

# Inflation, Tariffs and Tax Enforcement Costs

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*This paper derives the dependency of optimal tariff and inflation tax on tax collection and enforcement costs. The analysis is done for a small, open economy. The existence of such costs can justify tariff and inflation tax policies as optimal revenue-raising devices. This paper suggests that greater government demand for revenue will increase the use of inflation and tariffs as revenue devices. The analysis derives elasticity rules that tie optimal tariff and inflation rates to the costs of tax collection.*

## I. Introduction

Public finance literature has frequently concluded that efficiency considerations do not justify the use of tariffs as a means of raising revenue in a small open economy. Instead, one should apply consumption taxes [see Corden (1984) and the references listed there] Similar results were shown for optimal inflation tax: if one views money as input in the delivery of consumption goods, one should not use inflation tax as revenue device (see Hercowitz and Sadka (1984) and Kimbrough (1985)).<sup>1</sup> However, we can not escape the observation that small economies, frequently LDC, use both tariffs and inflation tax as revenue devices. Crude empiricism suggests that less efficient tax collection and tax enforcement authorities, as well as larger government revenue needs,

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The author would like to thank Michael Darby, Allan Drazen, Sebastian Edwards, Jacob Frenkel, Aryel Hillman, Michael Mussa and an anonymous referee for helpful comments; any errors, however, are mine. Financial support by the Graduate School Business, University of Chicago, is gratefully acknowledged. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the NBER.

1. The optimum quantity of money rule literature goes back to Friedman (1969). Similar results in a general equilibrium framework have been obtained by Jovanovic (1982). For related analyses of inflation in a public finance context see, for example, Phelps (1973), Frenkel (1976), Siegel (1978), Drazen (1977) and Helpman and Sadka (1997).

tend to increase the applicability of inflation tax and tariffs as revenue devices (on the use of inflation tax, see Fischer (1982)).

The gap between traditional public finance literature and the empirical regularities is a result of the tendency of the literature to overlook the role of costs of tax collection and enforcement. While the potential importance of these costs is widely recognized, traditional public finance literature has focused on policies that minimize the deadweight losses in the various markets, without accounting explicitly for the on the spent on the collection and enforcement of a given tax structure.<sup>2</sup> The purpose of this paper is to derive explicitly the dependence of optimal policies on collection costs. Inflation tax and tariffs have relatively low collection costs because inflation tax is an implicit tax and tariffs are collected at a centralized place -- the port of entry of imports. Optimality is achieved by equating across feasible taxes the sum of the marginal deadweight loss and the marginal collection costs associated with extra revenue. Consequently, one will expect that if the collection costs associated with consumption taxes are significant, inflation tax and tariffs will also be used as revenue sources.

Section II solves for the optimal tariff for a general utility. Section III studies the role of costs of tax collection in determining optimal inflation. This is done for the case of an economy where money serves as an "input" in the delivery of consumption. To simplify solves the implied elasticity rule that ties optimal interest rate to costs of tax collection. While the analyses in Sections II - III are somewhat disjointed, both sections exemplify the common principle associated with optimal taxes in the presence of collection costs. This principle is elaborated in Section IV, which contains concluding remarks. The Appendix describes the solution for the more general problem, allowing the simultaneous determination of inflation and tariffs.

## II. Costs of Tax Collection and Optimal Tariff

Suppose that a representative consumer has the following utility :

$$U(X, Y) + V(G) \tag{1}$$

where  $Y$  is the imported good and  $X$  is the domestic good, and  $G$  is the public good

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2. For public finance analyses of optimal taxes see Diamond and McFadden (1974), Diamond and Mirrlees (1973), Mirrlees (1976) and Sandmo (1976).

supplied by the government. The consumer is endowed with  $\bar{X}$  units. The government can raise taxes via two channels: a tariff at a rate  $\tau$  and consumption tax at a rate  $\bar{\theta}$ . Denoting by  $q$  the external terms of trade, the consumer budget constraint is given by

$$\bar{X} = [X + q(1 + \tau)Y] [1 + \bar{\theta}]$$

Because there are no assets in this simple economy, consumption tax  $\bar{\theta}$  is equivalent to an endowment tax  $\theta$ , defined by  $\theta = \bar{\theta}/(1 + \bar{\theta})$ , where the equivalent budget constraint is now

$$\bar{X}(1 - \theta) = X + q(1 + \tau)Y. \quad (2')$$

We introduce costs of tax collection by assuming that there are real costs associated with the collection of revenue via consumption (or endowment) tax and that tariff revenue can be raised costlessly. As a result net government revenue is

$$G = \bar{X}\theta(1 - \phi) + \tau qY \quad (3)$$

Where  $\phi$  denotes the cost of collecting consumption taxes, defined in percentage term. The problem of the government is to choose taxes so as to maximize the indirect utility of a representative consumer subject to the budget constraint.

