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Optimal Endogenous Growth in a Two-Sector Model with Learning-by-Doing and Spillovers

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Abstract

I derive the optimal choice of public policy, explicitly characterize the optimal rate of endogenous economic growth, and compare it to the competitive growth rate in a two-sector model with learning-by-doing and spillovers, based on the Krugman-Lucas model, with the innovation of an endogenous labor supply. I find that the optimal growth rate exceeds the competitive growth rate, since both the optimal level of labor supply and share of labor in the progressive sector exceed the competitive levels. Furthermore, looking at optimal as opposed to competitive paths is shown to have important implications for divergence of growth rates and comparative advantage in the open economy. Implications for trade, labor migration, and economic integration are discussed. Finally, I introduce asset pricing in the model; I demonstrate that, along the optimal paths, there is no a priori reason to expect the positive correlation between growth rates and interest rates which is a feature of the competitive equilibrium.

I. Introduction

This paper presents a two-sector model of endogenous growth with learning-by-doing and spillovers. The motivation is to present some important conclusions

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stemming from an analysis of the *optimal* choice of public policy and the *optimal* rate of endogenous economic growth which arise in a two-sector economy characterized by Marshallian external economies and spillovers. The economic environment of the model is not new – it essentially is a modification of the standard learning-by-doing model of endogenous growth, pioneered by Lucas [1988] and Boldrin and Scheinkman [1988]. Two key insights emerge from this literature: (i) when learning-by-doing effects are strong, existing patterns of comparative advantage tend to be reinforced over time and (ii) the economy's rate of growth will increase upon opening to trade if it exports commodities which generate the largest learning-by-doing effects and will fall if it exports commodities in which learning has been exhausted.

Developments in the literature have generally taken the path of spelling out more precisely where the learning effects are coming from and how they will be disseminated through the economy. The state of the art is represented in recent contributions by Stokey [1988] and Young [1991] which explicitly model the introduction of new goods and the transmission of learning effects between one generation of product and the next. However, as complex as the economic analysis has become (or perhaps because of it), these recent contributions, nor indeed the original papers by Lucas [1998], Boldrin and Scheinkman [1988], nor the seminal analysis of Krugman [1987] upon which they are based, examine the nature of *optimal* policy intervention in an environment characterized by learning-by-doing and spillovers. Intuitively, it is apparent that in the presence of externalities the decentralized path of the economy will not be efficient; introspection reveals that the optimal policy will require subsidization of the sector which generates the external learning effects. What I do in this paper is to sharpen this intuition by explicitly solving for the optimal policy intervention and obtaining closed-form expressions for the optimal and decentralized growth rates, thus enabling me to undertake comparative statics exercises. To obtain the cleanest results and accordingly sharp intuition, I deliberately simplify the model structure to its bare bones, especially in terms of the functional forms for preferences (Cobb-Douglas), production technology (Ricardian), and the learning technology (linear). Indeed, such a simple structure or something very like it must be assumed if closed-form expressions are to be obtained. I hope by the end of the day to convince the reader that the intuition garnered along the way will be worth the price of the simplification.

In addition to the shift of focus to optimal policy and hence optimal growth, the analysis does modify and enrich the benchmark model in a number of ways. First, I endogenize the labor supply decision of the representative household and hence allow the economy's labor supply to be determined endogenously rather than assumed to be inelastic, as in all previous models. Second, I consider as well the path of asset prices and hence of interest rates in equilibrium, using the Lucas [1978] asset pricing approach, absent from previous models and hinted at but not undertaken by Young [1991].

II. The Model

Consider, then, an economy, initially closed, populated by a large number of identical, infinitely-lived households, whose number is normalized for simplicity to unity.[1] Time is discrete. The representative household has preferences over the consumption of two nonstorable output goods, Y and X, and leisure, $(1 - L)$, where L is labor supply and the time endowment per period has been normalized to unity, as follows,

$$\sum_{t=0}^{\infty} \beta^t \left[\{1 - L(t)\}^{\omega} \left\{ X(t)^{\theta} Y(t)^{1-\theta} \right\}^{1-\omega} \right], \tag{1}$$

where β, $0 < \beta < 1$, is the discount rate, and where ω and θ are parameters such that $0 < \omega < 1$ and $0 < \theta < 1$. The two goods, Y and X, are produced according to the following Ricardian production functions,

$$Y(t) = A(t)\,\lambda(t)\,L(t) \tag{2}$$

$$X(t) = B(t)\left\{1 - \lambda(t)\right\} L(t) \tag{3}$$

where λ, $0 < \lambda < 1$, is the share of the representative household's labor supply devoted to the production of Y and where A, $A > 0$, and B, $B > 0$, are indices of labor productivity in Y and X, respectively. I suppose that there exist learning-by-doing

