

Theoretical and Empirical Analysis of Consistency in the Exchange Rate Expectation Formation Process

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Abstract

This paper undertakes a two step test of consistency in the foreign exchange rate expectation formation process. In step one the General Extrapolative Model (GEM) is used with level of exchange rate survey forecasts. In step two the changes in levels of exchange forecasts are used to test consistency applying the cointegration methodology, thus taking non-stationarity into account. The thorny issue of the risk premium is avoided by using survey data on actual experts expectations. The GEM upholds (rejects) consistency in the short (long) forecast horizon, but the cointegration results confirm consistency and hence rationality in expectation formation across all horizons.

I. Introduction

The proper measurement of the investors exchange rate expectation formation process is a critical issue in answering the question of efficiency of the foreign exchange market. These expectations are based on experts probabilistic evaluation of future changes in devaluation and intervention policies. The

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question here is twofold: First what is the exact expectation formation process and second, does it lead to consistent and hence rational exchange rate expectation formation. Neither of these questions have been adequately addressed though there is an elaborate literature on the foreign exchange market.¹ Our attempt here is to address the latter problem.

Consistency is a necessary condition if expectations are to be rational. It is weaker than rationality since it does not require expectations to match the stochastic process generating actual exchange rates. It is also free from the two major limitations of rationality tests, namely Stochastic Bubbles and the Peso Problem. Moreover, rationality tests are based on untenable information and methodological foundations (Pesaran [1987]). We claim a rigorous test of consistency in expectation formation should be the starting point.

In this study we extend the work of Froot and Ito [1989], which was the first attempt at examining consistency in the expectation formation process. They apply a statistical iteration procedure, not an economic expectation formation model. The contribution of this paper to the present literature is twofold. We examine consistency in the expectation formation process using the cross equation restriction methodology pioneered by Meiselman [1962] and subsequently developed by Mincer [1969]. This is the first attempt at using standard economic expectation formation models to test consistency. This methodology, as advocated by Pesaran [1989] provides a simpler and more generally applicable derivation of the cross equation restrictions obtained by Froot and Ito. It is particularly useful in situations where observations at different dates for the same future period is available, like our survey data set. In step two we first examine and then take into consideration the non-stationarity (random walks) in the levels of exchange rates. Using the recently available cointegration methodology, we examine consistency / rationality / efficiency in the foreign exchange market. The direction of causality runs from consistency to rationality leading to efficiency.

A major bone of contention in consistency (rationality) studies is the presence/absence of the risk premium in the forward exchange rate. According to Frankel and Froot [1987] "Most of the empirical literature testing the unbi-

1. For an excellent review see Booth and Longworth [1986] and Liu and Maddala [1992].

asedness of the forward exchange rate has found it necessary to either arbitrarily assume away the existence of the risk premium, if the aim is to test whether investors have rational expectations, or else to assume that expectations are in fact rational, if the aim is to test propositions about the behavior of the risk premium." Survey data on exchange rate expectations provide a solution to this problem. They directly measure the expectations of the market participants, and is thus absolutely free of the risk premium. Two surveys and a broad spectrum of forecast horizons, from one week to one year ahead expectations, help ensure that our results are not due to particularities of a single small sample.

This paper is subdivided into 4 sections. Section I defines consistency as it relates to the exchange rate expectation formation process. Section II describes the survey data used. In section III we derive the necessary consistency conditions assuming extrapolative expectations, followed by estimation and results. In section IV we first examine the non-stationarity of the levels of spot and expected exchange rate series. The necessary consistency conditions under random walks is then derived and tested. Estimation and results are presented followed by our concluding remarks.

I. Consistency

Consistency in the expectation formation process is the equivalence between short and long horizon expectations. Agents expectations are consistent when their expectations at different forecast horizons lead to equivalent predictions of the level of the exchange rate into the future. In our context short term expectations will be inconsistent relative to long term expectations if a possible shock to the exchange rate leads agents to expect a higher long run future spot rate when iterating forward their short term expectations than when thinking directly about the long run. A failure of short term expectations to be consistent would imply that even the agents (experts) are not willing to live with the long run implications of their short run forecasts.

In the exchange rate expectation formation process let Y^* be the expected future spot exchange rate.

${}_t Y_{t+j}^*$ = Expectations of Y for future period $(t+j)$ made at period t .

${}_{t+i} Y_{t+j}^*$ = Expectations of Y for future period $(t+j)$ made at period $(t+i)$.

$E_t({}_{t+i}Y_{t+j}^*)$ = Expectations formed at time t about the future expectations of Y^* for period $(t+j)$ to be made at time $(t+i)$.

Here $t < i < j$.

Expectation formation is consistent if²

$${}_tY_{t+j}^* = E_t({}_{t+i}Y_{t+j}^*)$$

On a time scale let t stand for today, $i = 1$ for tomorrow and $j = 2$ for day after tomorrow

$${}_tY_{t+2}^* = E_t({}_{t+1}Y_{t+2}^*)$$

II. Survey Data

The use of survey data in the exchange rate expectation formation process is of recent origin.³ Our data set comprises exchange rate surveys from two different sources.⁴

Economist Financial Report: A news letter from London associated with *The Economist*. Every six weeks since mid-1981 the report has polled currency room traders and economists at fourteen major banks for their expectations of the value of the dollar against five currencies (British Pound, French Franc, Swiss Franc, German Mark and the Japanese Yen) in 3, 6, & 12 month horizons. Our data set comprises observations from June 1981 to August 1988, for a total of 60 observations.