To simplify the exposition, we take  $G$  to be given exogenously at its optimal target. The consumer sets  $X$ ,  $Y$  as to maximize (1) subject to its budget constraint; equation (2'). The first order conditions are given by

$$U_X = \lambda \quad (4)$$

$$U_Y = \lambda q(1 + \tau) \quad (5)$$

where  $\lambda$  is the associated Lagrange multiplier.

Let us assess the welfare change  $\left(\frac{\Delta U}{U_x}\right)$  resultant from a change in tax policy  $(\Delta \tau, \Delta \theta)$  which is designed so as to keep net revenue given  $(\Delta G=0)$ . Using the first order conditions

$$\frac{\Delta U}{U_X} = \Delta X + \frac{U_Y}{U_X} \Delta Y = \Delta X + q(1 + \tau) \Delta Y. \quad (6)$$

Because the consumer is moving on his budget constraint, we learn from 2' that

$$\Delta X + q(1 + \tau) \Delta Y = -\bar{X} \Delta \theta - qY \Delta \tau \quad (7)$$

Thus, by combining equations 6 and 7

$$\left. \frac{\Delta U}{U_X} \right|_{\Delta G=0} = -\bar{X} \Delta \theta - qY \Delta \tau. \quad (6')$$

The government change  $(\theta, \tau)$  keeping its revenue the same  $(\Delta G=0)$ , so equation 3 implies :

$$-\bar{X} \Delta \theta - qY \Delta \tau = -\bar{X} \Delta (\theta \phi) + \tau q \Delta Y \quad (8)$$

Combining 6' , 8 we get

$$\left. \frac{\Delta U}{U_X} \right|_{\Delta G=0} = -\bar{X} \Delta (\theta \phi) + \tau q \Delta Y \quad (9)$$

The change in tax policy acts upon welfare in two ways : First, it changes the resources devoted to tax collection, which are reflected by the first term of equation 9 ; next, it affects the distorted activity (consumption of  $Y$ ). The marginal welfare gain resulting from this change is the distortion  $(\tau q)$  times the change in the distorted activity  $(\Delta Y)$ , as reflected in the second term of equation 9. To gain further in-sight into the optimal tariff determination, note that for a constant marginal cost of tax collection, 8 indicates that

$$\Delta \theta = -\frac{qY \Delta \tau + \tau q \Delta Y}{\bar{X} (1 - \phi)} \quad (10)$$

Applying 10 to 6', one gets

$$\left. \frac{\Delta U}{U_X} \right|_{\Delta G=0} = \frac{Yq\phi}{1-\phi} \Delta \tau + \frac{\tau q}{1-\phi} \Delta Y \quad (11)$$

Optimal tariff is set such that the right hand side of 11 is zero, thus :

$$\frac{\Delta Y}{\Delta \tau} = - \frac{\phi Y}{\tau} \quad (12)$$

or, in elasticity terms :

$$\frac{\tau}{1+\tau} \epsilon = \phi \quad (13)$$

where  $\epsilon = -d \log Y / d \log(1 + \tau)$  denotes the elasticity of demand for importables, defined to be positive. Thus, optimal tariff is given by<sup>3</sup>

$$\tilde{\tau} = \frac{\phi}{\epsilon - \phi} \quad (14)$$

Consequently, positive collection costs can justify the active use of tariff in a small, open economy. This result provides a formal verification of the argument in Corden (1984), which suggests that relatively low collection costs for trade taxes are the essential requirement for trade taxes to be optimal revenue-raising devices in the small economy model. The above discussion can be readily extended in several directions. For example, suppose that  $\bar{X}$  is produced by labor ( $\bar{X} = L$ ), that leisure enters the utility ( $U = U(X, Y, \bar{L} - L)$ ), and that the tariff is associated with enforcement cost  $\phi\tau$ . The optimal tariff can be shown to be

$$\tilde{\tau} = \frac{\phi - \phi_\tau + (1 - \phi) \frac{\theta}{1 - \theta} n_L}{\epsilon(1 - \phi_\tau) - [\phi - \phi_\tau + (1 - \phi) \frac{\theta}{1 - \theta} n_L]} \quad , \quad \text{where } n_L \quad (14')$$

is the supply elasticity of labor ( $n_L = d \log L / d \log(1 - \theta)$ ). Thus, a higher  $n_L$  as well as a higher collection cost differential ( $\phi - \phi_\tau$ ) will increase the tariff rate<sup>4</sup>.