1. In assuming for simplicity a stationary population, I follow the literature. However, this assumption is not innocuous; as valuably pointed out to me by John Conlisk and spelled out consequently in Dehejia [1992], allowing exogenous population growth leads to explosive *per capita* output growth, a troublesome but inevitable implication of the structure when nondiminishing returns to learning are assumed.

in the production of Y which augments labor productivity in Y, as measured by A, and further I suppose that this learning-by-doing spills over to the production of X and augments B, but to a lesser extent. The externality and the spillover are modelled as a Marshallian external economy; hence, the benefits cannot be internalized and appropriated by private agents, which is why it will turn out that there is under-accumulation along the competitive path.[2] A simple way to capture this idea is the following,

$$A(t+1) = A(t) + \gamma Y(t); \tag{4}$$

$$B(t) = A(t)^{\mu}; \tag{5}$$

where γ, $\gamma > 0$, is a parameter capturing learning in the Y-sector and μ, $0 < \mu < 1$, is a parameter capturing the spillover to the X-sector. Substituting for (2), (4) can be rewritten as:

$$\{A(t+1) - A(t)\} / A(t) = \gamma \lambda(t) L(t). \tag{4'}$$

Given the Ricardian structure of the economy, it is easily shown that the relative price of Y in terms of X, p, and national income measured in terms of X, I, are given by the following expressions:

$$p(t) = A(t)^{\mu - 1}; \tag{6}$$

$$I(t) = A(t)^{\mu} L(t); \tag{7}$$

The policymaker's optimal control problem at time 0 is to select a sequence $\{\lambda(t), L(T)\}$: $t = \{0, 1, 2, \ldots, \infty\}$ that maximizes (1) subject to (2) – (5), with $A(0) > 0$ given. The Euler equations characterizing the optimal choice are as follows, which come from substituting (2) – (5) into (1) and differentiating:

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2. This formulation of the Marshallian externality which assumes spillovers is clearly most relevant in a context in which the two tradeable goods are technologically similar but seems less so when they are not. Thus, for instance, it seems plausible that learning in the computer industry may have a positive spillover to the automobile industry, but less plausible that it may have a significant spillover to the handicrafts industry. For evidence on externalities and spillovers in manufacturing, see Caballero and Lyons [1991].

$$-\theta / \{1-\lambda(t)\} + (1-\theta) / \lambda(t) + \beta\theta\mu / \lambda(t) + \beta(1-\theta) / \lambda(t) = 0; \qquad (8)$$

$$-\omega / \{1-L(t)\} + (1-\omega)\{\theta(\beta\mu+1) + (1-\theta)(\beta+1)\} / L(t) = 0; \qquad (9)$$

Notice now what the severe assumptions on functional forms buys us: the optimal choices of λ and L are time-invariant; optimality requires that they be set at the chosen level at time 0 and maintained thereafter. This in turn will imply that it is optimal for this economy to jump onto its balanced growth path at time 0 without transitional dynamics. This feature enormously simplifies the comparative statics analysis but will not typically be satisfied with general preferences and technology. Equations (8) and (9) can in turn be solved to yield explicit solutions for λ and L as follows,

$$\lambda^*(\overset{+}{\beta}, \overset{-}{\theta}, \overset{+}{\mu}) = \{(1-\theta) + \beta(\theta\mu + 1 - \theta)\} / \{1 + \beta(\theta\mu + 1 - \theta)\}; \qquad (10)$$

$$L^*(\overset{+}{\beta}, \overset{-}{\theta}, \overset{+}{\mu}, \overset{-}{\omega}) = \{(1-\omega)\,\theta(\beta\mu+1) + (1-\theta)(\beta+1)\} / \{\omega + (1-\omega)\,\theta(\beta\mu+1) + (1-\theta)(\beta+1)\}; \qquad (11)$$