New-York Money Market Survey (NYMMS): Conducted by Money Market Survey. About thirty traders each week report their expectations of the value of the dollar against four currencies namely British Pound, Swiss Franc, German Mark and the Japanese Yen at one and four week ahead horizons. Our

2. For further elaboration see (Pesaran [1987], pp. 274), Froot and Ito [1989] and Arora and Dutt [1993].

3. See Dominguez [1986], Frankel and Froot [1987], Froot and Frankel [1989], Froot and Ito [1989], Fischer [1989], Ito [1990], Liu and Maddala [1992] and Arora and Dutt [1993].

4. The author is grateful to Professor Kenneth A. Froot (M.I.T) for providing the data set.

data set comprises observations from October 1984 to January 1988 for the one week ahead horizon for a total of 165 observations, and from July 1985 to January 1988 for the four week ahead period with 129 observations.

We treat survey responses as though they are a perfect measure of the unobservable market expectations. The median investors response is assumed to be an unbiased estimate of aggregate expectations. The surveys may also contain measurement errors because only a subsample of the investor population is represented. As with many sampling methods, the measurement error will be purely random provided the sample groups expectation does not differ systematically over time from those of the population. Our estimation process allows for these sources of measurement errors because the survey responses will be used only on the left-hand side and so any measurement errors will end up in the contemporaneous residuals and will not affect our tests of consistency.

III. Extrapolative Expectations

We assume a general distributed lag specification for expectation formation.

$${}_t Y_{t+k}^* = \gamma_k + \sum_{i=1}^{\infty} \omega_{ki} Y_{t-i+1} + \mu_{kt} \quad (1)$$

Here expectations of the future period Y_{t+k} formed at time t are extrapolations from past observations on the value of Y as $Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3} \dots$ and μ_{kt} are the unobserved components arising from measurement errors or omitted variables. This model has been discussed extensively in the literature and provides an alternative representation of Meiselman's Error Learning Model (ELM) eq.(2)⁵

$${}_t Y_{t+k}^* - {}_{t-1} Y_{t+k}^* = \delta_k (Y_t - {}_{t-1} Y_t^*) + \dots + \xi_{kt} \quad (2)$$

The revision coefficient δ_k characterize the pattern of revision of future expectations in response to the current error of expectations and ξ_{kt} is the error process. There is a one-to-one correspondence between the revision coefficients δ_k and the weights ω_i . Forecasting one period ahead in eq.(1):

5. This test is recommended by Pesaran [1989] and Liu and Maddala [1992]. Also see Pesaran [1987] for an extensive study of this literature.

$${}_t Y_{t+1}^* = Y_1 + \omega_{11} Y_t + \omega_{12} Y_{t-1} + \dots + \mu_{1t} \quad (3)$$

Taking expectations of the same forecast horizon one period back

$$E_{t-1}({}_t Y_{t+1}^*) = \gamma_1 + \omega_{11} E_{t-1}(Y_t) + \omega_{12} E_{t-1}(Y_{t-1}) + \dots + E_{t-1}(\mu_{1t}) \quad (4)$$

Assuming consistent expectation formation

$${}_{t-1} Y_{t+1}^* = \gamma_1 + \omega_{11} {}_{t-1} Y_t^* + \omega_{12} Y_{t-1} + \dots + {}_{t-1} \mu_{1t}^* \quad (5)$$

Subtracting eq.(5) from eq.(3), we get the ELM

$${}_t Y_{t+1}^* - {}_{t-1} Y_{t+1}^* = \omega_{11} (Y_t - {}_{t-1} Y_t^*) + \mu_{1t} - {}_{t-1} \mu_{1t}^* \quad (6)$$

Comparing eq.(6) with eq.(2) at the one period ahead forecast horizon ($k = 1$), the cross equation coefficient constraints which will hold under consistent expectation formation is

$$\delta_1 = \omega_{11} \quad (7)$$

In the two period ahead forecasting case⁶

$$\delta_2 = \omega_{11}^2 + \omega_{12} = \omega_{11} \delta_1 + \omega_{12} \quad (8)$$

Thus for the general case

$$\delta_k = \sum_{i=1}^k \omega_{1i} \delta_{k-i} \quad (9)$$

where $\delta_0 = 1$ and $k = 1, 2, 3, \dots$

A direct test of consistency is to measure ω_{ki} in eq.(1) and δ_k in eq.(2) and test if the cross equation restrictions of equation (9) hold for different forecast horizons. It is particularly suitable in situations where observations on expectations formed at different dates for the same future date are available. The cross equation restriction in eq.(8) will hold true if expectations are consistent *i.e.* $\delta_1 = \omega_{11}$. The theoretical constraint for the two period ahead horizon δ_2 is [$\delta_2 = \omega_{11}^2 + \omega_{12}$] which is the same as eq.(8) if and only if eq.(7) holds true. Thus we

6. Extension to the two period ahead forecast horizon is straightforward.

have two testable restrictions in case of the two period horizon. Since we have expectation data made at point "t" for three future horizons [three (ES3), six (ES6) and twelve (ES12) months] we can undertake the above mentioned cross equation restriction tests.

