Journal of International Economic
Integration 6(1) Spring 1991, 64−90

The Optimal Borrowing Tax

James E. Anderson*

Abstract

Optimal public finance analysis of intertemporal models has been devoted to special cases where the distortion of intertemporal choice is avoided. In contrast, practical public finance has significant distortion of intertemporal choice. This paper characterizes optimal intertemporal taxation in a simple model. With distortionary taxation necessary, fluctuations of productivity of external real interest rates are potent possible reasons to distort intertemporal choice. Borrowing/lending taxation is shown to be generally required for efficiency, even in the special separable case usually thought to imply no intertemporal distortion. The time profile of the optimal borrowing/lending tax is characterized in a special case, and the resulting relation between optimal public debt and the trade account is analyzed.

I. Introduction

Optimal tax analysis of intertemporal models has typically been devoted to special cases where the distortion of intertemporal choice is avoided. For example, in one benchmark class of analytic models, the optimal consumption or wage tax for revenue purposes is uniform over time(Barro (1979), Kydland-Prescott(1982), Razin-Svensson (1983)), due to a separability assumption.[1] In other words, the *optimal* revenue-raising policy appears to involve no (or little) distortion of intertemporal mar-

* Department of Economics, Boston College, Chesthut Hill, MA, 02167, U.S.A.

1. The condition is that the representative consumer's utility function be weakly separable with respect to the partition between the sequence of goods and the sequence of leisures and that subutility functions be homothetic. Thus intertemporal aggregates exist in consumption and leisure, and the tax structure analysis boils down to essentially a one period case. The condition is very restrictive for an intertemporal study. The separability condition implies that the marginal rate of substitution between consumption today and tomorrow is independent of the amount of leisure either today or tomorrow.

ginal rates of subsitution away from marginal rates of transformation, and a single, constant-over-time distortion of the marginal rate of subsitution between leisure and consumption away from the marginal product of labor, using either a consumption or a wage tax. Since uniformity economizes on information, may help with *time-inconsistency* problems of the credibility of announced government tax policy, and presumptively is equitable in an intergenerational setting, its theoretical appeal is great.

In contrast, from the general setup of the Ramsey problem, it is clear that optimal taxation *generally* requires distortion on all margins, while leaving the exact form of the tax structure implicit in the inverse of an elasticity matrix. The problem of the current optimal taxation literature (see for example Swaroop (1989)) is to go between these extremes and somehow characterize the optimal tax structure. This paper analyzes the nature and significance of distortion on the intertemporal margin in an optimal tax model. (A companion paper analyzes the time profile of the optimal consumption tax.) The related work of Frenkel and Razin (1987) initiates the positive analysis of borrowing taxation, but stops short of considering optimality.

From a practical point of view, governments do generally distort the intertemporal choices of agents. Significant distortions arise when external capital mobility is taxed (often by means of dual exchange rates) or otherwise limited in the presence of internationally integrated capital markets. One potentially important rationale is that governments do so as part of their fiscal policy. It is thus useful to explore the fiscal policy implications of external borrowing taxation in a simple model. Many other possible reasons for governments to limit or tax external capital movements (the noncooperative game involved in *capital flight* is the most obvious example), but this one deserves attention.

To rule out any other motives for borrowing taxation consumers and the government both have perfect foresight, external capital markets are perfectly competitive with infinitely elastic supply of credit, and the government credibly commits to its announced tax policy. The accumulation of capital is suppressed. Thus the very common practice of using temporary capital controls or *financial* exchange rates can be justified even in an economy with no distortions other than those necessitated by optimal consumption taxation (usefully thought of as optimal crowding out). While the model is easiest to think of as applying to small open economies, Appendix 2 shows that the external capital market is not formally necessary, since the same results are obtained in a closed economy with a *storage* technology.

The basic model is laid out in Part Ⅱ. Part Ⅲ shows that even under the Razin-Svensson sufficient conditions for uniform-over-time consumption or wage taxation a borrowing tax is required for efficiency, in turn implying a time-varying consumption tax.[2,3] The borrowing tax permits the government to tax leisure, albeit combined with a surcharge on the consumption tax and with a subsidy to the endowment of time at the same rate, and this is welfare improving. A numerical illustration provided below suggests the efficiency gain from distorting on the intertemporal margin may be substantial.

Part Ⅳ shows that the combination of a borrowing tax and a consumption tax is equivalent to a combination of a wage and a consumption tax structure. (This covers the same ground as Frenkel and Razin, in a somewhat different model.) Part Ⅴ presents propositions on the temporal structure of the optimal borrowing tax given a constant consumption tax. Under the conditions below, periods of high productivity also face high borrowing (lending) taxes, so that taxation smooths the ratio of current borrowing or lending to expenditure. External real interest rate fluctuations are damped (amplified) by the borrowing tax as the elasticity of intertemporal substitution is less (greater) than one.

Finally, in Part Ⅵ, propositions are offered on the rich menu of possible correlations of optimal government budget and trade deficits when the optimal borrowing tax is imposed. Conditions sufficient for perfect positive or negative correlation are presented. In contrast, under the Ricardian equivalence model, periods of low public borrowing, *cet. par.*, are associated with high lump-sum taxation. Consumers offset the effect of high current taxes on real income by high private bor-

2. In a companion paper, Anderson-Young (1990) show that the optimal time-varying consumption tax structure is based on a combination of two factors : (i) equiproportionate consumption crowding on the contemporaneous margin, and (ii) total-expenditure-crowding on the intertemporal margin. A plausible conjecture that led to the present paper was that a borrowing tax would restore a constant consumption tax by appropriately adjusting the intertemporal margin. Instead, even the cases which produce a constant consumption tax when it is the only instrument have time-varying taxes when a borrowing tax is available. Under the Razin-Svensson separability condition, the combination of the two time-varying taxes results in a *full tax* on consumption which is, at the optimal setting, constant over time.
3. Part Ⅳ develops an equivalence among wage, consumption, and borrowing taxes such that optimal settings of any pair are equivalent. While *uniform* wage and consumption taxation are equivalent, at the uniform position it will always pay to depart from uniformity and institute either a borrowing tax or simultaneous wage subsidy and consumption taxes. We focus on the former because it is more realistic and highlights the direct distortion of the intertemporal margin which is implicit in using the other pair of tax vectors.

rowing, causing negative correlation of public and private deficits, *cet. par*. Part Ⅶ is the conclusion.

Ⅱ. The Basic Model

Section A describes the representative consumer's intertemporal budget constraint and characterizes his indirect utility function. Seciton B describes the government's revenue constraint. The optimal tax problem is discussed in Section C.

A representative consumer model is chosen for two reason. First, with a focus on efficiency rather than equity, it is an appropriate strategy to evade the multiple equilibrium possibilities of overlapping generations models. Second, intergenerational transfers are practically relevant, and in a limiting case can make the two models equivalent.

A. The Representative Consumer Problem

There is a representative consumer with a planning horizon of N+1 periods, labelled $s=0,1,\cdots\cdots,N$.[4] The consumer derives utility from the consumption of a composite tradable good X, and leisure, L in each time period.

The representative consumer's preferences over sequences of commodities, and leisure $\{(X_s, L_s)\}$ is given by a utility function which is additively separable with respect to time :

$$(1)\quad \Sigma d^s u(X_s, L_s)$$

where d^s is the period s subjective discount factor. Throughout, this function is assumed to exhibit non-satiation and be concave. Summations without limits are understood to be from $s=0$ to $s=N$.