3. Equation 14 assumes implicitly an internal solution, where  $\tau \geq 0$ .

4. If we allow the costs of tax collection to depend on the tax rates ( $\phi = \phi(\theta)$ ;  $\phi_\tau = \phi_\tau(\tau)$ ), we should modify  $\tilde{\tau}$  by replacing  $(\phi, \phi_\tau)$  with  $(\phi(1 + \delta), \phi_\tau(1 + \delta_\tau))$ , where  $(\delta, \delta_\tau)$  are the supply elasticities of the cost of tax collection with respect to the tax rates.

In deriving (14') we obtain a useful extension of equation 14. It can be shown that the optimal tax structure is attained when a marginal change in taxes ( $\Delta\theta$ ,  $\Delta\tau$ ; for  $\Delta G = 0$ ) implies

$$\theta\Delta L - \Delta[\phi\theta L] = \tau q(-\Delta Y) - \Delta(-\phi_{\tau}\tau qY) \quad (9')$$

The first term in each side of 9' is the marginal welfare effect of a change in a distorted activity. The second term in each side of 9' is the marginal welfare effect of a change in the resources spent on tax collection. Optimal tax structure implies that we equate across taxes the sum of the marginal deadweight loss plus the marginal collection costs. We close this section with a simple Cobb-Douglas example, from which we can derive the reduced form of the optimal tax structure. Suppose that a representative consumer has the following utility:

$$\alpha \log X + \beta \log Y + \tau \log G; \quad \alpha + \beta = 1 \quad (15)$$

Direct optimization reveals that for  $\phi \leq \frac{\alpha\tau}{\beta + \tau}$

$$\tilde{\tau} = \frac{\phi}{\alpha - \phi} \quad (16a)$$

$$\tilde{\theta} = \frac{\tau}{1 + \tau} - \phi \frac{\beta}{(\alpha - \phi)(1 + \tau)} \quad (16b)$$

$$\tilde{g} = \frac{\tau}{1 + \tau} (1 - \phi) \quad (16c)$$

where for simplicity we assume a fixed marginal collection cost, and that ' $\sim$ ' stands for the optimal value. For values of  $\phi$  higher than  $\frac{\alpha\tau}{\beta + \tau}$  we get

$$\tilde{\tau} = \tau/\beta \quad (17a)$$

$$\tilde{\theta} = 0 \quad (17b)$$

$$\tilde{g} = \frac{\tau\beta}{\beta + \tau} \quad (17c)$$

Note that in the absence of collection costs ( $\phi \equiv 0$ ) only consumption tax is used, at a rate that reflects the priority given to government activity ( $\tilde{\theta} = \frac{r}{1+r}$ ,  $\tilde{\tau} = 0$ ).

Positive collection costs justify the use of tariff as a revenue device.

### III. Costs of Tax Collection and Inflation Tax

In this section we derive the dependence of inflation tax on tax collection costs. For simplicity of exposition, we focus on the case of a one good open economy, fully integrated with the international capital market. The analysis can be readily extended to allow for optimal tariff as well as other taxes. Consider a two period endowment model, where the consumer preferences are described by :

$$U = u(L_0, X_0) + pu(L_1, X_1) \quad (18)$$

where  $L_t$  stands for leisure in period  $t$ , and  $X_t$  for the consumption in period  $t$ ,  $t = 0, 1$ . The presumption made in this paper is that money provides services by reducing the cost of exchanging goods. The use of real balances promotes more efficient exchange and in so doing saves costly resources. These resources might include time and capital which would be used to coordinate various transactions<sup>5</sup>. To simplify exposition, the paper studies the case in which the exchange activity is time intensive. A possible way of capturing this notion is by assuming that leisure is a decreasing function of the velocity of circulation. I.e., a drop in the velocity of circulation is associated with a higher intensity of money per transaction, allowing one to save on the use of time in facilitating transactions, thereby increasing leisure.<sup>6</sup> Thus :

$$L_t = L_t(v_t); L'_v < 0 \quad (19)$$

where  $v_t = p_t X_t / M_t$ ,  $P_t$  being the price of good  $X$  in period  $t$ <sup>7</sup>.