where the stars denote the optimized values and where the signs of the partial derivatives are shown in brackets above the respective arguments. By contrast to the optimal solution, under the decentralized competitive solution, households ignore the external effect that their choices have on productivity next period, and thus the equilibrium choices of λ and L satisfy the equilibrium conditions (derived in the appendix), $\lambda = (1-\theta)$ and $L = (1-\omega)$.[3] By inspecting equations (10) and (11), it is evident that $\lambda^* > (1-\theta)$ and $L^* > (1-\omega)$, since the policymaker accounts for the positive externality due to learning-by-doing and the spillover.[4] The comparative statics results are eminently intuitive: a greater degree of impatience, a greater marginal valuation on the laggard good, and a lower degree of learning

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3. Notice from (10) and (11) that the competitive solutions coincide with the optimal solutions when $\beta = 0$. Intuitively, when the future does not matter, it is optimal to ignore the externality since it does not yield a benefit until next period.
4. The optimal choices of λ and L can be supported in competitive equilibrium by an equivalent economy-wide optimal subsidy to labour and an incremental optimal subsidy to the high-learning sector, if lump sum financing is assumed. These computations have been omitted for reasons of space but are available upon request from the author.

spillovers all imply lower optimal values for λ^* and L^*;[5] in addition, a higher marginal valuation on leisure will reduce λ^*. These results tie nicely into the infant industry literature (see, for instance, Bardhan [1970, 1971], Clemhout and Wan [1970], Teubel [1973], and Succar [1987]) in which the validity of the case for subsidization of sectors subject to learning effects is examined; indeed, although my discussion has been cast purely in terms of externalities and spillovers, the result that λ^* and L^* exceed their decentralized levels can be given an infant industry interpretation, where Y, the progressive sector, is the *infant*.

The implications for optimal growth are immediate. Substituting (10) and (11) into (4′), notice that so long as λ and L are constant – which they are guaranteed to be – $A(t)$ grows at a constant rate and hence $I(t)$ can be shown to grow at the following constant rate,

$$\varphi(\overset{+}{\beta}, \overset{-}{\theta}, \overset{+}{\mu}, \overset{-}{\omega}, \overset{+}{\gamma}) = \mu\gamma\lambda^* L^*; \tag{12}$$

where $\varphi \equiv \ln I(t+1) - \ln I(t)$. In this economy, as discussed, it is optimal to jump at time 0 to the constant, steady-state growth rate given in (12). By contrast, the competitive growth rate is given by,

$$\varphi' = \mu\gamma(1-\theta)(1-\omega) \tag{12′}$$

where it is evident that $\varphi > \varphi'$. Hence, the optimal growth rate exceeds the competitive growth rate, because, at the optimum, labor supply is higher and a greater fraction of that labor supply is devoted to production of the progressive good. Interestingly, while the optimal growth rate exceeds the competitive rate, it is not the maximal feasible growth, $\mu\lambda$, which would be achieved by setting $\lambda = L = 1$. Hence, just as in the one-sector model of Barro [1990], this model reveals the nonmonotonicity of the growth-welfare relationship. Starting from the competitive growth rate, welfare at first increases as growth increases, reaches a maximum at the growth rate given in (12), and decreases thereafter.

The comparative dynamics results on the growth rate can be read off from the signs of the partial derivatives in (12) and are intuitively plausible. Thus, for instance, and economy with a higher marginal valuation on leisure, ω, or on the

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5. This result extends Succar's [1987] finding that the optimal path of output subsidies is higher when spillovers are present than when they are not; my result indicates that the optimal allocations of λ and L (and hence the corresponding subsidies) increase *monotonically* in μ, the spillover parameter.

laggard good, θ, would grow more slowly, as would an economy with a greater degree of impatience as reflected in a lower value for the discount rate β. But these results do not carry any welfare implications, since in each case the growth rate φ in (12) represents the first-best optimum.