In the open economy the composite good is tradable at a fixed foreign currency price in period s, π_s. The governement taxes consumption of the good at the rate τ_s so the consumer's price in foreign currency terms is $p_s = \pi_s(1+\tau_s)$.[5] The consumer owns H_s units of time, offering $H_s - L_s$ units of labor at price w_s. The wage rate is externally fixed by the value of marginal product relation $w_s = a_s\pi_s$ with no wage taxation (without loss of generality, due to the equivalency relation offered in Part

4. It is inessential whether N is finite in this model.
5. The domestic producer's price is always equal to π.

IV). a_s is the constant marginal and average product of labor in Ricardian production technology. Labor income is $w_s(H_s - L_s)$.

The consumer's external pre-tax flow between periods is Z_s. It equals current expenditure plus financial tax payments less current wage income at time s. Z_s represents borrowing, or foreign repayment of principal plus interest on previous loans when positive ; and lending, or domestic repayment of principal plus interest when negative. The exact temporal structure of the loan contracts is suppressed for simplicity.[6] Periods when Z is positive (negative) are conventionally labelled *borrowing* (*lending*) periods. Z_s is also equal to the private portion of the international trade deficit (surplus if Z is negative) at time s, by the one period budget constraint of the consumer and the equality of wage payments with the value of output.

The consumer can freely *borrow* (*lend*) on external markets at a fixed interest rate. Assume that the borrowing tax is equivalent to *points* charged on cash flow.[7] An external loan to the consumer of Z_s of foreign exchange nets the consumer $Z_s(1-b_s)$ of current purchasing power, where b_s is the number of points charged by the domestic government. An external loan from the consumer which incurs a foreign obligation of Z_s(i.e., $Z_s<0$) requires an outlay from the consumer of $(1-b_s)Z_s$, where $b_s<0$.[8]

This treatment of borrowing or lending taxation is easiest to envision as a *financial* exchange rate system, with borrowing or lending (or principal+interest repayment) at the financial exchange rate f_s while trade account transactions occur at the non-financial account exchange rate e_s. Under this interpretaion, $-b_s = \frac{f_s - e_s}{e_s}$, the financial exchange rate premium. In turn, the dual capital control system is implied by interpreting the financial exchange rate as including a shadow price.

Alternatively, e may equivalently be regarded as the controlled price. This implies that a sequence of b's can be reproduced by a sequence of net import taxes

6. There is no loss of generality in this treatment under the present assumptions.

7. For example, for a one period loan of Z_0 dollars there is a points charge such that the consumer receives less than Z_0 dollars and must repay the obligation of Z_0 with payment Z_1 equal to one plus the interest rate times Z_0.

8. Implicit in the treatment of intertemporal payments is a possible *double taxation* of capital transactions with the borrowing being taxed with *points* and the repayment being similarly taxed. The stucture can be converted into a more conventional one in which the sequence of b's implies a sequence of one-period effective interest rates. The present structure has the great analytic advantage of isolating the intertemporal expenditure shifting behavior of consumers.

(deficit periods) and net export subsidies (surplus periods).[9] In turn the dual solution is to subject the trade account to exchange controls.

The consumer's expenditure on goods, p_sX_s, is constrained by wage income plus the net cash from *borrowing*, $w_s(H_s - L_s) + Z_s(1-b_s)$ in any period s. The intertemporal structure requires that the consumer's sequence of Z'_s is constrained by $\Sigma\delta^s Z_s \leqq 0$, with the fixed external discount factor $\delta^s = (1+r)^{-s}$, where r is the external interest rate.

Formally, the consumer maximizes the utility function (1) subject to the sequence of one-period budget constraints :

$$(2)\ p_sX_s \leqq w_s(H_s - L_s) + Z_s(1-b_s)\ ;\ s=0,\cdots\cdots,N$$

and the intertemporal zero net indebtedness constraint :

$$(3)\ \Sigma\delta_s Z_s \leqq 0.$$

A useful alternative form of (2) is obtained by dividing through by $(1-b_s)$. Since borrowing and lending are both possible, (2) can then be solved for Z and the result substituted into (3). Then imposing non-satiation, the consumer is constrained only by

$$(4)\ \Sigma\delta'_s\{p_sX_s - w_s(H_s - L_s)\} = 0,\ \text{where}$$

$$(5)\ \delta'_s = \delta^s/(1-b_s).$$

(4) is finally rearranged as :

$$(6)\ \sum_0^N \delta'_s(p_sX_s + w_sL_s) = W',$$

where consumer wealth W' is given by

$$(7)\ W' = \Sigma\delta'_s w_sH_s.$$

The consumer's problem is restated as

$$(8)\ \max_{\{X_s, L_s\}} \Sigma d^s u(X_s, L_s)\ \text{subject to}\ \Sigma\delta'_s(p_sX_s + w_sL_s) = W'.$$

The optimal sequence of X, L depends on the sequence of prices and wealth : $\{X_s(\{p_t, w_t, \delta'_t, W'\}), L_s(\{p_t, w_t, \delta'_t, W'\})\}$. Substituting into the utility function yields the indirect utility function

$$(9)\ V(\{p_t\}, \{w_t\}, \{\delta'_t\}, W') = \Sigma d^s u(X_s(\{p_t, w_t, \delta'_t, W'\}), L_s(\{p_t, w_t, \delta'_t, W'\})).$$

9. These are not import taxes or export subsidies, which have relative price effects.

B. The Government's Problem

For its own purposes the government spends a sequence of foreign currency expenditures $\{D_s\}$ with present value

(10) $\Sigma\delta^s D_s = D.$[10]

Government can freely borrow at the external discount factor δ, subject to a zero net indebtedness constraint. Tax revenues must be raised equal in present value to government obligations, D. Lump-sum taxation is infeasible. as is the taxation of leisure. Wage, consumption or borrowing taxes may be used. Becuase of an equivalence relation among these, only two are independent. The latter two are studied here with a demonstration of equivalence in Part Ⅵ.

It is convenient to arrange the analysis so that the government's tax problem is to crowd out sufficient private consumption in external price terms. The consumer's intertemporal budget constraint for this purpose is alternatively :

(11) $\Sigma\delta^s\ [p_sX_s + w_sL_s + b_sZ_s] \leqq \Sigma\delta^s w_sH_s.$

(11) is obtained from solving (2) for Z_s and substituting into (3). The government budget constraint in its revenue = expenditure form is :

(12) $\Sigma\delta^s\ [(p_s-\pi_s)X_s + b_sZ_s] = D.$

Subtracting (12) from (11) and using $w_s = a_s\pi_s$:

(13) $\Sigma\delta^s\pi_s(X_s + a_sL_s) + D \leqq \Sigma\delta^s a_s\pi_sH_s = W,$

which is the *crowding-out* form of the external budget constraint facing the government. The problem is to optimally use the consumption tax and the borrowing tax sequences to crowd out $\{X\}$ and $\{L\}$ to admit D. Define the *foreign exchange value*

10. Following the convention of the macro literature, we can suppose that the government is committed to a real consumption g_s of the composite good in each period, with foreign exchange value $D_s = \pi_s g_s$. Analysis at a deeper level proceeds as follows. A sequence of government consumption of the composite good is purchased to maximize a government utility function subject to a revenue constraint. A government in consumers' interests presumably uses these as inputs to supply a public good, and the shape of government utility reflects the technology of public goods production. The revenue level is then set such that the marginal benefit of public goods production equals the marginal cost of raising the revenue. All that is needed at this stage is a determinate solution value D, independently of the details of arriving at it.

of private consumption as

$$(14)\ C(\{p,w,\delta'\}, W') = \sum_{s=0}^{N}\delta^s\pi_s[X_s\{p_t, w_t, \delta_t', W'\} + a_s L_s\{p_t, w_t, \delta_t', W'\}]$$

Formally, the government's problem is to select the sequence of taxes $\{\tau\}$, $\{b\}$ which imply $\{p\}$, $\{\delta'\}$, to maximize the utility of the representative consumer subject to the revenue constraint.

$$(15)\ \max_{\{p,\delta'\}} V(\{p, w, \delta'\}, W') \qquad \text{subject to}$$

$$(16)\ C(\{p, w, \delta'\}, W') + D = \Sigma\delta^s w_s H_s = W.$$

I assume this problem has a well-behaved maximum (despite well-known possible difficulties). Sufficient conditions for a unique global maximum are guaranteed as direct utility is sufficiently concave, as measured by standard risk aversion concepts.

