax revenue minus the cost of the public input),  $r$  is the domestic rate of return to capital, and  $\tau$  is the tax rate on the rate of return to capital. It is also assumed that net payments to foreign capital are included in the foreign country's income.

Under the assumption that the government provides the public input free of charge to the private sector, the country's income expenditure identity is given by<sup>4</sup>

$$E(p, u) = R(p, g, K) - rK + gC^g(\omega). \quad (10)$$

From equation (3),

$$dr = R_{K^g}^* dg. \quad (11)$$

Differentiating equations (9) and (10), using equation (11), and the proper-

3. For section III, the analysis remains the same if instead of *ad-valorem* capital taxes we assume specific capital taxes and only part of the domestic capital is owned by foreigners. For section IV, the analysis remains the same, regardless of whether capital taxes are *ad-valorem* or specific and whether part of or all the domestic capital is owned by foreigners.

4. From the properties of the expenditure and GDP functions, it is known that  $p(E_p - R_p) = E(p, u) - R(p, g, K)$ . Thus, substituting equation (9) into (10) yields  $p(E_p - R_p) + r(1 - \tau)K = 0$ , which is the country's balance of payments equilibrium condition. Thus, instead of using equations (9) and (10) in the analysis, either equation (9) or (10) could be used with the balance of payments equilibrium condition.

ties of the GDP function with public input production, as given by lemma 1, yields<sup>5</sup>

$$du = (R_g^* - KR_{Kg}^* + gC_{\omega}^g R_{vg}^*)dg, \text{ and} \quad (12)$$

$$dB_1 = \tau r dK - (C^g + gC_{\omega}^g R_{vg}^* - \tau KR_{Kg}^*)dg = 0, \quad (13)$$

where by choice of units,  $E_u = 1$ .

Equations (12) and (13) show that an inflow of foreign capital does not affect welfare directly, but only indirectly through the change in the level of public input. Specifically, an inflow of capital increases net tax revenue, increases the provision of the public input which affects national income and thus welfare. Using equation (10), an increase in the level of the public input affects income by (i) changing the production of private goods by  $R_g$  which is positive (negative) if  $R_g^* > (<) C^g$ , (ii) by changing the income of factors derived from the production of the public input by  $C^g + gC_{\omega}^g R_{vg}^*$ , and (iii) by changing the income of foreign capital.<sup>6</sup> The net effect on national income is given by the right-hand-side of equation (12). From equation (4),  $gC_{\omega}^g = v^g$ . Substituting this result into equations (12) and (13) yields

$$\begin{bmatrix} 1 & -(R_g^* - KR_{Kg}^* + v^g R_{vg}^*) \\ 0 & C^g + v^g R_{vg}^* - \tau KR_{Kg}^* \end{bmatrix} \begin{bmatrix} du \\ dg \end{bmatrix} = \begin{bmatrix} 0 \\ \tau r \end{bmatrix} dK. \quad (14)$$

From the system of equations (14), the effect of an exogenous capital inflow on the amount of the public input is given as

$$\Delta_1 (dg / dK) = \tau r, \quad (15)$$

where  $\Delta_1 = C^g + v^g R_{vg}^* - \tau KR_{Kg}^*$ , and if it is positive, then the inflow of foreign capital increases the production of the public input. Equation (13), however, shows that  $\Delta_1$  is positive if  $(\partial B_1 / \partial g) < 0$  (i.e., if the increase in the production of the public input reduces net government revenue). Note that if

5. The notation  $xy$  denotes the inner product of the vectors  $x$  and  $y$ . For example,  $C_{\omega}^g R_{vg}^*$  is the inner product of the vectors  $C_{\omega}^g$  and  $R_{vg}^*$ .

6. When the unit cost of the public input exceeds the value of its marginal product, then a further increase in the public input reduces the production of the private goods (i.e.,  $R_g$  is negative). In this case, the public input is over-produced.



$(\partial B_1 / \partial g) \geq 0$ , then the government could increase the provision of the public input, which increases private production, without reducing the net government revenue. Thus, if  $(\partial B_1 / \partial g) \geq 0$ , the provision of the public input should increase until the increase in the public input reduces net revenue. For this reason, it is assumed that at the initial equilibrium an increase in the public input reduces net revenue. That is,  $\Delta_1$  is positive. Equation (13) also shows that an inflow of foreign capital increases net government tax revenue (i.e.,  $(\partial B_1 / \partial K) = \tau r > 0$ ).