We estimate eq.(1), the general extrapolative form of the exchange rate expectation formation process. Parameter estimates are obtained using Ordinary Least Squares (OLS). This gives us the unrestricted sum of squares (URSS). We start with the simplest case in which agents use only the most recent change in the spot rate to predict the subsequent change, so $i = 1$. The estimated equation is

$${}_tY_{t+1}^* = \gamma_1 + \omega_{11}Y_t + \mu_{1t} \quad (10)$$

where ${}_tY_{t+1}^*$ is the three month ahead expectations regressed on the log of the spot exchange rate. Then we estimate the ELM

$${}_tY_{t+1}^* - {}_{t-1}Y_{t+1}^* = \delta_1(Y_t - {}_{t-1}Y_t^*) + \xi_{1t} \quad (11)$$

We can estimate eq.(11) since we have expectation data formed at time t for two different horizons (three and six months). The data is set up for eq.(11) below:

Y_t = spot exchange rate data starting from September 1, 1981.

${}_tY_{t+1}^*$ = three month ahead expectation data starting September 1, 1981 (series ES3 lagged by two observations).

${}_{t-1}Y_t^*$ = three month ahead expectation data from June 1, 1981 for September 1, 1981 (series ES3).

${}_{t-1}Y_{t+1}^*$ = six month ahead expectations starting June 1, 1981 for December 1, 1981 (series ES6).

The procedure is identical for the six and twelve month horizon. Consistency imposes testable restrictions on the parameters of eq.(10) and eq.(11) namely eq.(7). We estimate eq.(10) for the three month ahead expectation horizon and eq.(11) for the three and six month ahead expectation horizon. Eq.(11) allows the error correction to be done as new information is made available. Then we constrain δ_1 of eq.(11) with ω_{11} of eq.(10) to test for the equality of the theoretically derived cross equation constraints that should hold if expectation formation is consistent over all future horizons.

Table 1 reports the regression of the five currencies included in the economist survey. The forecast horizon is three months and we obtain the testable parameters of Y_t (ω_{11}). We get the unrestricted sum of square errors (URSS). Table 2 is the regression of the revision in expectations (${}_t Y_{t+1}^* - {}_{t-1} Y_{t+1}^*$) on the error in

Table 1
Vector Autoregression (VAR) on Economist Survey Date (ESD)

$${}_t Y_{t+1}^* = \gamma_1 + \omega_{11} Y_t + \mu_{1t}$$

CUR	F.H.	γ_1	ω_{11}	R^2
B.P.	3 mon	0.00055	1.011517	0.98
F.F.	3 mon	-0.00484	0.9915809	0.99
S.F.	3 mon	0.01275	0.9826129	0.98
G.M.	3 mon	0.032522	1.004841	0.98
J.Y.	3 mon	-0.148884	0.9664369	0.98

Table 2
Error Learning Model (ELM) With ESD

$${}_t Y_{t+1}^* - {}_{t-1} Y_{t+1}^* - \delta_1 (Y_t - {}_{t-1} Y_t) + \xi_{kt}$$

CUR	F.H.	δ_1	R^2	URSS
B.P.	3 & 6 mon	0.872452	0.90	0.024701
F.F.	3 & 6 mon	0.882674	0.88	0.022746
S.F.	3 & 6 mon	0.986380	0.89	0.034980
G.M.	3 & 6 mon	0.871818	0.90	0.044772
J.Y.	3 & 6 mon	0.952858	0.88	0.0306

Table 3
Estimating the ELM with Constraint $\delta_1 = \omega_{11}$

$${}_t Y_{t+1}^* - {}_{t-1} Y_{t+1}^* = \omega_{11} (Y_t - {}_{t-1} Y_t) + \xi_{kt}$$

CUR	F.H.	R^2	RSS	"F" STAT	"W" STAT
B.P.	3 & 6 mon	0.88	0.030591	13.85	14.23
F.F.	3 & 6 mon	0.87	0.025514	7.79	8.06
S.F.	3 & 6 mon	0.89	0.034984	0.0067	0.0069
G.M.	3 & 6 mon	0.90	0.024434	2.46	2.54
J.Y.	3 & 6 mon	0.88	0.030672	0.093	0.093

the current period forecast ($Y_t - {}_{t-1}Y_t^*$) from which we obtain the error revision coefficient δ_1 . In Table 3 we restrict the revision coefficient δ_1 of the ELM with the GEM coefficient ω_{11} and run least square regression to obtain the restricted sum of squares (RSS). The testable hypothesis is

$$H_0 : \delta_1 = \omega_{11}$$

$$H_a : \delta_1 \neq \omega_{11}$$

We obtain mixed results. Consistency is upheld at the three month ahead expectation horizon for the Swiss Franc ($F = 0.006$), German Mark ($F = 2.46$), and the Japanese Yen ($F = 0.093$) as the "F" and the Wald statistics are significant at the 1% and 5% level. Not so for the British Pound ($F = 13.85$) and the French Franc ($F = 7.79$).