C. The Optimal Tax Structure

The optimal consumption tax profile which solves problem (15)−(16) satisfies the necessary conditions :

$$(17)\ V_p = \lambda C_p$$

$$(18)\ V_{\delta'} - \lambda C_{\delta'} \leqq 0 \text{ for all t.}$$

A negative δ' corresponds to $1\text{-}b < 0$, which is infeasible.
Subscripts denote differentiation save for the time subscripts s and t. (17) is intuitively interpreted as :

$$(19)\ V_p/C_p = \lambda \text{ for all t.}$$

A similar interpretation arises from interior values of (18). The right hand side of (19) is the social marginal utility of a gift of foreign exchange to the government at time zero. The left hand side is the social marginal utility (at time zero) of the private foreign exchange released by a change in the tax at time t. (19) implies that optimal taxation requires smoothing the latter completely over time. In contrast, the private marginal utility of foreign exchange at any time t is always V_w, since the intertemporal smoothing behavior of private agents automatically (from the planner's point of view) takes care of it. The difference between private and social marginal utility arises because of the need to use distortionary taxation ; (19) implies that the optimal time profile of consumption taxation makes the private and social marginal utility of foreign exchange proportional in all periods.

Ⅲ. The Optimality of a Borrowing Tax

A. The Borrowing Tax

The main technical issue of this section is whether the optimum could ever be achieved without the use of a borrowing tax, even in models known to yield constant consumption taxation when that is the only instrument. Formally, can (17) and (18) be satisfied with $b_t = 0$ for all t? The answer is no.

To evaluate (17) and (18) it is necessary to develop the properties of V and C. Using standard properties of the indirect utility function, it is staightforward that :

$$V_p = -V_{W'} X_t \delta'_t$$
$$V_b = -V_{W'} Z_t \delta'_t / (1-b_t).$$

The effect of the taxes on C is more complex. The Appendix develops the two-stage budgeting form of the consumer problem, and then applies it to the special Cobb-Douglas case to develop expressions for C_b and C_p. For the Cobb-Douglas case of this paper, which yields a constant consumption tax when that is the only instrument, the single-period utility is :

$$u(X_s, L_s) = \frac{1}{1-\rho} (X_s^{1-\alpha} L_s^{\alpha})^{1-\rho}$$

Combining this with (1), defining utility as the discounted sum of one-period utilities, the coefficient $1/\rho$ is the elasticity of intertemporal substitution, while $1-\alpha$ and α are the parametric consumption and leisure shares in each period. The key derivatives of C are, from the Appendix :

$$(20)\ C_{bt} = \frac{\delta'_t}{(1-b_t)} \frac{C}{W'} (-Z_t(1-b_t) + \frac{I_t}{\rho}) - \delta'_t (\frac{\pi_t}{p_t}(1-\alpha)+\alpha) \frac{I_t}{\rho}.$$

$$(21)\ C_{pt} = \frac{1-\rho}{\rho} \frac{C}{W'} \delta'_t X_t - \delta^t X_t \{\frac{\pi_t}{p_t} + \frac{1-\rho}{\rho} (\frac{\pi_t}{p_t}(1-\alpha)+\alpha)\}$$

Now I will show that the necessary conditions for optimal b and τ (the ad valorem commodity tax rate) cannot be satisfied at a zero borrowing tax. The first order condition for optimal τ using (21) in (17) implies :

$$(22)\ \frac{V_{W'}}{\lambda} = -\frac{1-\rho}{\rho} \frac{C}{W'} + (1-b_1) + \{\frac{1}{1+\tau_t} + \frac{1-\rho}{\rho} (\frac{1-\alpha}{1+\tau_t} + \alpha)\} \text{ for all t.}$$

The first order condition in optimal b using (20) in (18) implies :

$$(23)\ -Z_t \frac{V_{W'}}{\lambda} + Z_t(1-b_t) \frac{C}{W'} - \frac{I_t}{\rho} \{\frac{C}{W'} - (1-b_t)(\frac{1-\alpha}{1+\tau_t} + \alpha)\} \leqq 0 \text{ for all t.}$$

Theorem 1 : The optimal borrowing tax is not zero.

Proof : At b=0, W=W′, and the optimal consumption tax is uniform, from (22). Then $C/W=\frac{1-\alpha}{1+\tau^{*}}+\alpha$ from Appendix equation (A.5), where the required level of the tax τ^{*} is determined from the government budget constraint C+D=W. Substituting back into (22) and (23) :

(22′) $\frac{V_w}{\lambda}=\frac{1}{1+\tau^{*}}$. Using (22′), (23) becomes

(23′) $-Z_t\frac{1}{1+\tau^{*}}+Z_t\left(\frac{1-\alpha}{1+\tau^{*}}+\alpha\right)\leqq 0,$

which is necessarily violated for any period in which $Z_t>0$.‖

Theorem 1 implies that it is not optimal to forego the chance to tax leisure via the borrowing tax, even though the leisure tax is simultaneously tied to a surcharge on the consumption tax and a subsidy to the endowment of time.[11] This holds even when the optimal consumption tax rate is constant in the absence of a borrowing tax. There are two cases of constant elasticity of intertemporal substitution for which the consumption tax is constant (see Anderson−Young). The Cobb-Douglas case of (22) is one, while the other case is when the intertemporal and contemporaneous substitution elasticities are the same. Thus even in this special case, the borrowing tax is required. *A fortiori*, it will be required with more general utility functions.

B. The Implications for Consumption Taxation

Note also that for the full optimum, *the commodity tax must be time varying*. From (22), b constant with τ constant is possible, but from (23) this requires I / Z to be constant, which is impossible save for Z everywhere equal to zero. On the other hand, when ρ equals one, the *full tax* on consumption is constant over time. At $\rho=1$, (22) becomes :

(22″) $\frac{V_{w'}}{\lambda}=\frac{1-b_t}{1+\tau_t}.$

The right hand side of (22″) is the ratio of $\delta^t\pi_t$, the present external cost of the consumption good at time t, to $\frac{\delta^t p_t}{1-b_t}$, the present domestic cost of the consumption good at time t, which is the inverse of the *full consumption tax* at time t. Intuitively,

11. Consider (2) divided through by $1-b_s$:

$$\frac{p_s}{1-b_s}X_s+\frac{w_s}{1-b_s}L_s=\frac{w_s}{1-b_s}H_s+Z_s$$

Theorem 1 and (22″) together mean that the untaxability of leisure which defines the Ramsey problem can be partially overcome by taxing borrowing while at the same time smoothing out the implied extra distortion of consumption by varying the consumption tax. In the special case of (22″) this smoothing is complete.