Similarly, it is easily shown that, along the optimal path, the relative price of the progressive good falls at the following constant rate,

$$\pi = (\mu - 1)\gamma \lambda^* L^*, \tag{13}$$

where $\pi \equiv \ln p(t+1) - \ln p(t)$, which of course exceeds the rate at which p falls in the competitive equilibrium,

$$\pi' = (\mu - 1)\ \gamma(1-\theta)\ (1-\omega) \tag{13'}$$

Hence, along the optimal path, the economy becomes progressively more competitive in the progressive good faster than it does along the competitive path. The rate at which the progressive good cheapens, along either the optimal or competitive path, of course depends on the same underlying parameters as determine the growth rate. This means that the model has simple and powerful implications for comparative advantage, which in this setting depends both on initial conditions and the parameters.[6]

To think about the trade implications, consider two economies which are initially closed, from time 0 to time T, and then move to free trade at time, T, and remain in free trade thereafter, where time T is exogenously given, Furthermore, to rule out strategic behavior, suppose that the opening to trade at T is unforeseen by policymakers in both countries; that is, from 0 to $(T-1)$, there is no anticipation in

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6. The trade implications of this model contrast interestingly with those of the *uneven development* models (see, for instance, Krugman [1981] and Dutt [1986]), in which externalities play a crucial role in assuring that small differences in initial conditions can generate large and cumulate divergences in growth rates and comparative advantage. In this model, by contrast, the effect of initial conditions is always dominated eventually by parametric differences; thus, given a long enough period in autarky before the two countries open to trade, the effect of initial conditions will be obliterated by parametric differences. The reason is that, in this model, the economy jumps onto a balanced growth path with a constant rate of growth at time 0; hence, a higher growth rate will allways eventually dominate a lower starting value. In the models by Krugman [1981] and Dutt [1986] in which growth effects die away over time, initial conditions can win out.

either country that they will open to trade at T.[7] Then, it is evident that comparative advantage at date T, which determines which country will specialize in and export which commodity, is determined both by initial conditions and parametric differences. Suppose that initial conditions and parameters, with the exception of β, are identical in the two countries; let country 1's discount rate be β, country 2's discount rate be β', where $\beta' \equiv (\beta - \varepsilon)$, $\varepsilon > 0$, ε arbitrarily small, that is, country 1's policymaker is less impatient than country 2's by an arbitrarily small degree. Then, the competitive allocations and hence the competitive paths of prices and outputs would be the same in these two economies, and thus, at date T, these two economies will have no reason to trade with each other. By contrast, along the optimal paths, country 1 will clearly grow faster, and the progressive good will become cheaper faster, than in country 2, and hence, at T, country 1 will specialize in and export Y and country 2 will specialize in and export X. Growth will jump in country 1 as it specializes in Y and will cease in country 2 as it specializes in X.[8] Thus, considering optimal growth paths as opposed to competitive ones in a learning-by-doing growth model allows us to explain divergence in growth rates and gains from trade between economies which are otherwise identical except for the degree of impatience reflected in the discount factor β. Note as well, following the discussion in Clarida and Findlay's [1991] analysis, that this result can be given a political economy interpretation. Suppose that the *true* β in countries 1 and 2 is identical; however, policymarkers in country 1 have a higher β than those in country 2. Then, country 1 will grow faster than country 2 in autarchy, and 1 will have a comparative advantage in, and export, Y in free trade equilibrium. Hence, if policy is determined with reference to the policymaker's β and not the representative household's, this model predicts divergence in growth rates and gains from international trade, which

7. Clearly, if policymakers in either country either could choose date T or anticipate it, the policymaker may have an incentive to *engineer* comparative advantage in the progressive good by increasing λ and L above their optimal levels or by delaying opening to trade until the country has *naturally* achieved comparative advantage. I do not take up these questions of strategic policy in this paper.

8. This strong result depends on my assumption that, while knowledge spills over between sectors in an economy, it does not spill over between countries (corresponding to Krugman's [1987] $\delta = 0$ case). While this may seem unreasonable in a world in which productivity improves due to profit-maximizing R&D investments, which may be imperfectly appropriable due to imperfect international patent protection, it seems more plausible in a world in which productivity improves due to on-the-job learning and spillovers depend on geographical proximity.

cannot be explained by appealing to the decentralized, competitive growth paths of these economies.