Next we try to test for consistency over the six and twelve month forecast horizon. We use an autoregressive structure in our expectation formation equation implying that agents use the most recent changes in the spot rate and last periods spot rate (Y_{t-1}) to form their expectations.

$${}_{t-1}Y_{t+1}^* = \gamma_1 + \omega_{11}Y_t + \omega_{12}Y_{t-1} + \mu_{1t} \quad (12)$$

We are using the same three month ahead expectation horizon but with one more lag. This gives us the parameters ω_{11} of Y_t and ω_{12} of Y_{t-1} . The cross equation restriction here is eq. (8).

$$\delta_2 = \omega_{11}\delta_1 + \omega_{12} \quad (8)$$

where the revision coefficient δ_2 is the proportion of the error ($Y_t - {}_{t-1}Y_t^*$) which enters the revision of expectations made of a six month ahead future date today and expectations of the same date made six months ago (difference between twelve and six month expectation horizon). The revision coefficient δ_1 , is obtained from Table 2.

Table 4 shows the least square regression of the three month ahead expectation on the spot and one period lagged spot rate. Table 5 is the regression of the revision in expectations made today about a six month ahead future date (six month forecast horizon) and expectations of the same date made six months back (twelve month ahead forecast), on the expectation error i.e. dif-

ference between actual spot rate today and expectation of today's rate made six months back. We get the URSS her. In Table 6 we regress the ELM with the constrained value of the revision coefficient δ_2 . This gives us the RSS. Consistency is rejected for all the currencies at the long forecast horizon. None of the

Table 4
VAR on ESD With Higher Lags

$${}_t Y_{t+1}^* = \gamma_1 + \omega_{11} Y_t + \omega_{12} Y_{t-1} + \mu_{1t}$$

CUR	F.H.	γ_1	ω_{11}	ω_{12}	R^2
B.P.	3 mon	0.011667	0.986144	0.445407	0.98
F.F.	3 mon	-0.008116	0.989831	0.182710	0.99
S.F.	3 mon	0.011753	0.981065	0.54941	0.98
G.M.	3 mon	0.030644	1.00252	0.138126	0.98
J.Y.	3 mon	-0.169586	0.962505	0.257275	0.99

Table 5
ELM with ESD

$${}_t Y_{t+1}^* - {}_{t-1} Y_{t+2}^* = \delta_2 (Y_t - {}_{t-1} Y_t^*) + \xi_{2t}$$

CUR	F.H.	δ_2	R^2	URSS
B.P.	6 & 12 mon	0.871818	0.90	0.044772
F.F.	6 & 12 mon	0.893607	0.91	0.040283
S.F.	6 & 12 mon	0.877635	0.90	0.063426
G.M.	6 & 12 mon	0.908192	0.91	0.045929
J.Y.	6 & 12 mon	0.886713	0.92	0.039109

Table 6
ELM with Constraint

$${}_t Y_{t+2}^* - {}_{t-1} Y_{t+2}^* = (\omega_{11} \delta_1 + \omega_{12}) (Y_t - {}_{t-1} Y_t^*) + \xi_{2t}$$

CUR	F.H.	R^2	RSS	"F" STAT	"W" STAT
B.P.	6 & 12 mon	0.68	0.156227	139.42	144.48
F.F.	6 & 12 mon	0.88	0.553600	21.35	22.11
S.F.	6 & 12 mon	0.89	0.034984	13.98	14.53
G.M.	6 & 12 mon	0.87	0.070385	30.31	31.40
J.Y.	6 & 12 mon	0.82	0.091667	76.64	79.30

"F" or the Wald statistics are significant, either at the 1% or 5% levels.

For the six and twelve month horizon we have another testable constraint.

$$\delta_2 = \omega_{11}^2 + \omega_{12} \quad (13)$$

The econometric procedure is the same as before and warrants no repetition.

None of the "F" and the Wald statistics are significant at the 1% and 5% level implying expectation formation at this long horizon is not consistent.

At a shorter forecast horizon, we find evidence of consistent expectations in three out of five currencies, but it breaks down completely over the longer term. The complex cross equation restrictions make it difficult to interpret the importance of the "F" and Wald statistic. To give some economic intuition to our results we follow Froot & Ito [1989]. They graphically represent the band-

Table 7
ELM with Constraint

$${}_t Y_{t+2}^* - {}_{t-1} Y_{t+2}^* = (\omega_{11}^2 + \omega_{12}) (Y_t - {}_{t-1} Y_t^*) + \xi_{2t}$$

CUR	F.H.	R ²	RSS	"F" STAT	"W" STAT
B.P.	6 & 12 mon	0.67	0.160494	147.7	153.03
F.F.	6 & 12 mon	0.83	0.081588	58.5	60.2
S.F.	6 & 12 mon	0.87	0.078003	13.6	14.1
G.M.	6 & 12 mon	0.85	0.080841	44.8	46.0
J.Y.	6 & 12 mon	0.82	0.095121	82.3	84.8

wagon effects. Starting from equilibrium, a sudden shock is delivered to the spot exchange rate, say it is appreciated by 1%. The expected future path is then graphically traced. In case of the Japanese Center for International Finance (JCIF) data, a 1% dollar appreciation leads to 0.08% expected appreciation in one month, but to a 0.16% depreciation over the next three months. Similar iteration to the six and twelve month horizon resulted in greater expected depreciation when compared to the direct six and twelve month ahead expectations. Thus a present shock generates a higher expected long run exchange rate, when short horizon expectations are iterated than when long horizon expectations are formed directly *i.e.* bandwagon effects. This is the direct implication of our inconsistent long forecast horizon results.