This result can be generalized somewhat. The implicit separability assumption of Razin-Svensson suffices for the optimality of a constant consumption tax in the absence of a borrowing tax, and also implies the *full tax* is always constant. This constancy of the contemporaneous wedge between consumption and leisure might be taken to imply that taxation of capital income is never optimal in this type of model. Instead, a borrowing tax combined with a time-varying consumption tax which smooths the full consumption tax satisfies all the conditions of the earlier literature and improves welfare.

It is worth noting that the combination of the borrowing and consumption taxes still does not achieve the first best attainable with a lump-sum tax. A wealth tax achieves the first order condition $V_w/\lambda = C/W$ instead of (22)-(23). The reason is that a borrowing tax is a levy on both consumption and leisure at the same rate, and is thus not a fully independent instrument. An independent leisure tax at each date is needed to restore lump-sum equivalence.

C. A Numerical Illustration

How important is the lack of use of a borrowing tax? The full answer to this question must be obtained in a full simulation model. The simple model of this paper offers some of the intuition needed to understand simulation models, and a numerical illustration of it argues that the issue could be important in situations involving large borrowing or lending, and when the elasticity of intertemporal substitution is large. The numbers should not be taken too seriously apart from making a case for further investigation of the use of a borrowing tax.

In a two period version of the model of this paper let $\alpha = .5$, $\delta = .9$, $d = .9$, and $D/W = .20$. The first assumption implies that half the endowment of time is spent on leisure, which is approximately true for those in the labor force. The second and third imply external and subjective interest rates of 11 percent. D/W is the government share of activity on a stock basis, at 20 percent comparable to the usual measure of the flow of government spending relative to GNP. Endowments are unchanged in all the simulations reported. Borrowing is thus initially zero for constant prices and productivity. Borrowing is generated in the simulations by imposing productivity and real interest rate (external price) changes. The appropriate debt benchmark in a two period model is problematic. The standard measure of external debt to GNP is used for comparison[12], with price and productivity movements over time designed to generate debt/GNP ratios similar to those of current highly indebted countries.

The pure consumption tax is compared to the optimal consumption/borrowing tax using an equivalent variation measure of welfare improvements. The lump-sum tax solution is the benchmark.[13] The relative inefficiency measure reported is the equivalent variation associated with the consumption-cum-borrowing tax divided by the latter variation.

Table 1 reports results for a unitary elasticity of intertemporal substitution. The first two columns give the rate of change of productivity over the two periods and the real interest rate. The third column of *Table 1* is the *debt-income ratio*, $Z_0/w_0(H_0-L_0)$, the proportion of private external debt or assets to initial period national factor income. In comparison, the debt-GNP ratios of highly indebted countries lie under 75%.[14] The fourth column reports the revenue collected with the borrowing/lending tax in the optimal solution. As the base grows, the revenue collected rises but is never a substantial source of funds.

Table 1
Relative Inefficiency of the Consumption Tax

Productivity Shift	Real Interest	Debt-Income Ratio	Debt Tax Revenue	Relative Cost
0.0	−.03	.046	.0011	.007
.03	.11	.17	.0046	.024
.50	.11	.248	.012	.056
−.30	.11	−.26	.005	.044
−.50	.11	−.522	.013	.148

12. Debt to wealth ratios in the present model are not comparable to a ratio of debt to capitalized GNP data because wealth in the model includes the value of all time, not just labor time.

13. Let $v(\{\pi, w, \delta\}, W-D)$ be the utility achieved with the lump sum tax D. The equivalent variation EV for the consumption/borrowing tax solution is defined by $v(\{\pi, w, \delta\}, W-D+EV) = v(\{\pi(1+\tau), w, \delta(1-b)\}, W')$.

14. See the *World Development Report*. 1988, p31.

The last column shows that the relative cost of abstaining from use of the borrowing tax rises with the amount of borrowing but remains modest for the unitary elasticity of substitution case.

The figures in *Table 1* may overstate or understate the advantage of the borrowing tax due to the assumed unitary elasticity of intertemporal substitution. Recent evidence (Hall, (1988)) suggests an elasticity close to zero. On the other hand, behavior toward risk (low risk premia, implying low risk aversion) suggests a high intertemporal substitution elasticity. The relative advantage of using the borrowing tax is very sensitive to the elasticity and ordinarily rises with it. The assumed productivity shift of the fourth row of *Table 1* is analyzed under elasticities of 5 and 1/5 in *Table 2*. The middle row of *Table 2* reproduces for reference the last three elements of the fourth row of *Table 1*.

Table 2

Sensitivity to Intertemporal Substitution Elasticity

Elasticity	Debt-Income Ratio	Debt Tax Revenue	Relative Cost
5	−1.36	.025	.263
1	− .26	.005	.044
1/5	−.178	.005	.409

Large intertemporal substitution elasticities enlarge the borrowing tax base, other things being equal, as shown in the first column. This leads to greater use of the borrowing tax, and greater welfare improvement from its use. But large elasticities have a separate influence on the relative cost, since for the same size base the advantage of using the borrowing tax grows with the elasticity. Very large relative costs can be generated with high elasticities.

Ⅳ. The Relation of Borrowing Wage, and Commodity Taxes

It is well-known that a uniform-across-time wage tax is equivalent to a uniform-across-time commodity tax, when both are constrained to raise the same amount of revenue. A surprise is that the optimal wage and consumption taxes will generally not be equivalent, because uniformity will generally be violated (see Anderson−Young). The optimality of the use of a borrowing tax is related to these propositions and makes it useful to develop an equivalence between consumption and borrowing, and wage and borrowing taxation at the optimum.

The simple equivalence of uniform wage and consumption taxation follows immediately from intertemporal aggregation and the composite commodity theorem. Let ω be the uniform rate of wage taxation, so that the consumer takes home $(1-\omega)w$. The consumer's budget constraint is alternately

$$(1+\tau)\Sigma\delta^s\pi_s X_s + \Sigma\delta^s w_s[L_s-H_s] = 0, \text{ or}$$
$$\Sigma\delta^s\pi_s X_s + (1-\omega)\Sigma\delta^s w_s[L_s-H_s] = 0.$$

The government budget constraint requires $\tau\Sigma\delta^s\pi_s X_s = D = \omega\Sigma\delta^s w_s[L_s-H_s]$. The composite commodities of intertemporal commodities and labor have their relative price distorted by the tax system, and the distortion is the same for either form of tax.

For non-uniform taxation the above demonstration does not go through. A borrowing (lending) tax, $b>(<)0$, from (2) is equivalent to a tax (subsidy) on consumption X plus a tax (subsidy) on leisure L plus a subsidy (tax) to the endowment of time H. The latter two components are equivalent to a wage subsidy (tax).

$$(2') \ \frac{P_t}{1-b_t}X_t + \frac{w_t}{1-b_t}[L_t-H_t] \leqq Z_t.$$

Evidently the trio of taxable activities (leisure being assumed untaxable save as labor is subsidized) X, H−L, Z has two independent instruments in the form of relative prices to distort. Any two of the three possible taxes may be used. The combination of a borrowing tax and a consumption tax (or other possible pairs of taxes) remains distortionary, however, since the two relative prices *are* distorted. While an effective borrowing tax can be implicitly constructed using only time-varying τ and ω, it aids clarity to focus on a borrowing tax directly, in company with a consumption or wage tax.