Even barring trade in goods, the model also has very strong implications for international migration and economic integration. Suppose once again two economies, otherwise identical (including population) except for the policymaker's β, and now, suppose that, at time T, the two economies open not to free trade in goods but allow free mobility of labor. Then, since from date 0 to date T country 1 has grown faster and its real wage, denominated in either good, is higher, labor will migrate from country 2 to country 1, which will only widen the gap in growth rates and wage rates between the two countries, recalling from (12) that the growth rate is linear in the labor force;[9] in the presence of increasing returns, the incentive to migrate does not diminish. Thus, the process will continue until country 2 is completely depopulated.[10] Thus, just as in Krugman [1979], history can be decisive in determining the equilibrium outcome in a world characterized by increasing returns.[11]

As for economic integration, suppose now two absolutely identical economies, including population level and the respective policymaker's β; then, even along the optimal paths, these economies will evolve identically, and there will be no incentive either for trade in goods or for migration. However, these economies can gain by integrating, since doing so will increase – in this simple case double – the available labor supply and hence double the rate of economic growth and increase the utility of the representative household. In such a world, these will clearly be strong pressures for economic integration, since even identical economies can gain by merging. Indeed, there is a case to be made that economic integration will

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9. This is because, if population is N, rather than normalized to unity, equation (12) becomes:

$$\varphi = \mu\gamma\lambda LN. \tag{F. 1}$$

In such a world, differences in population alone suffice to generate divergence in growth rates and gains from trade or labor migration.

10. This strong result depends on the assumption that labour is completely homogeneous between countries; if labour from country 2 could only be absorbed into country 1's economy with training and adjustment costs, presumably equilibrium will be attained at a positive level of population in country 2.

11. In the context of a model similar to this one, Krugman [1991] points out that, in a world with costless adjustment, it is self-fulfilling expectations, not history, which are decisive, whereas, with sufficiently costly adjustment, history is always decisive. Matsuyama [1991] points out that this result may depend crucially one linearity of the dynamics in Krugman [1991].

maximize world welfare in such a setting, and furthermore integration will be more appealing from the political-economy perspective to country 2's policymakers, since it will avoid the unpalatable consequences of depopulation with free labor mobility or cessation of growth with international trade.

Caution is required, however, in interpreting these results on labor mobility and integration as also for results in models by Rivera-Batiz and Romer [1991a, 1991b]. The strong results clearly follow from the strong assumptions, in particular the assumption that international transfers of know-how are ruled out. If, for instance, a country which has experienced greater learning-by-doing could license this know-how to the other country, the other country reap at least some of the learning gains without being forced into a *merger* with the high-learning country. As a perceptive referee has pointed out, what is really happening in the integrated case is that we move from no technology transfers to perfect technology transfers. If there are less drastic alternatives available, higher growth might still be available to the low-learning country without needing to sacrifice its national sovereignty, but one would need to factor into the analysis the payment of royalties to the licensor and the incentive for the licensor to transfer inferior know-how. These issues, however, take us beyond the scope of this analysis.

Finally, I consider the implications of the model for asset prices along the competitive and optimal paths. The simplest way to do this is to introduce a market for a one-period discount bond, denominated in units of the numeraire good, X, which is in zero net supply. Since this is an economy with nonstorable outputs, we know that, in equilibrium, the price of this bond will have to adjust so that the representative household just demands its own endowment and so that there is no trade in equilibrium, making all of the usual assumptions of the Lucas [1978] asset pricing model. Suppose that $q(t, 1)$ denotes the price, in terms of X, of a bond purchased at date t which pays one uint of X at date $(t + 1)$. It is shown in the appendix that, in equilibrium.

$$q(t,\ 1) = \beta\{X(t) / X(t+1)\} \tag{14}$$

which in turn implies from (3) and (4′) that

$$q(t,\ 1) = \beta\{A(t) / A(t+1)\}^{\mu} \tag{14′}$$

Equation (14′) has immediate implications for the path of asset prices along the competitive and optimal paths. In particular, it is easily shown that, to a good approximation,

$$\ln q(t,\ 1) = \ln \beta - \varphi, \tag{15}$$

or, alternatively,

$$i(t,\ 1) = \rho + \varphi \tag{15'}$$

since $q\,(t,\,1) \equiv \{1 + i\,(t,\,1)\}^{-1}$, where $i\,(t,\,1)$ is defined as the one-period interest rate, and ρ is the pure rate of time preference, where $\beta \equiv (1+\rho)^{-1}$. Notice that the one-period interest rate exceeds the pure rate of time preference by the magnitude of the growth rate. Hence, either along the competitive or optimal paths, asset prices (and interest rates) are constant in equilibrium. Inspecting equation (15), and recalling equations (10) and (11), it is evident that $q\,(t,\,1)$ is lower ($i\,(t,\,1)$ is higher) along the optimal path than along the competitive path.[12]