5. For such a model, see Dornbusch and Frenkel (1973).

6. Such a formulation can be found in Aizenman (1985), which derives the complementarity of commercial policy, capital controls and inflation tax for the case where those are the only available taxing devices.

7. We assume that only domestic money is used on co-ordinating domestic transactions. The underlying structure of the economy described here is that of a centralized market only in the case of financial transactions (bonds). There is no centralized exchange of goods among domestic consumers. The asymmetry between financial transactions and the domestic exchange of goods among consumers is reflected in the specification of the velocity of money, which is defined only for transactions that involve consumption.

An intertemporal model is chosen to generate a meaningful opportunity cost of holding money. For simplicity of exposition, we take the case of only two periods. Our model can be readily extended for a general periods model, without altering the main results. We consider the case of a floating exchange rate system, in which there exists a traded bond,  $B$ , paying real interest rate  $r^*$ , where  $*$  stands for foreign values. An endowment tax at a rate  $\theta$  is applied in both periods. The budget constraint in period zero is given by:

$$P_0 X_0 + M_0 + P_0 B = P_0 (1 - \theta) \bar{X}_0 + \bar{M}_0 \quad (20)$$

where  $\bar{M}_0$  denotes the initial supply of money balances,  $\bar{X}_0$  the endowment of good  $X$ , and  $\theta$  corresponds to the endowment tax.

To simplify exposition, we assume zero initial holdings of traded bonds. In the next period our consumer is facing a budget constraint given by:

$$P_1 X_1 + M_1 = M_0 + P_1 (1 + r^*) B + P_1 (1 - \theta) \bar{X}_1 \quad (21)$$

Our consumer finances consumption and the use of money balances from his initial endowment. This endowment includes money balances carried over from period zero, endowment of good  $X$ , and the income paid on the traded bond. We denote by  $i$  the nominal interest rate defined by the traded bond: one monetary unit purchases  $\frac{1}{P_0}$

bonds in period 0, which pay  $\frac{P_1}{P_0} (1 + r^*)$  in monetary terms in period 1. Thus:

$$1 + i = \frac{P_1}{P_0} (1 + r^*) \quad (22)$$

We can collapse 20, 21 into a unique intertemporal budget constraint:

$$\bar{X}_0 (1 - \theta) + \frac{\bar{M}_0}{P_0} + \frac{\bar{X}_1 (1 - \theta)}{1 + r^*} = X_0 + \frac{X_1}{1 + r^*} + \frac{i M_0 + M_1}{P_0 (1 + i)}. \quad (23)$$

Let us denote real balances in period  $t$  ( $M_t / P_t$ ) by  $m_t$ , and by  $Z$  the discounted value of  $(Z_0, Z_1)$ :

$$Z = Z_0 + \frac{Z_1}{1 + r^*}$$

We can re-write the budget constraint as

$$\bar{X}(1 - \theta) + \bar{m}_0 = X + \frac{i}{1+i} m_0 + \frac{m_1}{1+r^*} \quad (24)$$

The net endowment of goods [ $\bar{X}(1 - \theta)$ ] and of initial money balances ( $\bar{m}_0$ ) finances private consumption and the cost of using money balances, as reflected by the corresponding opportunity cost ( $\frac{i}{1+i}$  and  $\frac{1}{1+r^*}$ ).

The net government revenue in periods zero and one is given by

$$M_0 - \bar{M}_0 + \theta(1 - \phi) P_0 \bar{X}_0 \quad (25a)$$

$$M_1 - M_0 + \theta(1 - \phi) P_1 \bar{X}_1 \quad (25b)$$

As in Section II, we assume that endowment taxes are associated with collection costs  $\phi$ . To simplify, we take  $\phi$  to be constant at the margin. The private budget constraint is given by equation (24), which takes government policies as given. Private agents maximize their utility subject to this constraint. For the resultant optimal behavior of the private sector, 25 implies the corresponding government revenue. Because our system is homogeneous, the real equilibrium will not be affected by an anticipated equi-proportional rise in  $(M_0, M_1)$ . To fix ideas, consider the case in which the value of  $M_0$  is given  $(M_0, \bar{M}_0)$  and the government sets  $M_1$ . In such a case money balances will increase by  $M_1 - M_0$  in period 1 as a result of the issue of new money to finance part of government's purchases of goods and services. Let us denote by  $\mu$  the rate of monetary expansion ( $\mu = (M_1 - \bar{M}_0) / \bar{M}_0$ ). Combining 25a, 25b we obtain as the net present value of government revenue (in terms of  $X_0$ )

$$G = \bar{X} \theta (1 - \phi) + \frac{m_1}{1+r^*} + \frac{m_0 i}{1+i} - \bar{m}_0 \quad (26)$$

