Public Inputs and the Pattern of Trade
between Underemployed Economies

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Abstract

This paper investigates the relationship between the supply of a pure public input and the pattern of trade between underemployed economies.

I. Introduction

Although large proportion of public budgets are directed towards public input provision, yet most theoretical studies do not consider the relationship between the supply of public inputs and the pattern of international trade.

Abe [1990] is the only available study where the relationship between the supply of a public input and pattern of international trade is considered.[1] Abe considers the trade pattern between economies which fully utilise all resources. However, significant labour unemployment is present in most real economies, it is therefore appropriate to consider the trade pattern between underemployed economies.[2] The pur-

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1. Most open economy studies which include government spending on public inputs are concerned with the shape of the production possibility curve. For example, Manning and McMillan [1979], Okamoto [1985], Tawada [1980, 1982], Tawada and Okamoto [1983], and Tawada and Abe [1984]. Tawada and Abe [1984] and Tawada and Okamoto [1983] have also examined the validity of various theorems in international trade theory in the presence of government spending on public inputs.

2. The term underemployed economies has been used by Batra and Beladi [1990]. The under-

pose of this paper is to examine the role of public inputs in determining the pattern of trade between underemployed economies.

The paper is organised as follows. The relationship between the supply of a pure public input and the pattern of trade between underemployed economies is examined in section two. The last section contains concluding remarks.

II. Public Input as a Determinant of Trade between Underemployed Economies

The purpose of this section is to develop a simple framework which allows an investigation of the relationship between the supply of a pure public input and the pattern of trade between underemployed economies.

Consider a self sufficient economy which produces two private goods: an intermediate good (R) and a final good (Y).[3] The final good is produced by means of a pure public input (G), the intermediate good, capital, and labour. Whereas, the intermediate good is produced by means of the public input, capital, and labour. The pure public input is provided free of charge by the government. The public production is financed by means of a flat rate on domestic income: government converts its tax revenue into a public input without additional cost.[4] A widely cited (see for instance, Laffont [1975] and Negishi [1973]) example of such public input is government financed applied scientific research whereby information on new production techniques is made available to all firms simultaneously.

The economy under consideration is assumed to be underemployed. Underemployment in most open economy studies is assumed to be due to rigid wages, see Bhagwati and Srinivasan [1983], and Batra and Beladi [1990] for example. The present study follows the literature in adopting this assumption. It is notable that in the present setting despite wage rigidity, international trade does not lead to complete specialisation.

The production functions for the final good (Y) and the intermediate good (R) are

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employment refers to labour unemployment due to rigid wages. Batra and Beladi have examined trade pattern between underemployed economies. However, they do not consider the role of public inputs in determining the comparative advantage.

3. This set-up is similar to the one utilised by Kemp and Ohyama [1978].

4. This is an assumption widely used in the literature, see Barro [1990] for instance.

given below:

$$Y = G^\alpha F(K_y, L_y, R); \quad 1 > \alpha > 0$$

$$R = G^\beta H(K_r, L_r); \quad 1 > \beta > 0$$

where α and β are constants; K_y and K_r are capital used in the production of final and intermediate good respectively; L_y and L_r are labour used in the production of final and intermediate good respectively.

The functional form of the above production technologies implies that the public input is cooperative with private inputs in the production of Y and R and there are diminishing returns with respect to the public input. $F(.)$ and $H(.)$ are linearly homogeneous with respect to the relevant inputs and the competitive firms take the supply of public input as given. The production functions described above are therefore the industry production functions. There are economies of scale in the present case but these economies are external to both the firm and the industry. The final good is the numéraire and public input is also measured in its units.

The relevant cost functions are derived below:

$$Y: c(r, w_o, p)/G^\alpha = \text{Min} \; [w_o L_y + rK_y + pR : Y = G^\alpha F(K_y, L_y, R)]$$

with respect to L_y, K_y, and R.

$$R: e(r, w_o)/G^\alpha = \text{Min} \; [w_o L_r + rK_r : R = G^\beta H(K_r, L_r)]$$

with respect to L_r and K_r.

where w_o and r are the minimum wage rate and the rate of return on capital; p is the price of intermediate good; $c(r, w_o, p)/G^\alpha$ and $e(r, w_o)/G^\alpha$ are unit cost functions for the final and intermediate goods respectively.