It should be noted that the optimality of a borrowing tax is not in conflict with the well-known Diamond-Mirrlees principle of preserving productive efficiency. Their principle for a small open economy implies free commodity trade (domestic marginal rates of transformation equal to marginal rates of transformation through trade), *save for non-produced imports.* The latter clause is operative for the borrowing tax : there is no domestic source of net lending. Thus the borrowing tax is effectively a type of consumption tax. With a richer characterization of domestic agents' expenditure shifting needs, there would be some domestic lending in periods when the net national economy is borrowing : the optimal policy would then be a borrowing tax, not an external borrowing tax.

V. The Time Structure of the Optimal Borrowing Tax

The optimal policy of jointly setting τ and b is complex to analyze. To develop intuition, and because of its possible practical significance, the time structure of the optimal borrowing tax for a given constant consumption tax is characterized in this Part. It is assumed that the constant consumption tax is set (or reduced) low enough so that positive revenue must be raised from the borrowing tax. This is necessary to make the *partial* Ramsey problem interesting.

A key element in the optimal tax structure turns out to be the borrower /lender status of the country. Section A defines a borrowing (lending) regime as opposed to a borrowing (lending) period. Section B uses the definition to illuminate the tax structure.

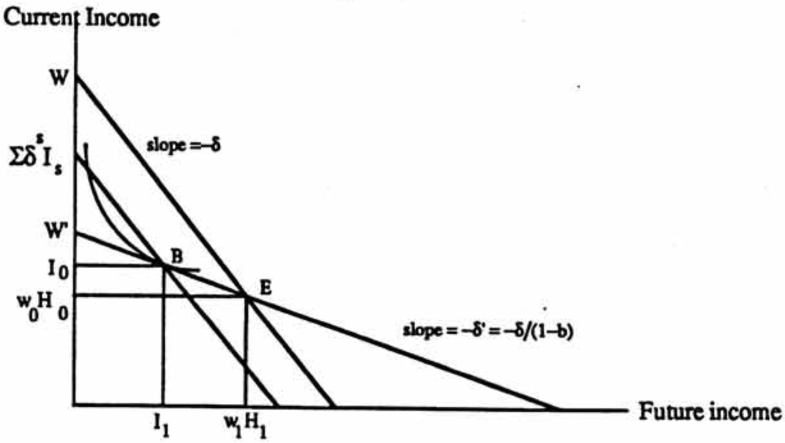
A. Borrower/ Lender Regimes and Tax Structure

The ratio of the present value of expenditure discounted using the external factor δ to tax-distorted wealth is $\Sigma\delta^s I_s / W'$.

Definition : There is a borrowing (lending) regime as $\Sigma\delta^s I_s / W' > (<) 1$.

The logic of the definition is based on a simple 2 period intertemporal exchange diagram. *Figure 1* depicts the borrowing regime.

Figure 1
The Borrowing Regime

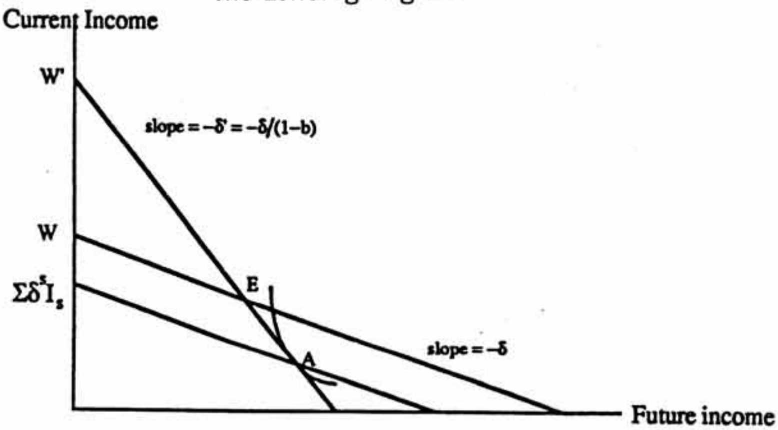


Let $b_0=0$, so that wealth in internal or external prices is measured in terms of period 0 foreign exchange. The endowment point E is such that consumers prefer to borrow in the present in the absence of borrowing taxes.[15] The positive revenue requirement $bZ>0$ implies that $-\delta > -\delta/(1-b)$. A representative indifference curve is drawn tangent to the budget line W'. The taxed consumption point is at B. At external prices, the consumer expends $\Sigma\delta^s I_s = W - \Sigma\delta^s b_s Z_s$, which allows the required amount of borrowing tax revenue. The revenue is raised by taxing the repayment ($b_1 < 0$, $b_1 Z_1 = b_1(I_1 - w_1 H_1) > 0$).

The lending regime is depicted in *Figure 2*. In the absence of taxation the consumer would choose a bundle on the budget line W below and to the right of the endowment point E. The positive revenue requirement forces consumption to a point like A. The revenue is raised by taxing foreigners' loan repayment ($b_1 > 0$, $Z_1 > 0$).

Figure 2

The Lending Regime



The two diagrams show that borrowing taxation involve tilting expenditure away from (toward) the present, and $\Sigma\delta^s I_s > W'$, as the consumer is a current period borrower (lender). For more than two periods, no such simple division into two phase can generally be made. The *definition* uses the logic of *Figures 1* and *2* to define a borrowing (lending) regime as one in which on average, taxation is tilted against (toward) earlier expenditure, hence $\Sigma\delta^s I_s / W' > (<) 1$.

The borrower/lender regime of the country signs the average taxation of expenditure via the borrowing tax. Note that :

$$\frac{\Sigma\delta^s I_s}{W'} = \frac{\Sigma\delta^s I_s}{\Sigma\delta_s' I_s} = 1-\beta, \text{ where } \beta = \frac{\Sigma\delta_s' b_s I_s}{\Sigma\delta_s' I_s}, \text{ the average taxation of expenditure.}$$

Thus $\beta>(<)$ 0 in a lending (borrowing) regime.

B. The Optimal Borrowing Tax Stucture

The time structure of the optimal borrowing tax is implicit in (23) at the interior maximum, reproduced for convenience below :

$$(23)\quad -Z_t\frac{V_{W'}}{\lambda}+Z_t(1-b_t)\frac{C}{W'}-\frac{I_t}{\rho}\{\frac{C}{W'}-(1-b_t)(\frac{1-\alpha}{1+\tau_t}+\alpha)\}=0 \text{ for all t.}$$

Note that in (23), due to the additive intertemporal separability of the utility function, only the contemporaneous taxes appear, save for terms involving the entire sequence of taxes. At the optimum, the set of first order conditions, one each for $s=0,\cdots,N$ have in common the sequence terms. Then different b's arise due to different contemporaneous productivities (a) or external prices (π), and what look like comparative static derivatives can give the response of optimal b to changes in a or π.

The social foreign exchange cost of private expenditure is : $S=V_{W'}/\lambda$. Substituting $\frac{C}{W'}=(\frac{1-\alpha}{1+\tau}+\alpha)(1-\beta)$ into (23) and simplifying, the first order condition for optimal b is :

$$(24)\quad S=(\frac{1-\alpha}{1+\tau}+\alpha)(1-\beta)(1-b_t)-(b_t-\beta)\frac{I_t}{(I_t-w_tH_t)\rho}).$$

Differentiating (24) *given the sequence of π,a,H,b* (which means given real wealth) the *comparative static* derivative in productivity is :[15]

$$(25)\ ab_a=wb_w=\frac{wS_w}{S_b}.$$

For clarity in analyzing (25) it is useful to make two assumptions.

Assumption 1 : $b_t-\beta>(<)$ 0 for a borrowing (lending) period.