Combining 24, 26 we get

$$\bar{X}(1 - \theta \phi) = G + X \quad (27)$$

equation 27 is the fundamental budget constraint. Net present value of private plus public consumption equals the net present value of the endowment, adjusted by the

resources spent on tax collection. For a given, known government policy private agents maximize utility  $U$  subject to equation 24, resulting in the following first order conditions :

$$\begin{aligned} \text{a. } U_{X_0} &= \lambda & \text{d. } U_{M_1} &= \frac{\lambda}{P_0(1+i)} \\ \text{b. } U_{X_1} &= \frac{\lambda}{1+r^*} & & \\ \text{c. } U_{M_0} &= \frac{\lambda i}{P_0(1+i)} & & \end{aligned} \quad (28)$$

where  $\lambda$  is the budget constraint multiplier and  $U_Z$  the total derivative of  $U^8$ .

Thus :

$$U_{X_t} = U_{X_t} + U_{L_t} \cdot \frac{dL_t}{dv_t} \cdot \frac{P_t}{M_t} \quad (t = 0, 1) \quad (29a)$$

$$U_{M_0} = -u_{v_0} X_0 P_0 / (M_0)^2 \quad (29b)$$

$$U_{M_1} = -\rho u_{v_1} X_1 P_1 / (M_1)^2 \quad (29c)$$

To gain further insight into the government's problem, consider a marginal change in the vector of government policies,  $\Delta(\theta, \mu)$ , keeping government revenue given ( $\Delta G = 0$ ). Such a change would affect welfare (as measured in  $U_{X_0}$  terms) by<sup>9</sup>

$$\frac{\Delta U}{U_{X_0}} = \Delta X_0 + \frac{U_{X_1}}{U_{X_0}} \Delta X_1 + \frac{U_{M_1}}{U_{X_0}} \Delta M_1 + \frac{U_{P_0}}{U_{X_0}} \Delta P_0 + \frac{U_{P_1}}{U_{X_0}} \Delta P_1 \quad (30)$$

8. Notice that because the analysis is conducted in two periods there is no future in period 1. It can be shown that in a model with  $n$  periods,  $n \geq 2$ , in period  $t$  ( $t < n$ ) exists that  $U_{M_t} = \frac{\lambda i_t}{P_t(1+i_t)}$ . Thus, 28c represents the more general expression, whereas 28d represents the 'terminal' condition. The main results of the paper can be shown to hold for a general  $n$  periods model.

9. An alternative, equivalent presentation of (30) is in terms of real balances. It can be shown that in such a case

$$\frac{\Delta U}{U_{X_0}} = \Delta X_0 + \frac{U_{X_1}}{U_{X_0}} \Delta X_1 + \frac{U_{m_0}}{U_{X_0}} \Delta m_0 + \frac{U_{m_1}}{U_{X_1}} \Delta m_1.$$

Similarly, we can state the first order conditions in terms of real balances; where it can be shown that

$$U_{m_1} = \frac{\lambda}{1+r^*}; \quad U_{m_0} = \frac{\lambda i}{1+i}.$$

The analysis in the paper is conducted in terms of nominal balances mainly because the government instrument is the rate of growth of nominal money balance ( $\mu$ ).