The system of equations (14) also gives the effect of capital inflow on welfare as follows:

$$\Delta_1 (du / dK) = \tau r (R_g^* - KR_{Kg}^* + v^g R_{vg}^*). \quad (16)$$

An inflow of foreign capital that raises the net government tax revenue and increases the production of the public input increases welfare if (i) it reduces the payment to the foreign capital, and (ii) increases the cost of producing the public input. But, since  $v^g$  also includes capital and since  $K - K_g = K^p$ , the sum of  $v^g R_{vg}^*$  and  $(-KR_{Kg}^*)$  is positive if the effect of the increase of the public input on the payments of the other factors (i.e., except capital) employed in the public sector minus the effect on the payments of capital employed in the private sector is positive. A sufficient, but not necessary, condition for the sum of  $(-KR_{Kg}^*)$  and  $v^g R_{vg}^*$  to be positive is that the reward of capital decreases and the reward of the other factors increases with the increase in the production of the public input. If, however, the effect of the increase of the public input on the payments of the other factors employed in the public sector minus the effect on the payments of capital employed in the private sector is negative, then welfare may decrease with the inflow of foreign capital. This occurs, for example, if the public input increases the reward of capital and reduces the rewards of some other factors.

**Proposition 1:** Assume that net government revenue decreases with an increase in the level of the public input. If the effect of the increase in the public input on the payments of the other factors employed in the public sector minus the effect on the payments of capital employed in the private sector is positive (negative), then the foreign capital inflow increases (may decrease) the country's welfare.



#### IV. Tariffs and Foreign Capital Inflow

Many developing countries rely heavily on tariffs as a source of revenue. This revenue is not generally distributed to consumers as a lump-sum, but is used by governments to finance the provision of public inputs. This section considers the welfare effect of an inflow of foreign capital in the presence of a tariff when the government uses the tariff revenue for the provision of a public input. It is assumed that only two goods, one imported and one exported, exist, and that the rate of return to capital is not taxed. Since the model assumes that changes in factor supplies do not affect factors rewards, there are at most only two factors. Thus, it is assumed that in the production of the two goods only labor and capital are used. In this case, the government's budget constraint requires that

$$B_2 = tZ_p(u, q, g, K) - gC^g(\omega) = 0, \quad (17)$$

where  $B_2$  is net government revenue,  $t (= p - q)$  is the tariff rate,  $p$  is the domestic price and  $q$  is the world price of the imported good, and  $Z_p (= E_p - R_p)$  is the amount of the imported good. The country's income expenditure identity remains the same as is given by equation (10).

Differentiating equation (17), and using lemma 1, equation (4), and equation (12) provides the following system of equations

$$\begin{bmatrix} 1 & -(R_g^* - KR_{Kg}^* + v^g R_{vg}^*) \\ tE_{pu} & tZ_{pg} - C^g - v^g R_{vg}^* \end{bmatrix} \begin{bmatrix} du \\ dg \end{bmatrix} = \begin{bmatrix} 0 \\ tR_{pk}^* \end{bmatrix} dK. \quad (18)$$

From equations (18), the effect of an inflow of foreign capital on the amount of the public input is given by

$$\Delta_2 (dg / dK) = tR_{pk}^*, \quad (19)$$

where  $\Delta_2$  is the determinant of the left-hand-side coefficient matrix of equations (18). If  $\Delta_2$  and  $R_{pk}^*$  are both negative, then an inflow of foreign capital increases the provision of the public input. If in the system of equations (12) and (17), we treat  $u$  and  $B_2$  as endogenous and  $g$  and  $K$  as exogenous variables, then  $(dB_2/dK) = -tR_{pk}^*$ , and  $(dB_2/dg) = \Delta_2$ . Thus, the inflow of foreign capital increases net government revenue if the imported good is labor

intensive (i.e.,  $R_{pK}^* < 0$ ).<sup>7</sup> Also,  $\Delta_2$  is negative if the increase in the amount of the public input reduces net government revenue. Using the same reasoning as in the previous section, it is assumed that at the initial equilibrium  $\Delta_2$  is negative. Intuitively, equation (19) implies that if the imported good is capital intensive (i.e.,  $R_{pK}^* > 0$ ), then the inflow of foreign capital increases the production of the imported good, and decreases imports and tariff revenue. The decreased revenue decreases the provision of the public input.