Assumption 1 necessarily holds for the two period case, and is *on average* true for many periods.[16]

15. The technique uses infinitesimal calculus for ease of presentation, but there is actually no need for the relevant values of b and either π or a to lie close together, so long as the derivative sign conditions hold in the relevant range of values. Thus the propositions below are not dependent on a continuum of values.

16. Consider first the two period case. $b_0=0$ by convention, and b_1, is $<(>)0$ as there is current borrowing (lending). β is a positive-weighted average of 0 and b_1, hence has smaller absolute value than b_1. Thus $b-\beta$ has the sign of b, which is positive (negative) for borrowing (lend-

Assumption 2 : $S_b < 0$[17,18].

In the usual comparative static method a term like S_b is negative due to the second order condition. In the present problem $S_b < 0$ is not necessary for the second order condition, though it acts in the right direction. For low enough intertemporal substitution elasticity $1/\rho$, or for a lending period, $S_b < 0$. In borrowing periods with high elasticity, it is more problematic.

The sign of b_w is then the sign of S_w, given by :

$$(26)\quad S_w = -\frac{\frac{1-\alpha}{1+\tau}+\alpha}{\rho}\frac{w_tH_tI_t}{(I_t-w_tH_t)^2}(b_t-\beta)(1-\frac{w_t\partial I_t}{I_t\partial w_t}).$$

From (A.4) note that given the sequence of prices(real wealth)

$$\frac{w_t\partial I_t}{I_t\partial w_t} = -\alpha\frac{1-\rho}{\rho},\quad \text{hence } (1-\frac{w_t\partial I_t}{I_t\partial w_t}) = 1-\alpha+\frac{\alpha}{\rho} > 0.$$

Substituting in (26) and using Assumption 1 yields :

Theorem 2 : The borrowing (lending) tax falls with rises in productivity, $b_a < (>)\ 0$.

For the borrowing (lending) tax, the result occurs because the rise in w on balance raises the critical term $\frac{I_t}{I_t-w_tH_t}$ in (24), hence lowers (raises) the value of the right hand side, requiring a fall (rise) in b to restore the optimality condition. *The optimal policy smooths the ratio of current borrowing or lending to current expenditure.*

Now consider the optimal response to external price fluctuations. Since the model implies that $w = a\pi$, a rise in π causes an equiproportionate rise in both w and p. This rise in the price index ϕ is equivalent to a rise in the discount factor at time t or a fall in the real interest rate.

Using the steps above, $\text{sign}(b_\pi)=\text{sign}(S_\pi)$. Differentiating (24), and using (A.4)

---

ing) periods. For more than two periods, note that β is a positive-weighted average of positive (borrowing period), and non-positive (lending periods plus period 0) elements b_t. Then some element b_t in both the borrowing and lending periods must exceed in absolute value the average, β : and indeed the conditional (on lending or borrowing periods) average of them must do so. But no appealing conditions appear to exist to guarantee the useful property that $(b_t-\beta)Z_t > 0$.

17. $S_b = -(1-\beta)+\frac{b_t-\beta}{1-b_t}\frac{I_tw_tH_t}{\rho^2Z_t^2}$ differentiating (24) and using $Z-I= -wH$ and $\partial I_t/\partial b_t$ given real wealth $= -\frac{1}{\rho(1-b_t)} < 0$

18. In the lending regime, $(b-\beta) < 0$, hence S_b is guaranteed to be less than zero.

$$(27)\ \pi S_{\pi} = -\left(\frac{1-\alpha}{1+\tau}+\alpha\right)(b_t-\beta)\frac{w_tH_tI_t}{(I_t-w_tH_t)^2\rho}\frac{1-\rho}{\rho}\ .$$

Then :

Theorem 3 : The optimal borrowing or lending tax rises (falls) in absolute value with a rise in external prices as the elasticity of intertemporal substitution is less (greater) than one.

The intuition is straightforward. Nominal expenditure I(hence the borrowing Z) rises or falls with the rise in ϕ as the elasticity of intertemporal substitution is less than or greater than one. *Theorem 3* implies that *the optimal policy smooths nominal borrowing* Z. For example a rise in ϕ with elasticity less than one will increase borrowing Z but be offset by the rise in b required to fulfill (24).

Note that *Theorem 3* implies that *the optimal response to external real interest rate fluctuations is to amplify (damp) them as the elasticity of intertemporal substitution is greater (less) than one.*

Ⅵ. Optimal Public and Private Debt

It is interesting to consider the effect of changes in productivity and external prices on new public debt, private debt, and the international trade accout. Under a constant consumption tax and an optimal borrowing tax, is the new effect to make optimal public debt pro- or counter-cyclical, defining counter-cyclical policy as a rise in public deficit when productivity falls or external prices rise? What about the trade deficit? Below are sufficient conditions for pro- or counter-cyclical behavior of the public debt and trade deficits ; and for either perfect positive or negative correlation of the public and trade deficits.

Public new debt in any time period s is derived from the basic relation giving contemporaneous consolidated external budget constraint :

$$(28)\ \pi_sX_s + w_sL_s + D_s = w_sH_s + Z_s + F_s.$$

F_s is the government *new borrowing* as s, the government budget deficit, and D_s is the government expenditure. The present value of the stream $\{D_s\}$ is D used above. For simplicity D_s is assumed to be constant over time.[19] Alternatively, (28) combined

19. Alternative models of government behavior have been usefully studied in the literature, but draw attention from the main point of the analysis.

with the consumer's budget constraint (2) gives the government budget constraint : government expenditure less tax collections equals *new debt*. Note that the total *new debt*, equal to the trade account deficit, is

$$K_s = Z_s + F_s.$$

For the parametric special case, F_s is obtained from (28) using

$$\pi_s X_s + w_s L_s = I_s\left(\frac{1-\alpha}{1+\tau}+\alpha\right) :$$

$$F_s = D_s - \left(w_s H_s + Z_s - I_s\left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right), \text{ or}$$

$$(29)\ F_s = D_s - \left(w_s H_s \frac{-b_s}{1-b_s} + I_s\left(\frac{1}{1-b_s} - \left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right)\right), \text{ using } Z_s = \frac{I_s - w_s H_s}{1-b_s}.$$

Similarly the trade account deficit is obtained as :

$$(30)\ K_s = D_s - \left(w_s H_s - I_s\left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right).$$

At the optimum, the effect of different w or π will shift F and K both directly via shifts in wH and I, and indirectly, via the accompanying shift in b. Section A contains the analysis of productivity changes while Section B contains the analysis of external price changes.

A. Productivity Changes

From (29), the effect of a (hence w) on F involves direct effect plus effects via b. As before, the derivatives, below are for given real wealth.

$$(31)\ aF_a = \frac{-Zwb_w}{(1-b)^2} - \left(\frac{1}{1-b} - \left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right)(wI_w + I_b wb_w).$$

From (A.4), the right bracket is

$$\left(-\alpha\frac{1-\rho}{\rho} - \frac{1}{\rho}\frac{wb_w}{1-b}\right) I.$$

Substituting into (31),

$$(31')\ aF_a = \frac{-Zwb_w}{(1-b)^2} - \left(\frac{1}{1-b} - \left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right)\left(\alpha\frac{1-\rho}{\rho} + \frac{1}{\rho}\frac{wb_w}{1-b}\right) I.$$

The first term is always non-negative, by (26) and the Assumption 1 that $(b-\beta)Z>0$. The second term has either sign. For unambiguous results it is necessary to impose strong restrictions. As the elasticity of intertemporal substitution is low, wb_w is small. The dominant term of aF_a becomes $-\left\{\frac{1}{1-b}-\left(\frac{1-\alpha}{1+\tau}+\alpha\right)\right\}$. In lending periods ($b<0$) F_a is positive provided b is large enough relative to τ. In borrowing periods ($b>0$) aF_a is negative. Summarizing :

Theorem 4 : For low intertemporal substitution elasticity and low enough τ, optimal public debt moves pro-(counter-) cyclically with productivity as the relevant periods have private lending(borrowing).