Although prices are exogenously given to each agent, a change in the prices would affect welfare via its direct effect on velocity and indirect effect on leisure. From 18, 19 we get

$$U_{P_0} = u_{v_0} \frac{X_0}{M_0} \quad (31a)$$

$$U_{P_1} = \rho u_{v_1} \frac{X_1}{M_1} \quad (31b)$$

It is useful to apply the first order condition (28, 29) into 30 in order to derive the welfare change in terms of observable variables. We can simplify further by using 31 to determine that

$$\frac{\Delta U}{U_{X_0}} = \Delta X + \frac{i}{1+i} \Delta m_0 + \frac{\Delta m_1}{(1+r^*)} \quad (32)$$

The policy applied by the government has the effect of changing  $\mu$ , without affecting  $M_0$ . Assuming standard specification for the demand for money, such a policy would tend to raise price period 1 such that  $d \log M_1 \simeq d \log P_1$ , with negligible effects on  $m_1$ . It would affect  $m_0$  via its price effect, induced due to higher anticipated inflation which would, in turn, tend to reduce the demand for money in period 0. To simplify exposition, we presume that  $\Delta m_1 \simeq 0$ . Then

$$\frac{\Delta U}{U_{X_0}} = \Delta X + \frac{i}{1+i} \Delta m_0 \quad (33)$$

Applying the aggregate budget constraint 27 to 33,  $\Delta G = 0$  implies:

$$\left. \frac{\Delta U}{U_{X_0}} \right|_{\Delta G=0} = -(\Delta \theta) \bar{X} + \frac{i}{1+i} \Delta m_0 \quad (34)$$

The resultant welfare change is composed of two terms. The first corresponds to the marginal change in resources spent on tax collection, the second refers to the marginal change in the distorted activity, weighted by the distortion ( $\frac{i}{1+i}$ ). Given the government budget constraint 26 we can also determine that  $\Delta G = 0$  and  $m_0 = m_0$  implies:

$$(1 - \phi) \bar{X} (\Delta \theta) = \Delta \left( \frac{m_0}{1+i} \right) \quad (35)$$

Applying 35 to 34 we get

$$\left. \frac{\Delta U}{U_{X_0}} \right|_{\Delta G=0} = -\Delta \left( \frac{m_0}{1+i} \right) \frac{\phi}{1-\phi} + \frac{i}{1+i} \Delta m_0 \quad (36)$$

Alternatively :

$$\left. \frac{\Delta U}{U_{X_0}} \right|_{\Delta G=0} = \frac{\Delta m_0}{1+i} \left( i - \frac{\phi}{1-\phi} \right) + \frac{m_0}{(1+i)^2} \Delta (1+i) \frac{\phi}{1-\phi} \quad (36')$$

Optimality requires that the interest rate be set such that  $\left. \frac{\Delta U}{U_{X_0}} \right|_{\Delta G=0} = 0$ , thus 36' necessitates that :

$$n \left[ i - \frac{\phi}{1-\phi} \right] = \frac{\phi}{1-\phi} \quad (37)$$

where  $n$  corresponds to the elasticity of the demand for money with respect to the gross interest rate<sup>10</sup>  $(1+i)$ . Alternatively, optimal interest rate is given by

$$\tilde{i} = \frac{\phi}{1-\phi} \left[ 1 + \frac{1}{n} \right] \quad (38)$$

In the absence of costs of tax collection (i.e.,  $\phi = 0$ ) optimal interest rate is zero, and we can apply Friedman's optimal quantity of money rule. Positive costs of tax collection will justify the application of positive interest rates. For example, if the elasticity of money with respect to the net interest rate ( $n'$ ) is 0.25, values of  $\phi$  given by (.04, .07, .17, .25) correspond to optimal values of  $i$  equal to (.05, .1, .5, 1). Equation 38 can also be used to infer from known values of  $i, n$  a crude approximation of the implied  $\phi$ . As is evident from 38, less elastic demand raises optimal  $i$ , in accordance with the Ramsey's results.<sup>11</sup>

10.  $\eta$  is defined by  $\eta = -d \log m_0 / d \log (1+i)$ . Note that if one denote the elasticity with respect to the interest rate by  $\eta'$  ( $\eta' = -d \log m_0 / d \log i$ ), one get that  $\eta'(1 + \frac{1}{i}) = \eta$ .

11. If one allows  $X$  to be produced by labor ( $\bar{X} = L$ ), then the optimal interest rate can be shown to be

$$\tilde{i} = \left[ \frac{\phi}{1-\phi} + \frac{\eta_L \frac{1-\theta}{1-\theta}}{(1-\phi)[1-\eta_L \frac{\theta}{1-\theta}]} \right] \cdot \left[ \frac{1}{\eta} + 1 \right]$$

where  $\eta_L = d \log L / d \log (1-\theta)$ . Thus, a higher labor supply elasticity implies a higher optimal interest rate.