One interesting implication is that, looking only at competitive paths, the model predicts, at least in the closed economy, a negative correlation between growth rates and asset prices in a cross-country regression, that is, a positive correlation between growth rates and interest rates. However, along the optimal paths, if we suppose that differences in growth rates arise from differences in the policymaker's β, as discussed above, there is no clear-cut prediction on the correlation between growth rates and interest rates, since substituting the optimal values for λ and L form equations (10) and (11) into equation (15) and differentiating with respect to β, the sign of the derivative is indeterminate. Hence, if differences in growth rates stem largely from differences in the policymaker's degree of patience, there is no *a priori* reason expect the positive correlation between interest rates and growth rates that is predicted by looking at the competitive paths.

I would point out that these results on the relation (or lack of it) between growth rates and asset prices along the optimal growth path are devoid of policy implications; since, by construction, the growth paths are optimal, the differences in interest rates

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12. In the open economy, the model's prediction about interest rates will depend on whether or not borrowing and lending are allowed between countries. If they are not, interest rates (in line with growth rates) will further diverge when the two economies engage in free trade of goods. I do not take up in this paper an examination of the implications when borrowing and lending are allowed.

along these path must also be optimal, so long as international borrowing is ruled out, as I have done. Once again, an examination of the consequences or relaxing this assumption is beyond this paper's mandate.

III. Conclusion

It is to be hoped that the preceding analysis has cast some light and further sharpened the reader's intuition on the nature of optimal policy intervention and optimal endogenous growth in a model with external learning-by-doing and intersectoral spillovers. The strategy has been to simplify the economic structure, drastically where necessary, to obtain crisp closed-form expressions and balanced growth paths and hence easily interpreted comparative statics results. Stepping back from the specific model presented here, however, one may introspect on which of the lessons drawn in such a simple setting will carry over to more general and realistic models.

First, it seems evident that, in any economic environment characterized by intersectoral differences in productivity driven by external learning-by-doing, optimal policy intervention will require the redeployment of some quantity of the economy's resources away from the *laggard* toward the *progressive* sectors, which can surely be expected to deliver a boost to the rate of economic growth. However, the growth path will most likely not be the simple step function of the model here, but rather will be smoothed out as labor, capital, and other resources move slowly between the two sectors, perhaps according to some cost of adjustment technology. Furthermore, when a country opens to trade with comparative advantage in the *laggard* sector, it will not see its growth cease but only fall somewhat as resources are redirected by the international price signal from the *progressive* to the *laggard* sector, unlike in the Ricardian world modelled here, the progressive sector will not altogether disappear (at least for a terms of trade change which is not too large), because of the increasing rate of product transformation implied by moving along a concave (as opposed to linear) transformation frontier.

Indeed, it is easy to think of a relatively straightforward extension of this model to the neoclassical Heckscher-Ohlin setting. Suppose an economy with two sectors and two factors, labor and capital. Suppose that the capital-intensive sector is also the *progressive* sector in terms of learning-by-doing. Then, it is immediate that relatively capital-abundant economies, upon opening to trade, will experience a jump in their growth rate as both and capital are redeployed toward the sector with comparative advantage, and similarly growth will fall in labor-abundant economies. Thus, the analysis can be neatly tied into the arguments driven by factor proportions in neoclassical trade theory.

Second, the model could be modified to incorporate the possibility of factor unemployment in the transition process as an economy opens to trade. Suppose, for instance, that there is some transitory stickiness in factor rewards in one or possibly both of the sectors; then, in the transition process as the economy is shifting resources from one sector to the other, there could be temporary unemployment and possibly therefore temporarily lower growth until the new steady-state is reached at which factor prices have adjusted toward their new equilibrium levels.