Using equations (18), the welfare effect of an inflow of foreign capital is given by

$$\Delta_2(du/dK) = tR_{pK}^*(R_g^* - KR_{Kg}^* + v^g R_{vg}^*). \quad (20)$$

The terms in parenthesis on the right-hand-side of equation (20) are the same as the terms in parenthesis on the right-hand-side of equation (16). Thus in the presence of a tariff, even if the imported good is capital intensive (i.e.,  $R_{pK}^* > 0$ ), welfare may increase with an inflow of foreign capital if the sum of the terms in parenthesis on the right-hand-side of equation (20) is negative. This may occur when the effect of the decrease in the public input on the payments of labor employed in the public sector minus the effect on the payments of capital employed in the private sector is negative. Intuitively, when the imported good is capital intensive, an inflow of foreign capital increases the production of the imported good, and reduces imports and tariff revenue. Lower tariff revenue decreases the production of the public input. If, for example, the decrease in the production of the public input reduces the return to capital and increases the return to labor, then welfare may increase.

**Proposition 2:** *Assume that net government revenue decreases with an increase in the level of the public input. An inflow of foreign capital may increase the host-country's welfare, even if the imported good is capital inten-*

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7. The analysis calls for the imported good to be capital (labor) intensive when the increase in capital increases (decreases) its production. Equivalently, using Samuelson's reciprocity condition (i.e.,  $R_{pK}^* = R_{Kp}^*$ ), the imported good is capital (labor) intensive when an increase in its price increases (decreases) the reward of capital. This occurs when the share of capital in the production of the imported good exceeds its share in the production of the exported good.

sive and imports are restricted with a tariff. This may occur if the effect of the reduction in the public input on the payments to labor employed in the public sector minus the effect on the payments of capital employed in the private sector is negative.

## V. The Two-Good, Two-Factor Case

To derive some more specific results, consider a model where two goods and two factors, labor and capital, exist. This model is described in the Appendix B. Define  $\theta^{iL} = wC_w^i/C^i$ ,  $\theta^{iK} = rC_r^i/C^i$ , and  $e^i = (\partial C^i/\partial g)(g/C^i) > 0$  for  $i = 1, 2$ , where  $w$  is the wage rate, and  $L$  is labor. When  $\theta^{1L} > \theta^{2L}$ , then good 1 is labor intensive and good 2 is capital intensive. Equation (B3) shows that the effect of the public input on the return to capital and labor depends on the factor intensity of each good and on the elasticity of each cost function with respect to the public input. An increase in the amount of the public input may increase the rewards of both factors, or decrease the reward of one factor and increase the reward of the other factor. Equation (B3) shows that an increase in the provision of the public input increases the return to capital if good 1 is labor (capital) intensive and  $(e^2/e^1) > (<)$   $(\theta^{2L}/\theta^{1L})$ .

**Proposition 3:** *An inflow of foreign capital that increases the production of the public input increases (reduces) the reward of capital if good 1 is labor (capital) intensive and  $(e^2/e^1) > (<)$   $(\theta^{2L}/\theta^{1L})$*

When  $e^1 = e^2 = e$ , then an increase in the public input raises the rewards of both factors. That is,  $\hat{r} = \hat{w} = \hat{e}g$  (see equation (B3)). In the present two-good, two-factor case, the last two right-hand side terms of equations (16) and (20) become  $v^g R_{vg}^* - KR_{Kg}^* = (wL^g \hat{w}/\hat{g}g) - (rK^p \hat{r}/\hat{g}g) = (wL^g - rK^p)(e/g)$ , where  $L^g$  is the amount of labor used in the production of the public input, and  $K^p$  is the amount of capital available for the production of the private goods. In this case, if  $wL^g > (<) rK^p$ , then the foreign capital inflow increases (may decrease) the country's welfare, for the case of capital taxes and decrease (may increase) welfare when the capital intensive imported good is restricted with a tariff.

When good 1 is labor intensive (i.e.,  $\theta^{1L} > \theta^{2L}$ ), and  $(e^1/e^2) > (\theta^{1L}/\theta^{2L})$ , or

when good 2 is labor intensive and  $(e^2/e^1) > (\theta^{2L}/\theta^{1L})$ , then an increase in the production of the public input raises the wage rate and reduces the reward to capital. In this case, the traditional welfare effects of the inflow of foreign capital emerge. That is, welfare improves when the return to capital is taxed, and welfare decreases when the capital intensive imported good is restricted with a tariff.

On the other hand, if good 1 is capital intensive and  $(e^1/e^2) > (\theta^{1K}/\theta^{2K})$ , or if good 2 is capital intensive and  $(e^2/e^1) > (\theta^{2K}/\theta^{1K})$ , then an increase in the production of the public input decreases the wage rate and increases the return to capital. In this case, the change in the production of the public input due to the inflow of foreign capital may decrease welfare when capital is taxed, and may increase welfare when the capital intensive imported good is restricted with a tariff.