#### IV. Concluding Remarks

This paper has derived the functional dependency between costs of tax collection, optimal tariff and inflation tax. It was shown that positive collection costs can justify the application of both policies as revenue devices. This, in turn, implies that in case of higher revenue needs or less effective tax collection, inflation taxes and tariffs will be used more frequently. This also implies that liberalization and stabilization attempts should be approached in the broader context of government capacity to replace inflation and tariff with alternative source of funds, or government capacity to cut public sector activities.<sup>12</sup> Our results were conditional on the assumption that enforcement costs of tariffs and inflation tax are small relative to alternative taxes. This might not hold in a country that tended to "abuse" the above policies, through smuggling activities (in the case of a tariff) or currency substitution (in the case of inflation).

The general principle that characterizes optimal taxes in the presence of collection costs is that we equate across taxes the sum of the deadweight loss and the marginal collection cost. Alternatively, we equate across taxes the difference in marginal collection costs to the corresponding difference in marginal deadweight losses. Thus, the welfare ranking and the application of various taxes as revenue devices is highly dependent on the structure of collection and enforcement costs, and would involve higher tax rates on activities with lower collection costs.

#### APPENDIX

In this Appendix we generalize the analysis of Section III by considering optimal taxation in the presence of enforcement costs for a multi-product monetary economy. Thus, we solve simultaneously for optimal inflation, the tariff and the endowment tax. To simplify presentation we assume an endowment model where there are  $n$  goods ( $X_1, \dots, X_n$ ), whose foreign price is normalized to 1. Let  $\theta_i$  denote the tax rate on good  $i$ , and let  $\phi_i$  be the collection costs associated with  $\theta_i$ . Let  $\theta_0$  be the endowment tax, associated with collection cost  $\phi_0$ . Let  $\theta_\pi$  define the implicit inflation tax,

$$\theta_\pi = \frac{i}{1+i}.$$

Following Section III, we assume a two-periods model. The government budget constraint is:

$$(A1) \quad G_0 = \bar{X}\theta_0(1 - \phi_0) + \sum_{i=1}^n X_i \theta_i (1 - \phi_i) + m_0 \theta_\pi - \bar{m}_0 + \frac{m_1}{1+r^*}$$

12. For a related discussion, see Frenkel (1983) and Edwards (1984).

where  $\bar{X}$  is the net present value of endowment and  $X_i$  is the net present value of consumption of good  $i$ <sup>13</sup>. Equation (A1) is a generalization of (26) for a multi-good, multi-taxes analysis. Note that equilibrium in the money market requires  $m_0 = \bar{m}_0$ .

It is useful to treat  $m_0$  and  $m_1$  as  $X_{n+1}$  and  $X_{n+2}$  where  $\phi_{n+1} = \phi_{n+2} = 0$ , and  $\theta_{n+1}$  and  $\theta_{n+2}$  are obtained from the government budget constraint ( $\theta_{n+1} = \theta_\pi - 1, \theta_{n+2} = \frac{1}{1+r^*}$ ). Using this notation we can restate the government budget constraint by

$$(A1') \quad G_0 = \sum_{i=1}^{n+2} X_i \theta_i (1 - \phi_i) + \bar{X} \theta_0 (1 - \phi_0)$$

The problem of optimal taxes can be stated in terms of the indirect utility function, forming the Lagrangean :

$$(A2) \quad L = V[\theta_0, \dots, \theta_n, \theta_\pi; I] + v[\bar{X} \theta_0 (1 - \phi_0) + \sum_{i=1}^{n+2} X_i \theta_i (1 - \phi_i) - G_0]$$

where 'I' denotes real income ( $I = \bar{X}(1 - \theta_0) + \bar{m}_0$ )

The private Budget constraint is

$$(A3) \quad \bar{X}(1 - \theta_0) + \bar{m}_0 = \sum_{i=1}^n X_i (1 + \theta_i) + m_0 \theta_\pi + \frac{m_1}{1+r^*}$$

The problem facing the government is to choose  $(\theta_0, \dots, \theta_n, \theta_\pi)$  so as to maximize (A2).