Next a non-parametric test of consistency is conducted. We check the autocorrelation structure of the stochastic process in the Error Learning Model eq. (11). From eq. (2) and eq. (6) the error sequence at the single forecast horizon ($k = 1$), under consistent expectation formation is

$$\xi_{1t} = \mu_{1t} - {}_{t-1}\mu_{1t}^* \quad (14)$$

For the two period ahead forecast is

$$\xi_{2t} = {}_t\mu_{1, t+1} - {}_{t-1}\mu_{1, t+1}^* + \omega_{11}\mu_{1t} - \omega_{11}{}_{t-1}\mu_{1t}^* \quad (15)$$

For the general case

$$\xi_{kt} = \sum_{i=1}^k \delta_{i-1} V_{t, t+k-i} \quad (16)$$

where

$$V_{t, t+i} = {}_t\mu_{1, t+i} - {}_{t-1}\mu_{1, t+i}^* \quad (17)$$

and $V_{t, t+i}$ stands for the revision in expectations of $\mu_{1, t+i}$ between periods $(t-1)$ and t . Consistent expectations would imply a random (serially uncorrelated) error structure *i.e.* $\xi_{1t}, \xi_{2t}, \dots, \xi_{kt} = 0$.⁷ Preliminary evidence of significant serial correlation in the error process is obtained from the low Durbin-Watson Statistic in Table 8. An array of confirmatory tests are conducted to further check the structure of the error process. We estimate:

1) Autocorrelation Coefficient From Durbin-Watson (D.W) Statistic:

$$d \approx 2(1 - \hat{\rho}_1) = 2 - 2\hat{\rho}_1$$

$$\hat{\rho}_1(A) \approx \frac{(2-d)}{2} = (1 - \frac{d}{2}) \quad (18)$$

2) Iterative Cocrane-Orcutt Method: Here we first estimate ξ_{kt} and then estimate $\hat{\rho}_1$ from

7. The author wishes to thank an anonymous referee for recommending this test and the cointegration analysis.

$$\hat{\rho}_1(B) = \frac{\sum \xi_{kt} \xi_{k(t-1)}}{\sum \xi_{kt}^2} \quad (16)$$

3) Testing the Autocorrelation Structure of the Error Term ξ_{kt} :

$$\xi_{kt} = \hat{\rho}_1(C) \xi_{k(t-1)} + e_t \quad (20)$$

The autocorrelation coefficient is less than one in absolute value, implying significant serial correlation for all currencies and across all forecast horizons. This rejects consistency (rationality) in expectation formation in its entirety. Liu and Maddala [1992] also report significant serial correlation.

IV. Cointegration Analysis

The results reported above though in conformity with the literature could be biased because of two reasons:

1) The ad hoc assumption that experts use a distributed lag expectation specification, though there is no conclusive evidence that they do not.⁸ These

Table 8
Error Structure Randomness Test

CUR	F.H.	D.W. STAT.	$\hat{\rho}_1(A)$ Eq.(18)	$\hat{\rho}_1(B)$ Eq.(19)	$\hat{\rho}_1(C)$ Eq.(20)
B.P	3 & 6 months	1.05	0.46	0.46	0.46
	6 & 12 months	1.13	0.41	0.41	0.41
F.F	3 & 6 months	1.33	0.33	0.33	0.33
	6 & 12 months	1.25	0.37	0.37	0.37
S.F	3 & 6 months	1.37	0.21	0.21	0.21
	6 & 12 months	1.40	0.25	0.25	0.25
G.M	3 & 6 months	0.83	0.58	0.58	0.58
	6 & 12 months	1.15	0.42	0.42	0.42
J.Y	3 & 6 months	1.36	0.27	0.27	0.27
	6 & 12 months	1.25	0.22	0.22	0.22

Notes: CUR = Currency, F.H. = Forecast Horizon, B.P = British Pound, F.F = French Franc, S.F = Swiss Franc, G.M = German Mark and J.Y = Japanese Yen.

models have been used extensively in the literature, but their assumed limited information set of the agent is inadequate. On the other hand the rational expectation model attributes to agents, information which they may not possess.

2) The traditional regression analysis used here will be inappropriate if the exchange rates, both spot and expectation series are non-stationary processes *i.e.* follow a random walk.

We take this into consideration and first test for non-stationarity of the spot and expectation series. The necessary consistency conditions are then derived and examined. We assume expectations are formed using Meiselman's Error Learning Model. Here if expectation formation is consistent then the stochastic process it generates will be serially uncorrelated. The Martingale difference method can be used as a general solution for consistent expectation formation in terms of the revision process that appears in the procedure for updating expectations. Comparing eq.(2) and eq.(6) for $k = 1$, we get eq.(14). Thus if the expectation formation process is consistent, then μ_{1t} and ${}_{t-1}\mu_{1t}^*$ will follow a martingale and therefore, ξ_{1t} will follow a martingale difference sequence and⁹

$$\xi_{1t} = \mu_{1t} - {}_{t-1}\mu_{1t}^* = 0 \quad (21)$$

Consistent expectation formation requires that the spot and forecasted series generate a random (serially uncorrelated) error sequence as in eq. (21). If the spot exchange rate series Y_t is non-stationary, say integrated of order one and the expectation data Y_e is a consistent (rational) forecast of the spot rate series, then two necessary conditions must hold:

- 1) Y_e must be integrated of order one: $Y_e - I(1)$.
- 2) Y_e must be cointegrated with Y_t .