Now consider the trade account deficit under productivity change. Counter-cyclical behavior implies a rise in the deficit when productivity falls, or $K_a < 0$. Differentiating (30)

$$aK_a = wK_w = -w_tH_t + I\left(\frac{1-\alpha}{1+\tau}+\alpha\right)(wI_w + I_b wb_w)$$

As noted above, $(wI_w + I_b wb_w) = \left(-\alpha\frac{1-\rho}{\rho} - \frac{1}{\rho}\frac{wb_w}{1-b}\right) I.$

From *Theorem 2*, the sign of b_w is minus the sign of Z. Then :

Theorem 5 : At the optimal borrowing tax structure, the optimal trade deficit moves counter-cyclically to productivity

(a) for elasticity of intertemporal substitution weakly greater than one in a lending period,

(b) for low intertemporal substitution elasticity.

Proof : (a) follows immediately from substituting into the expression for aK_a the expression for $\frac{w\partial I}{I\partial w}$ and $\frac{I_b}{I}$. Under the conditions $\rho \leqq 1$ and $Z < 0$, the term multiplying I is negative. (b) follows from, at large ρ :

$$aK_a = -w_tH_t + \alpha\left(\frac{1-\alpha}{1+\tau}+\alpha\right) I_t.$$

Even at $\tau=0$, a negative sign requires $Z(1-b_t)-(1-\alpha)I_t > 0$, that borrowing exceed commodity expenditure, (i.e., that labor income be negative) which is impossible.‖

The more interesting possiblity is that the movement be pro-cyclical. In periods of private borrowing, and with $1 > 1/\rho$, this may arise, but sufficient conditions for it do not appear to be interpretable.

Note that *Theorems 4* and *5* imply that *under low intertemporal substitution elasticity, in private borrowing periods the optimal public and trade deficits are perfectly negatively correlated.* On the other hand *in lending periods with low substitution elasticity, the two deficits are perfectly positively correlated at their optimal settings.* Thus the optimum allows a rich menu of relations between the three borrowing accounts.

B. External Price Changes

Now consider external price changes, which imply equiproportionate changes in the price index ϕ. A rise in ϕ is in turn equivalent to a fall in the real interest rate.

Note that for the trade deficit, (30) implies that the behavior of K is proportional to the behavior of I. Form (A.4), for given real wealth the total elasticity I with respect to π is

$$\left(\frac{\pi}{I} I_\pi + \frac{I_b}{I} \pi b_\pi \right) = 1 - \frac{1}{\rho} - \frac{1}{\rho(1-b_s)} \pi b_\pi.$$

Counter-cyclical borrowing is defined as a rise in new debt, or an increase in the trade deficit, as π rises ; or $K_\pi > 0$.

Theorem 6 : (a) The optimal trade account moves pro-cyclically for elasticity of intertemporal substitution greater than one in a lending period, and counter-cyclically for elasticity of substitution less than one in a borrowing period.

(b) The optimal trade account moves counter-cyclically for elasticity of substitution small.

Proof : (a) follows from *Theorem 3* and the expression for $dI/d\pi$. (b) follows from forming πb_π and taking the limit as ρ increases, which is zero. The limit of the first term is one.‖

For the cases of $1 > 1/\rho$ and borrowing, or $1 < 1/\rho$ and lending, the conditions for one sign or the other are not readily interpretable.

Turning to public debt, besides the direct effect of ϕ on I there is also the indirect effect of ϕ on b. Differentiating (29) :

$$\pi F_\pi = \frac{w_s H_s - I_s}{(1-b_s)^2} \pi b_\pi - \left(\frac{1}{1-b_s} - \left(\frac{1-\alpha}{1+\tau} + \alpha \right) \right) \left(\frac{\pi}{I} I_\pi + \frac{I_b}{I} \pi b_\pi \right),$$

where $\left(\frac{\pi}{I} I_\pi + \frac{I_b}{I} \pi b_\pi \right) = 1 - \frac{1}{\rho} - \frac{1}{\rho(1-b_s)} \pi b_\pi.$

Theorem 7 : (a) For borrowing periods, optimal public debt moves pro-(counter−) cyclically to external prices as the elasticity of intertemporal substitution is less (greater) than one.

(b) For low elasticity of intertemporal substitution, optimal public debt moves counter-cyclically to external prices in borrowing periods, and pro-cyclically to external prices in lending periods.

Proof : The first term has the sign of $\frac{1}{\rho}-1$, from $wH-I=-Z$, equation (27) for πb_π and Assumption 1, $Z(b-\beta)>0$. In the second term, the bracket expression is positive in borrowing periods and negative in lending periods for low enough τ if it is assumed that $bZ>0$.[20]

(a) In borrowing periods, from (27), $\left(\frac{\pi}{I}I_\pi+\frac{I_b}{I}\pi b_\pi\right)$ has the sign of $1-\frac{1}{\rho}$.
Thus in borrowing periods, πF_π has the sign of $\frac{1}{\rho}-1$.

(b) For lending periods, when $\frac{1}{\rho}$ tends to zero, b_π goes to zero (see (27)) and from (A.4), the elasticity of I with respect to π (which raises ϕ proportionately) is $-\frac{1-\rho}{\rho}=1$ in the limit. For lending periods, provided b is sufficiently large relative to τ, the bracket term is negative, hence F_π is negative.‖

Under the conditions of the *Theorems 6(b)* and *7(b)*, the optimal public and trade deficits are perfectly negatively correlated under real interest rate fluctuations in lending periods and perfectly positively correlated in borrowing periods. Under the conditions of *Theorems 6(a)* and *7(a)*, the two deficits are perfectly positively correlated with elasticity of substitution less than one in a borrowing period. Thus a rich possible variety of deficit patterns may be consistent with optimality under real interest rate fluctuations.

Ⅶ. Conclusion

This paper establishes general inefficiency of not distorting intertemporal choice in the basic public finance (Ramsey) problem. Borrowing/lending taxes are generally desirable, even in the special separable preferences case which yields a uniform-over-time commodity tax when this is the only instrument.

The optimal borrowing tax may either damp or amplify external price (real interest rate) fluctuations, while it always rises with productivity. Optimal public and trade deficits may be negatively or positively correlated for productivity or real interest rate fluctuations.

The efficiency cost of not using borrowing taxes is the subject for another paper. Some simulations with the Cobb-Douglas case reported here suggest they could be substantial as borrowing is substantial.

Another worthwhile extension is to set up a two country version of the model,

20. Assumption 1 implies $bZ > \beta Z$, and β is positive (negative) in a lending (borrowing) regime.

and examine the nature of the jointly efficient borrowing taxes as compared to the Nash equilibrium borrowing taxes.

Appendix 1
Derivation of Properties of C

In the first stage, for each period s, expenditure I_s (including expenditure of leisure) is allocated according to :

$$(A.1)\ \max_{X_s, L_s} u(X_s, L_s), \text{ subject to } p_s X + w_s L_s \leqq I_s$$

Substituting the solution $x_s(p_s, w_s, I_s)$, $l_s(p_s, w_s, I_s)$ into u(X,L) yields the period s indirect utility function $v(p_s, w_s, I_s)$.