Using the properties of the indirect utility we obtain the following first order conditions

$$(A4) \quad 0 = L_{\theta_\pi} = -\alpha m_0 + \alpha \frac{\partial \bar{m}_0}{\partial \theta_\pi} + v[m_0 + \sum_{i=1}^{n+2} \frac{\partial X_i}{\partial \theta_\pi} \theta_i (1 - \phi_i)]$$

$$(A5) \quad 0 = L_{\theta_0} = -\alpha \bar{X} - \alpha \frac{\partial \bar{m}_0}{\partial I} \bar{X} + v[\bar{X}(1 - \phi_0) - \sum_{i=1}^{n+2} \frac{\partial X_i}{\partial I} \theta_i (1 - \phi_i) \bar{X}]$$

$$(A6) \quad 0 = L_{\theta_k} = -\alpha X_k + \alpha \frac{\partial m_0}{\partial \theta_k} + v[X_k(1 - \phi_k) + \sum_{i=1}^{n+2} \frac{\partial X_i}{\partial \theta_k} \theta_i (1 - \phi_i)] ,$$

for  $1 \leq k \leq n$ .

These equations may be transformed using the Slutsky equation  $\frac{\partial X_i}{\partial \theta_k} = S_{ik} - X_k \frac{\partial X_i}{\partial I}$

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13. Notice that in writing (A1) we assume that  $\theta_i \geq 0$  ( $\theta_i < 0$ ) will imply that in (A1) we should replace  $\theta_i(1 - \phi)$  with  $\theta_i(1 + \phi_i)$ , reflecting the cost of distributing a subsidy.

where  $S_{ik}$  is the derivative of the compensated demand. Assuming that cross effects are zero ( $S_{ik} = 0$  for  $i \neq k$ ) we can re-write (A4)–(A6) as :

$$(A4') \quad \frac{\alpha}{v} (m_0 - S_{n+1,n+1} + \frac{\partial X_{n+1}}{\partial I} m_0) = m_0 + (\theta_\pi - 1) S_{n+1,n+1} - H \cdot m_0$$

$$(A5') \quad \frac{\alpha}{v} (1 + \frac{\partial X_{n+1}}{\partial I}) = 1 - \phi_0 - H$$

$$(A6') \quad \frac{\alpha}{v} (1 + \frac{\partial X_{n+1}}{\partial I}) = 1 - \phi_k + \frac{S_{kk}}{X_k} \theta_k (1 - \phi_k) - H; \text{ for } 1 \leq k \leq n$$

$$\text{where } H = \sum_{i=1}^{n+2} \theta_i (1 - \phi_i) \frac{\partial X_i}{\partial I}$$

Applying (A5') to (A6') we obtain that for  $1 \leq k \leq n$

$$(A7) \quad 1 - \phi_0 = (1 - \phi_k) (1 - \frac{\theta_k}{1 + \theta_k} \epsilon_k) \text{ where } \epsilon_k = - \frac{S_{kk}}{X_k} (1 + \theta_k) \text{ is the compensated elasticity of demand for good } k. \text{ Alternatively, optimal tax on good } k \text{ is}$$

$$(A7') \quad \tilde{\theta}_k = \frac{\phi_0 - \phi_k}{\epsilon_k (1 - \phi_k) - (\phi_0 - \phi_k)} \text{ for } 1 \leq k \leq n.$$

Notice that for  $\phi_k = 0$  we obtain the result reported in (14). Applying (A5') to (A4') we obtain that

$$(A8) \quad (\frac{\alpha}{v} - 1) \frac{\epsilon m_0}{\theta_\pi} = \phi_0 - \epsilon m_0 \text{ where } \epsilon m_0 = - \frac{d \log m_0}{d \log \theta_\pi}$$

Applying the fact that  $i \eta = \epsilon m_0$  and (A5') we get

$$(A8') \quad \tilde{i} = \frac{\phi_0}{1 - \phi_0} [1 + \frac{1}{n}] + \sum_{k=1}^{n+2} \theta_k (1 - \phi_k) \frac{\partial X_k}{\partial I} \frac{(n + \phi_0)}{n(1 - \phi_0 - H)(1 - \phi_0)}$$

Equation (A8') is a multi-commodities extension of (38). The case analyzed in Section III corresponds to the case where  $\theta_k \frac{\partial X_k}{\partial I} = 0$  for  $0 \leq k \leq n + 2$ . Notice that (A7') and (A8') imply that if  $\phi_0 \rightarrow 0$ , only endowment tax is used. In general, a higher enforcement costs of endowment taxes will increase alternative taxes ( $\theta_1, \dots, \theta_n, \theta_\pi$ ), whereas a lower  $\phi_k$  and lower  $\epsilon_k$  will raise optimal  $\theta_k$  ( $1 \leq k \leq n$ ).

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