Our empirical test procedure goes as follows. We first test for the structure (stationarity / non-stationarity) of both the spot and expectation series across all horizons. If they are random walks (non-stationary), we then test for cointegration between the spot and expectation series. If they are cointegrated, they

8. Fischer [1989] assumes similar expectation formation specification in her tests of rationality in expectations of M1 for the one week ahead horizon.

9. For a similar analysis see Arora and Dutt [1993].

will generate a stationary error term *i.e.* the stochastic process will be integrated of order zero. This will confirm the consistency (rationality) of the exchange rate expectation formation process.

A preliminary test of non-stationarity of the spot and expectation data is to check the sample autocorrelation (correlogram) of the series and its first difference.¹⁰ We find evidence of non-stationarity in both. The next step is to differentiate between the trend and difference stationary processes. It is important because if the series are trend stationary, future uncertainty is bounded, but in case of difference stationarity a present shock will alter all future forecasted values. In case of difference stationarity, differencing the series produces stationary error structure, while for a trend stationary series, regression on a time trend will produce a stationary error structure. We find evidence of both the spot and forecasted series to be difference stationary, except in case of NYMMS data, where at very short forecast horizons there is some evidence of a time trend.

Direct Unit Root Tests: We apply the standard Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) technique, with extended lags to ensure a proper white noise error term.

$$e_t = \alpha + \beta_1 t + \beta_2 e_{t-1} + \xi_t \quad (22)$$

$$e_t = \alpha + \beta_1 t + \beta_2 e_{t-1} + \sum_{i=1}^p \lambda_i \Delta e_{t-i} + \xi_t \quad (23)$$

where Δ is the first difference operator. The null hypothesis that the series follows a difference stationary process with a unit root [the D.F. “ t ” statistic, $\tau_{\hat{\beta}_1}$ tests the null of $\beta_1 = 1$] is evaluated against the alternative of a trend stationary process by testing the null $Z(\phi_3) = H_0 : (\beta_1, \beta_2) = (0, 1)$. We also test for confirmation of the fact that the series is integrated of order one as opposed to higher orders from the first difference (FD) of the exchange rate series. The test hypothesis is:

$Z(\tau_{\hat{\beta}_2})$ tests $H_0 : \beta_1 = 1$ in eq. (22) and eq. (23).

$Z(\phi_3)$ tests $H_0 : (\beta_1, \beta_2) = (0, 1)$ in eq. (22) and eq. (23).

10. Results are not reported due to paucity of space, but is available from the author upon request.

Table 9
Unit Root Tests of the Spot and Expectation Series:
Econdat Data: Equation 22 and 23

Series	$\hat{\beta}_2$	Lag	$Z(\tau\hat{\beta}_2)$	$Z(\tau\hat{\beta}_2)$	$Z(\phi_3)$	$Z(\tau\hat{\beta}_2)$
			D.F	A.D.F		F.D
LNBS	0.97885	2		-1.22	4.89	-5.49
LNBE3	0.94336	1	-1.36		3.29	-5.11
LNBE6	0.95142	1	-1.38		3.53	-5.35
LNBE12	0.96118	1	-1.22		3.67	-4.77
LNFS	0.95026	1	-1.43		1.53	-4.43
LNFE3	0.95828	2		-1.92	3.37	-4.33
LNFE6	0.96069	2		-1.85	4.01	-4.25
LNFE12	0.96379	4		-2.66	4.93	-4.17
LNGS	0.94438	1	-1.23		2.56	-4.67
LNGE3	0.95537	4		-1.66	1.76	-4.55
LNGE6	0.96386	2		-2.45	1.79	-4.41
LNGE12	0.97443	4		-1.63	1.85	-4.16
LNSS	0.94500	2		-1.66	2.40	-4.75
LNSE3	0.95485	2		-1.65	1.64	-4.08
LNSE6	0.96135	2		-1.45	1.02	-4.55
LNSE12	0.96819	4		-2.65	1.13	-4.56
LNJS	0.90313	1	-1.68		5.02	-4.46
LNJE3	0.92618	1	-1.62		3.61	-4.32
LNJE6	0.93856	2		-2.00	4.55	-4.39
LNJE12	0.93837	5		-2.88	4.64	-4.24

Notes: LNBS = Log of spot exchange rate of the British Pound.

LNFS = " " French Franc.

LNGS = " " German Mark.

LNSS = " " Swiss Franc.

LNIS = " " Japanese Yen.

LNBE 3, 6 and 12 are the expectation series at those horizons.

Table 10
NYMMS Data: Equation 22 and 23

Series		$\hat{\beta}_2$	Lag	$Z(\tau\hat{\beta}_2)$	$Z(\tau\hat{\beta}_2)$	$Z(\phi_3)$	$Z(\tau\hat{\beta}_2)$
				D.F	A.D.F		F.D
1 WK	LNBS	0.93623	1	-2.91		2.71	-9.89
	LNBEI	0.85678	1	-2.56		7.63	-11.97
	LNGS	0.92357	1	-2.22		3.41	-9.32
	LNGEI	0.86715	2		-2.93	7.46	-8.79
	LNSS	0.92456	1	-2.91		4.26	-9.43
	LNSEI	0.88737	3		-2.90	21.12	-12.14
	LNJS	0.96238	1	-1.52		2.30	-6.85
	LNJEI	0.89735	2		-2.13	4.96	-7.90
4 WKS	LNBS	0.92132	1	-1.85		5.80	-9.66
	LNBE4	0.83549	1	-2.98		9.97	-10.09
	LNGS	0.90451	1	-2.90		4.84	-8.86
	LNGE4	0.81675	2		-2.53	6.73	-10.57
	LNSS	0.88356	1	-2.84		5.62	-8.97
	LNSE4	0.84367	3		-2.51	7.13	-10.20
	LNJS	0.95165	1	-1.98		2.62	-6.17
	LNJE4	0.93452	2		-2.14	2.90	-8.26