Third, the model as presented has examined issues of growth, trade, and integration between completely generic economies. However, it seems reasonably clear that the model assumptions better fit developed market economies (DMEs) rather than less developed economies (LDEs), since the assumptions of full employment and learning-by-doing in the *progressive* sector better seem to fit the DMEs. An interesting extension of the model, therefore, might be to model the interaction between a DME, which experiences learning-by-doing, and an LDE, which does not experience such learning or perhaps experiences a lower rate of learning but has an advantage along another dimension, say, an abundant and hence low-cost labor force or abundance in natural resources that augment factor productivity. It would be interesting to examine the growth effects of such an interaction on both the DME and LDE, and would in turn be nice extension of the older generation of *North-South* models of trade and growth to the newer generation of growth theory. This is left as the subject for a future paper.

Appendix

I derive the conditions for the decentralized, competitive equilibrium and the asset pricing equation in this appendix. The representative household maximizes its lifetime utility, given in equation (1), subject to the following dynamic budget constraint,

$$q(t,\ 1)\ Z(t+1) + X(t) + pY(t) \le w(t)\ L(t) + Z(t) \tag{A. 1}$$

where $Z\ (t{+}1)$ is the number of one-period discount bounds purchased at date t

which pay off at date $(t+1)$ and w is the wage rate. Notice as well that the period utility function can be rewritten as follows in logs,

$$u\{L(t), X(t), Y(t)\} = \omega \ln\{1 - L(t)\} + (1-\omega)\{\theta \ln X(t) + (1-\theta) \ln Y(t)\}, \quad \text{(A. 2)}$$

which can be substituted into (1) to facilitate easy calculation.

Suppose that conditions for an interior optimum are satisfied. Then equation (A. 1) will hold with equality. To derive the first order conditions for an interior optimum, set up the Lagrangean expression,

$$\mathcal{L} = \sum \beta^t \{u[L(t), X(t), Y(t)] + v(t)[wL(t) + Z(t) - q(t,1)Z(t+1) \\ - X(t) - pY(t)]\}$$

where $\mathcal{L}$ is the Lagrangean and $v(t)$ is the Lagrange multiplier associated with the constraint at date t. Then, the first order conditions with respect to $\{X(t), Y(t), L(t), Z(t+1)\}$ are:

$$(1-\omega)\theta / X(t) - v(t) = 0 \quad \text{(A. 4a)}$$

$$(1-\omega)(1-\theta) / Y(t) - v(t)p(t) = 0 \quad \text{(A. 4b)}$$

$$-\omega / L(t) - v(t)w(t) = 0 \quad \text{(A. 4c)}$$

$$-q(t, 1)v(t) + \beta v(t+1) = 0 \quad \text{(A. 4d)}$$

Taking a ratio of equations (A. 4a) and (A. 4b), and substituting for $X(t)$, $Y(t)$, and $p(t)$ from equations (3), (2), and (6), respectively, yields the following equilibrium condition for $\lambda(t)$:

$$\lambda(t) / \{1 - \lambda(t)\} = (1-\theta) / \theta \quad \text{(A.5)}$$

Taking a ratio of equation (A. 4c) and either equation (A. 4a) or equation (A. 4b) and substituting as before yields:

$$L(t) / \{1 - L(t)\} = (1-\omega) / \omega \quad \text{(A. 6)}$$

This justifies the appeal to these equilibrium conditions in the text.

I now derive the asset pricing equation, (14). Rearranging equation (A. 4d) yields:

$$q(t,\ 1) = \beta\{v(t+1)\,/\,v(t)\} \tag{A. 7}$$

Leading equation (A. 4a) by one period, and substituting the resulting expression and equation (A. 4a) into equation (A. 7) yields,

$$q(t,\ 1) = \beta\{X(t)\,/\,X(t+1)\} \tag{A. 8}$$

which is exactly equation (14) in the text.

Of course, to go from equation (14) as a first-order condition for a representative household to equation (14) as an equilibrium condition for asset pricing requires that we make the standard assumptions as laid out in Lucas [1978], that is, each household has identical preferences, identical endowments of time per period, and identical initial shares of the discount bond, $Z(0)$. Then, there will be no trade in equilibrium, and equation (14) will be satisfied as an equilibrium condition on asset prices, such that $q(t,\ 1)$ adjusts to ensure no trade in equilibrium.

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