**Proposition 4:** *Consider the two-good, two-factor model, and assume that net government revenue decreases with an increase in the level of the public input. An inflow of foreign capital increases (may decrease) welfare when the return to capital is taxed and decreases (may increase) welfare when the capital intensive imported good is restricted with a tariff, if the effect of the change in the public input on payments of labor employed in the public sector minus the effect on the payments on capital employed in the private sector is positive (negative).*

## VI. Concluding Remarks

This paper builds a general equilibrium trade model of a LDC facing imperfect capital mobility, and producing a public input. It examines the welfare effect of an inflow of foreign capital when the government finances the provision of a public input either (i) by taxing capital, or (ii) by imposing a tariff on imports. Within this framework, the paper shows that the inflow of foreign capital changes government revenue, changes the amount of the provided public input, and causes factors rewards to change. The change in factor rewards (i) may reduce the country's welfare when the government finances the provision of the public input by taxing capital, and (ii) may increase the country's welfare when the government finances the provision of the public input by imposing a tariff on the capital intensive

imported good.

In this paper, I make the assumptions (i) that the government finance the provision of the public input by taxing only capital, or by imposing a tariff on imports, (ii) that all capital located domestically is owned by foreigners, and (iii) that changes in factor supplies do not affect factor rewards. Within the conventional Heckscher-Ohlin model, the last assumption requires the number of goods to equal or exceed the number of factors. Relaxing any one of these assumptions will not change the essence of the results, but will make them more complicated.

## Appendix A

This Appendix provides a proof for lemma 1. Begin by assuming that  $R_{vv}^* = 0$ . Noting that  $R_v^* = \omega$ , and using equation (6), the following results emerge

$$v_p^p = -C_{\omega\omega}^g R_{vp}^{Z*} g, \text{ and } v_g^p = -g C_{\omega\omega}^g R_{vg}^* - C_{\omega}^g. \quad (A1)$$

From the definition of  $R(p, g, K)$ , and using (A1) leads to

$$\begin{aligned} R_p &= R_p^* + R_v^* v_p^p = R_p^* - \omega C_{\omega\omega}^g R_{vp}^{Z*} g = R_p^*, \text{ and} \\ R_g &= R_g^* + R_v^* v_g^p = R_g^* - \omega (C_{\omega\omega}^g R_{vg}^* g + C_{\omega}^g) = R_g^* - \omega C_{\omega}^g = R_g^* - C_g^g. \end{aligned} \quad (A2)$$

where the linear homogeneity of the  $C^g(\omega)$  has been used (*i.e.*,  $\omega C_{\omega\omega}^g = 0$ ). Note that  $\omega C_{\omega}^g = C^g(\omega)$ . Also, since  $R_{vv}^* = 0$ , then

$$R_K = R_K^*. \quad (A3)$$

Differentiating (A2) with respect to  $K$  yields

$$R_{gK} = R_{gK}^*, \text{ and } R_{pK} = R_{pK}^*. \quad (A4)$$

## Appendix B

Consider a model where two goods and two factors, labor and capital, exist. The zero profit conditions in the production of each good (*i.e.*, unit cost equals unit price) for this case are given by

$$C^1(w, r, g) = p^1, \text{ and } C^2(w, r, g) = p^2, \quad (\text{B1})$$

Totally differentiating equation (B1) gives

$$\theta^{1L} \hat{w} + \theta^{1K} \hat{r} - e^1 \hat{g} = \hat{p}^1, \text{ and } \theta^{2L} \hat{w} + \theta^{2K} \hat{r} - e^2 \hat{g} = \hat{p}^2, \quad (\text{B2})$$

where a (^) over a variable denotes a growth rate (i.e.,  $\hat{w} = dw/w$ ). When the prices of the goods are constant, equations (B2) give the effect of changing the public input on factors returns as follows:

$$|\theta| \hat{r} = (e^2 \theta^{1L} - e^1 \theta^{2L}) \hat{g}, \text{ and } |\theta| \hat{w} = (e^1 \theta^{2K} - e^2 \theta^{1K}) \hat{g}, \quad (\text{B3})$$

here  $|\theta| = \theta^{1L} \theta^{2K} - \theta^{1K} \theta^{2L} = \theta^{1L} - \theta^{2L} = \theta^{2K} - \theta^{1K}$ . Equations (B3) show that when  $e^1 = e^2 = e$ , then  $\hat{r} = \hat{w} = e \hat{g}$ .

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