Notes: Tables 9 and 10:

- 1) $Z(\phi_3)$ tests trend versus difference stationarity. The critical values at the 5% significance level is 6.73 and 6.49 for sample size 50 and 100 respectively. See Fuller [1976, pp.373] and Dickey and Fuller [1981, pp. 1063].
- 2) $Z(\tau\hat{\beta}_2)$ tests the null that the series is integrated of order one. Critical values are -3.50 and -3.18 at the 5% significance level for sample size 50 and 100 respectively and -3.44 (-4.03) at 5% (1%) level using the MacKinnon tables for sample size 100.
- 3) $Z(\tau\hat{\beta}_2)$ F.D. is the "t" test of the hypothesis that $\beta_2 = 1$, for the first difference of the series. Critical values are same as above.
- 4) S = Spot rate, E = Expectation series at one and four week horizon. Data for the French Franc was not available.

$Z(\tau\hat{\beta}_2)$ First Difference tests $H_0: \beta_1 = 1$ in eq. (22) and eq. (23).

Majority of the estimates of $\hat{\beta}_2$ fall between 0.85 and 0.97, consistent with realization of a random walk. The "t" statistic for the hypothesis $\hat{\beta}_2 = 1$ are all significant by conventional standards. Across the board for all expectation horizons and both data sets, the exchange rate series are non-stationary processes. From $(\tau\hat{\beta}_2)$ F.D. we find evidence of one and only one unit root *i.e.* the spot and expectation series are all integrated of order one. We also report the presence of a time trend in the NYMMS data (Table 10) at the one and four week ahead forecast period. At the longer horizon, $Z(\phi_3)$ rejects the null of a time trend (Table 9). This is in line with the popular belief that experts extrapolate in the shorter forecast horizon. Frankel and Froot [1988] forward the "chartist" versus "fundamentalist" view of expectation formation and the changing weights given to current events as the expectation horizon expands.

The next step is to check if the two series are cointegrated, *i.e.* there exists a linear combination of the series which is stationary or integrated of order zero. We use Engle and Granger's [1987] two step cointegration procedure as follows:

$$Y_t = \alpha + \rho_1 Y_e^* + \mu_{1t} \quad (24)$$

and the reverse regression

$$Y_e^* = \alpha + \rho_2 Y_t + \mu_{2t} \quad (25)$$

where Y_t and Y_e^* are the spot and exchange rate expectation series respectively. If the spot and expectation series are cointegrated, their linear combination $(Y_t - \rho_1 Y_e^*)$ and $(Y_e^* - \rho_2 Y_t)$ would be a valid stationary process. Ideally $\rho \approx \hat{\rho}_1 \approx \hat{\rho}_2$. According to Banerjee *et.al.* [1986] a high R^2 with $\hat{\rho}_1$ and $\hat{\rho}_2$ close to each other and a $D.W.$ statistic close to two implies zero bias and rejects the null of a unit root in the residuals.

We have overwhelming evidence of cointegration of the spot and expectation rate series. Here $\rho \approx \rho_1 \approx \rho_2$ for all cases, R^2 is close to unity and the $D.W.$ statistic is approximately equal to two. Thus bias due to super consistency in measurement of ρ is close to zero and a high $D.W.$ statistic indicates rejection of the null of a unit root in the residuals.

Step two of the Engle-Granger procedure is directly testing for stationarity of

Table 11
Cointegration Test: Step One:

Series	Equation (24)			Equation (25)		
	ρ_1	R^2	D.W.	ρ_2	R^2	D.W.
LNYE1(U.K)	0.97	0.98	1.99	1.00	0.98	1.99
LNYE4	0.97	0.98	1.98	0.97	0.95	1.99
LNYE3	0.96	0.97	1.96	1.01	0.98	1.97
LNYE6	0.98	0.98	1.95	0.98	0.97	1.97
LNYE12	1.03	0.94	1.93	0.88	0.94	1.95
LNYE1(W.G)	0.99	0.99	1.99	0.99	0.99	1.99
LNYE4	0.98	0.99	1.99	1.00	0.99	1.94
LNYE3	0.99	0.99	1.99	0.99	0.99	1.99
LNYE6	1.03	0.99	1.97	0.95	0.99	1.97
LNYE12	1.12	0.97	1.83	0.85	0.97	1.81
LNYE1(S.F)	0.99	0.99	1.94	1.00	0.99	1.96
LNYE4	0.98	0.99	1.90	1.00	0.99	1.90
LNYE3	1.02	0.99	1.99	0.98	0.99	1.99
LNYE6	1.04	0.98	1.96	0.99	0.98	1.95
LNYE12	1.14	0.97	1.98	0.94	0.97	2.00
LNYE1(J.Y)	0.99	0.99	1.99	1.00	0.99	1.99
LNYE4	0.99	0.99	1.99	0.97	0.99	1.99
LNYE3	1.02	0.99	1.99	0.98	0.99	1.99
LNYE6	1.04	0.99	1.91	0.94	0.99	1.90
LNYE12	1.01	0.98	1.86	0.88	0.98	1.86
LNYE3(F.F)	1.00	0.99	1.99	0.99	0.99	1.99
LNYE6	1.02	0.98	1.99	0.98	0.98	1.99
LNYE12	1.09	0.97	1.97	0.88	0.98	1.96