In the second stage, the consumer allocates expenditure between periods, solving :

$$(A.2)\ \max_{\{I_s\}} \Sigma d_s v(p_s, w_s, I_s) \text{ subject to } \Sigma\delta_s' = W'$$

The Lagrange multiplier associated with the constraint in (A.2) is the increase in total discounted utility made possible by an increase in W'. Recalling the derivation of the intertemporal indirect utility function, this equals $V_{W'}$. Thus the optimal intertemporal expenditure pattern satisfies :

$$(A.3)\ d^s V_I(p_s, w_s, I_s) = V_{W'}(\{\delta_s', p_s, w_s\}], W') \text{ for each s.}$$

The system (A.3) yields the set of I_s' implicitly as functions of the marginal utility of real wealth and the current period price vector.

For the Cobb-Douglas special case of this paper, the single period utility is :

$$u(X_s, L_s) = \frac{1}{1-\rho}(X_s^{1-\alpha}L_s^{\alpha})^{1-\rho}$$

In this model, ρ is the parametric coefficient of relative risk averision. $1/\rho$ is the intertemporal substitution elasticity. The Cobb-Douglas true cost-of-living index for period s is $\phi_s = p_s^{1-\alpha}w_s^{\alpha}$. The solution to the intertemporal allocation of expenditure is a CES form :

$$(A.4)\ I_s = \frac{W' d_s^{\frac{1}{\rho}} \delta_s'^{\frac{-1}{\rho}} \phi_s^{\frac{-(1-\rho)}{\rho}}}{\bar{P}};$$

$$\text{where } \bar{P} = \sum_0^N d_s^{\frac{1}{\rho}} \delta_s'^{\frac{-(1-\rho)}{\rho}} \phi_s^{\frac{-(1-\rho)}{\rho}}.$$

$\overline{P}^{\frac{-\rho}{1-\rho}}$ is the intertemporal true cost-of-living index.

The foreign exchange cost of private consumption in the Cobb-Douglas case is :

$$(A.5)\ C = \Sigma\delta^{s}\left(\frac{1-\alpha}{1+\tau_s}+\alpha\right) I_s,$$

using $X_s = (1-\alpha)I_s/p_s$, $\pi_s/p_s = 1/(1+\tau_s)$, and $L_s = \alpha I_s/w_s$. The effect on I_s of a change in b_t given W' is, from differentiating (A.4) :

$$(A.6)\ \frac{\partial I_s}{\partial b_t} = \frac{\partial I_s \partial \delta_t'}{\partial \delta_t' \partial b_t} = \frac{\delta_t'}{(1-b_t)}\left(\frac{-\delta_{st} I_s}{\rho\delta_t'}+\frac{(1-\rho)I_s I_t}{\rho W'}\right),$$

where δ_{st} is the Kronecker delta. The crowding out effect of a rise in b_t is then :

$$(A.7)\ C_{bt} = \Sigma\delta^{s}\left(\frac{\pi_s}{p_s}(1-\alpha)+\alpha\right)\frac{\partial I_s}{\partial b_t} + \Sigma\delta^{s}\left(\frac{\pi_s}{p_s}(1-\alpha)+\alpha\right)\frac{I_s w_t H_t \delta_t'}{W'(1-b_t)}$$

Substituting into (A.7) for (A.6), using (A.5) and $I_t = w_t H_t + (1-b_t)Z_t$, and simplifying,

$$(A.8)\ C_{bt} = \frac{\delta_t' \quad C}{(1-b_t)W'}\left(-Z_t(1-b_t)\frac{I_t}{\rho}\right) - \delta_t'\left(\frac{\pi_t}{p_t}(1-\alpha)+\alpha\right)\frac{I_t}{\rho}.$$

This yields equation (20) of the text.

Analogously to (A.6) the effect on I of the commodity tax is, using $\partial\phi/\partial p = 1-\alpha$:

$$(A.9)\ \frac{\partial I_s}{\partial p_t} = -\delta_{st}(1-\alpha)\frac{I_s}{p_s}\frac{1-\rho}{\rho} + (1-\alpha)I_s\frac{I_t}{p_t}\frac{1-\rho}{\rho W'}$$

Then substituing (A.9) into the derivative of C with respect to p_t where C is given in (A.5) and using $X_t = (1-\alpha)I_t/p_t$:

$$(A.10)\ C_p = \frac{1-\rho}{\rho}\frac{C}{W'}\delta_t' X_t - \delta^t X_t\left(\frac{\pi_t}{p_t} + \frac{1-\rho}{\rho}\left(\frac{\pi_t}{p_t}(1-\alpha)+\alpha\right)\right).$$

This yields equation (21) of the text.

Appendix 2
A Closed Economy Variant

A simple closed economy version of the model above yields exactly the same structure. The point is worth developing because in either form of the model it will pay to tax both investment and income or consumption ; which is at variance with previous analysis. Showing that it holds for a closed economy demonstrates that it is not external capital mobility *per se* which is responsible.

Suppose that a *storage* technology exists in which 1 unit of the composite good laid aside today yields $1+\chi$ units next period. The one period discount factor for such an economy will be $1/(1+\chi)$ so long as the feasibility constraint that current stocks exceed current borrowing is met. This can be guaranteed with sufficiently large initial stocks. The Ricardian technology fixes the relation between current w and current π(the producer's price), the storage technology at an interior fixes the relation between current and future commodity prices, hence all prices are fixed in terms of the current commodity(the numeraire). Then the price structure of (15)-(16) carries through to the closed economy.

The fixed discount factors may not be valid for all values of taxes, since the feasibility constraint may be violated, but this is actually a submerged issue for the small open economy as well : credit constraints may well be relevant. When such endogenous interest rates are present, it is obviously in the government's interest to institute borrowing taxation to smooth the marginal cost of borrowing, overcoming the obvious externality. For the open economy variant, the analogous tax has an *optimal tariff* component. This paper highlights the necessity of borrowing taxation at an optimum even when such factors are absent ; the optimal structure caused by them being a well-understood side issue.

References

Anderson, J.E. and L. Young (1990) "Optimal Taxation and Debt in an Open Economy", forthcoming in Journal of Public Economics.

Auerbach, A.J. (1983), *The Taxation of Capital Income*, Cambridge : Harvard University Press.

Dornbusch, R. (1983), "Real Interest Rates, Home Goods, and Optimal External Borrowing", *Journal of Political Economy, 91*, 141-153

Barro, R. (1979), "On the Determination of the Public Debt", *Journal of Political Economy, 87*, 940−971.

Frenkel, J. and A. Razin (1987), *Fiscal Policies and the World Economy*, Cambridge : MIT Press.

Hall, R.E. (1988), "Intertemporal Substitution in Consumption", *Journal of Political Economy, 96*, 339-57

Judd, Kenneth (1987), "The Welfare Cost of Factor Taxation in a Perfect Foresight

Model", *Journal of Political Economy, 95,* 675-709

Kydland, F., and E. Prescott (1982), "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy", in S. Fischer ed, *Rational Expectations and Economic Policy,* NBER.

Kotlikoff, L. (1985), "Taxation and Savings : A Neoclassical Perspective", *Journal of Economic Literature, 22,* 1576-1629

Razin, A. and L. Svensson (1983) "The Current Account and Optimal Government Debt", *Journal of International Money and Finance, 2,* 215-244

Persson, T. (1985), "Deficits and Intergenerational Welfare in Open Economies", *Journal of International Economics, 19,* 67-84

World Development Report 1988, New York : Oxford University Press, for the World Bank.