The zero profit conditions for the self sufficient economy are the following:

$$c(r, w_o, p)/G^\alpha = 1 \tag{1}$$

$$e(r, w_o)/G^\beta = p \tag{2}$$

For a given supply of public input, (1) and (2) determine the rate of return on capital and the autarky price of intermediate good independent of factor market

clearing conditions. Once the optimal r and p are determined, the factor market clearing conditions given below determine the output of the intermediate and final good along with labour employment

$$K_e = Y\{c_r(r, w_o, p)/G^\alpha\} + R\{e_r(r,w_o)/G^\beta\} \tag{3}$$

$$L = Y\{c_w(r, w_o, p)/G^\alpha\} + R\{e_w(r,w_o)/G^\beta\} \tag{4}$$

$$R = Y\{c_p(r, w_o, p)/G^\alpha\} \tag{5}$$

where K_e and L are capital supply and labour employment respectively; $c_r(.)/G^\alpha$ and $e_r(.)/G^\beta$ are capital requirement per unit of Y and R respectively; $c_w(.)/G^\alpha$ and $e_w(.)/G^\beta$ are labour requirement per unit of Y and R respectively; and $c_p(.)/G^\alpha$ is intermediate good requirement per unit of Y.

In the following section, the relationship between the supply of public input and pattern of trade is considered.

A. The Pattern of Trade

Consider two countries of the type described in the previous section. If both countries are identical in every respect, there is no basis for trade. Suppose that one country produces more public input: the autarky price of the intermediate good in the two countries will therefore be different. (1) and (2) can be used to derive the following derivative which describes the impact of an increase in the supply of public input on the autarky price of the intermediate good

$$\partial p / \partial G = [\alpha G^{\alpha-1}e_r(\cdot) - \beta G^{\beta-1}c_r(\cdot)]/[G^\beta c_r(\cdot) + c_p(\cdot)e_r(\cdot)] \tag{6}$$

If both industries benefit equally from the supply of public input (i.e., $\alpha = \beta$) then (6) can be re-written as[5]

$$\partial p / \partial G = \alpha G^{\alpha-1}[e_r(\cdot) - pc_r(\cdot)]/[G^\alpha c_r(\cdot) + c_p(\cdot)e_r(\cdot)] \tag{7}$$

$$= \alpha pG^{2\alpha-1}[K_r / pR - K_y / Y]/[G^\alpha c_r(\cdot) + c_p(\cdot)e_r(\cdot)]$$

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5. Properties of cost functions are used in the derivation of equation (7). For an excellent survey of duality theory see Diewert [1974].

If K_r/pR is greater (less) than K_y/Y then the above derivative is positive (negative). The following proposition follows immediately: *if two countries have identical preferences, production technologies, supply of primary inputs, minimum wage rates, and both industries benefit equally from the supply of public input, then the country which produces more public input exports the output of that industry which uses less capital per-unit of output; measured in value terms.* This follows from the fact that the rate of return on capital is higher in a country which produces more public input.

The result derived by Abe [1990] can be easily compared with the one derived in the present study. Abe [1990] has shown that when both private industries derive equal benefits from the supply of a public input, and factor intensity of the private and the public sectors is identical, then differences in the supply of a public input cannot influence the pattern of trade between two fully employed economies.[6]

It is interesting to note that the model presented in this paper can also be used to show that the differences in the minimum wage rates alone can also explain the pattern of trade between underemployed economies. The following derivative is derived by using (1) and (2)

$$\partial p / \partial w_o = - [e_r(\cdot)c_w(\cdot) - e_w(\cdot) \, c_r(\cdot)] / [G^\beta c_r(\cdot) + c_p(\cdot)e_r(\cdot)] \qquad (8)$$

$$= e_w(\cdot)c_w(\cdot)[K_y/L_y - K_r/L_r]/[G^\beta c_r(\cdot) + c_p(\cdot)e_r(\cdot)]$$

The following proposition follows immediately: *if two countries have identical preferences, production technologies, supply of public and primary inputs, then the higher minimum wage country exports the output of that industry which is more capital intensive.* This result follows from the fact that a higher minimum wage rate is associated with a lower rate of return on capital.

III. Concluding Remarks

This paper examines the relationship between the supply of a pure public input and the pattern of trade between underemployed economies. Each country pro-

6. In Abe [1990], the public input is produced by means of private inputs. On the other hand, the present study assumes that the public input is produced out of the final good. In other words, the present study implicitly assumes that the factor intensity of the private and the public sectors is identical.

duces two private goods: a final good and a intermediate good. In the present framework despite wage rigidity international trade does not lead to complete specialisation.

It is shown that when two countries have identical preferences, technology, minimum wage rates, supply of primary inputs, and both industries benefit equally from the supply of public input then the country which produces more public input exports the output of that industry which uses more capital per-unit of output; measured in value terms.

This paper also shows that if two countries have identical preferences, technology, and the supply of public and private inputs then the higher minimum wage country exports the output of that industry which is more capital intensive.

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