the error process. A preliminary check of the correlogram of the error structure indicates a stationary process.¹¹ We examine for unit roots in the error structure by regressing the first difference of the error term $\Delta\mu_t$ on its lagged values and μ_{t-1} .

$$\Delta\mu_{1t} = \mu_{1t} - \mu_{1(t-1)} = \alpha + \beta_1\mu_{1(t-1)} + \xi_{1t} \quad (26)$$

$$\Delta\mu_{2t} = \mu_{2t} - \mu_{2(t-1)} = \alpha + \beta_2\mu_{2(t-1)} + \xi_{2t} \quad (27)$$

We directly obtain "t" values of $\hat{\beta}_1$, $[\tau(\hat{\beta}_1)]$ and $\hat{\beta}_2$, $[\tau(\hat{\beta}_2)]$ which form evidence of the stationarity / non-stationarity of the error process.

Our "t" values clearly and unanimously reject the null of a unit root in the error process across the board for all currencies. This provides evidence that the spot and expectation series are cointegrated and hence expectation formation across all currencies and for all horizons is consistent (rational). These results are supported by Arora and Dutt [1993].

V. Conclusion

We study the consistency property in the exchange rate expectation formation process, which all rational forecasts have, but which itself does not require rationality. Alternative test procedures recommended by Pesaran [1989] is applied. Survey data helps avoid the risk premium issue altogether. Our results are far from conclusive, which is also the state of the relevant literature.

The General Extrapolative Forecasting Model provides evidence of bandwagon effects in the expectation formation process. Consistency is upheld at the shorter forecast horizon, but breaks down conclusively for the longer forecast periods. These results are supported by Froot and Ito [1989].

The randomness tests conducted on the error structure show significant serial correlation and hence reject consistency and rationality for all cases. Mixed results are reported by Liu and Maddala [1992]. They find significant (not significant) serial correlation for the monthly (weekly) data. Based on this they reject (do not reject) the rational expectations hypothesis for the monthly (weekly) horizon expectations.

Our cointegration results provide evidence of stationary error processes, implying consistent (rational) expectation formation across all horizons. This result is supported in its entirety by Arora and Dutt [1993], Fischer [1989] and also Liu and Maddala [1992] who provide evidence of rejection of the null of no

11. Results are not reported due to paucity of space, but is available from author upon request.

Table 12
Dickey-Fuller Test for Unit Roots in the Error Structure: Step Two

Series	Eq. (26)	Eq. (27)
	$\tau(\hat{\beta}_1)$	$\tau(\hat{\beta}_2)$
LNYE1 (U.K.)	-12.79	-12.77
LNYE4	-11.13	-11.28
LNYE3	-7.68	-7.76
LNYE6	-7.57	-7.64
LNYE12	-7.53	-7.90
LNYE1 (W.G.)	-12.80	-12.18
LNYE4	-11.30	-11.27
LNYE3	-7.70	-7.72
LNYE6	-7.64	-7.63
LNYE12	-7.15	-7.11
LNYE1 (S.F.)	-12.46	-12.58
LNYE4	-10.14	-10.79
LNYE3	-7.73	-7.74
LNYE6	-7.63	-7.72
LNYE12	-7.69	-7.72
LNYE1 (J.Y.)	-12.79	-12.79
LNYE4	-11.40	-11.31
LNYE3	-7.73	-7.73
LNYE6	-7.48	-7.45
LNYE12	-7.41	-7.38
LNYE3 (F.F.)	-7.77	-7.76
LNYE6	-7.72	-7.71
LNYE12	-7.63	-7.66

Notes: Critical values for $g(\hat{\beta}_1)$ and $g(\hat{\beta}_2)$ at 5 and 10% significance level are -3.50 (-3.18), -3.45 (-3.15) and -3.43 (-3.13) for sample size 50, 100 and 250 respectively. At the 5% (1%) significance level, the critical values are -3.85 (-4.44) using MacKin-non tables.

cointegration in both the weekly and monthly forecast horizons.

The probable reasons for lack of a conclusive outcome regarding consistent (rational) expectation formation are:

1) Agents use different models to form expectations of short versus long horizons (chartist vs. fundamentalist).

2) The possibility of survey sources systematically mismeasuring the markets true expectations.

3) Misspecified expectation formation model and that variables other than past exchange rate behavior matter in forming expectations.

4) News plays an important but as of yet undefined role in the expectation formation process.

5) The dynamics of a flexible exchange rate economy involve time frame expectations as an important factor in exchange rate determination. Here the Peso Problem has been cited as a probable cause for exchange rate expectations overshooting and hence inconsistency.¹²

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12. For elaboration on the Peso Problem see Krasker [1980], Hodrick [1987] and Obstfeld [1987]

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