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The Dornbusch Model, Trade Flow Lags, and Exchange Rate Overshooting

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This paper reconsiders the exchange rate overshooting proposition first advanced in Dornbusch's seminal work on exchange rate dymanics. The basic findings that monetary expansion causes exchange rate overshooting and that fiscal expansion does not are reconsidered in the context of Dornbusch's model modified to allow trade flows to respond with a lag to movements in the exchange rate. It turns out that, because of the trade flow lags, fiscal expansion always produces exchange rate overshooting (as does monetary expansion, as expected). This suggests that fiscal policy may be a more important source of exchange rate variability than is commonly believed.

This paper reconsiders the exchange rate overshooting proposition first advanced in Dornbusch's seminal work [2] on exchange rate dynamics. The basic findings that monetary expansion causes exchange rate overshooting and that fiscal expansion does not are reconsidered in the context of Dornbusch's model modified to allow trade flows to respond with a lag to movements in the exchange rate. The introduction of such a lag is a potentially important qualification in view of the substantial amount of empirical evidence [5] supporting this phenomenon. In fact, it turns out that, because of the trade flow lags, fiscal expansion always produces exchange rate overshooting. This suggests that fiscal policy may be a more important source of exchange rate variability than is commonly believed. With respect to monetary expansion, however, it remains true that overshooting occurs as in the Dornbusch model.[1]

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[1]See penati [7] for a recent summary of models dealing with fiscal policy and the exchange rate, none of which, however, is concerned with the issue of trade flow lags. Also see Sachs and Wyplosz [8] for a model imposing the requirement of a long–run balanced budget. Imperfect asset substitutability, not considered here, is a factor that can prevent exchange rate overshooting in response to monetary expansion. See Bhandari, Driskill, and Frenkel [1]. In addition, wealth effects on the demand for money, in conjunction with other effects, have been suggested as a cause of undershooting. See Driskill and McCafferty [3] and Engle and Flood [4].

I. The Dornbusch Model with Trade Flow Lags

Three basic building blocks of Dornbusch's model are given by the following relations:

$$r = r^{*} + \dot{e} \tag{1}$$
$$m - p = -\beta r + \phi \bar{y} \tag{2}$$

and

$$\dot{p} = \pi[(c + i + g + x) - \bar{y}] \tag{3}$$

where r = domestic interest rate; r^{*} = fixed world interest rate; e = log of the exchange rate on the foreign currency; m = log of domestic money supply; p = log of domestic goods price level; $\bar{y}$ = log of the natural rate of output; c = log of consumption spending on domestic output; i = log of investment spending on domestic output; g = log of government spending on domestic output; and x = log of exports. π is a positive speed of adjustment coefficient, and β and ϕ are positive structural parameters. Equation (1) is the perfect substitutability relation between domestic and foreign securities, where, under rational exchange rate expectations, the expected appreciation of the foreign currency is replaced with actual e. Equation (2) is Dornbusch's money market equilibrium condition, where the demand for money depends on the domestic interest rate and domestic income. Finally, equation (3) shows that the rate of inflation depends on the amount of excess demand in the goods market.[2] Notice that continuous full employment is assumed in this model, with domestic output always at its natural level, $\bar{y}$.

In order to close the model, it is necessary to specify the determinants of the components of aggregate demand. We follow Dornbusch in treating investment spending as a function of interest rates, as described by

$$i = \rho - \sigma r \tag{4}$$

Also government spending on domestic output, the fiscal policy variable in the model, is taken to be exogenous; but it will be permitted to change when fiscal policy is

[2]The form of the aggregate demand expression in (3) specified so as to produce a linear relationship between the log of aggregate demand and the interest rate, the relative price of foreign goods (ignoring lags), and income, as in Dornbusch [*op. cit.*, p.1164].

altered. Finally, consumption spending on domestic goods depends on the fixed level of domestic output ; but it also depends, along with exports, on the relative price of foreign goods, $e - p$. Furthermore, in order to capture the essence of trade flow lags, this relative price is taken to affect these two components of aggregate demand with a simple exponential distributed lag, as shown by

$$\dot{c} = \mu[\alpha + (\eta_M - 1)(e - p) + \gamma\bar{y} - c] \tag{5}$$

and

$$\dot{x} = \mu[x_0 + \eta_X(e - p) - x] \tag{6}$$

Here η_M and η_X denote the long-run demand elasticities for imports and exports, respectively ; x_0 is the initial steady-state level of exports ; γ is the income elasticity of demand for domestic goods ; and μ is a positive speed of adjustment coefficient.

Equation (5) in essence states that consumption spending on domestic output adjusts at a speed proportional to the gap between target spending on domestic goods $[\alpha + (\eta_M - 1)(e - p) + \gamma\bar{y}]$ and current spending on domestic goods, c. Notice furthermore that target spending depends on relative prices. An increase in $e - p$ raises the relative price of foreign goods and causes domestic consumers to buy fewer imports. Consequently, if their demand for imports is price elastic, they will reduce their nominal expenditures on imports and increase their expenditures on domestic output (and *vice versa* if $\eta_M < 1$). Thus, with respect to domestic goods consumption, the substitution effect exceeds or falls short of the income effect as η_M is above or below 1. Similarly, equation (6) states that foreigners will adjust their purchases of the country's exported goods at a speed proportional to the gap between target spending on exported goods $[x_0 + \eta_X(e - p)]$ and current spending on exported goods (x), where, of course, such target spending depends positively on relative prices. For simplicity, the speed of adjustment coefficients in (5) and (6) are taken to be identical.

Equations (1) - (6) constitute a complete dynamic model involving the six endogenous variables r, e, p, i, c, and x. This model may be simplified, however, by first writing (5) and (6) in the alternative form

$$c = \frac{\mu[\alpha + (\eta_M - 1)(e - p) + \gamma\bar{y}}{D + \mu} \tag{5'}$$

and

$$x = \frac{\mu[x_0 + \eta_x(e - p)]}{D + \mu} \tag{6'}$$

where D is the differential operator. Then substituting (4), (5′), and (6′) into (3) produces

$$\ddot{p} + \mu\dot{p} + \mu\pi\delta p = -\pi\{\mu(1-\gamma)\bar{y} - \mu[\alpha + \delta e + x_0] + \sigma\dot{r} - \mu(\rho - \sigma r) - \mu g\} \tag{3'}$$

where δ denotes $(\eta_x + \eta_m - 1)$, which is positive by virtue of the Marshall−Lerner condition. The new dynamic model now consists of equation (1), (2), and (3′) involving the three endogenous variables r, e, and p ; and the characteristic equation of this system turns out to be the folluwing :

$$\beta\lambda^3 + (\pi\sigma + \beta\mu)\lambda^2 + (\beta\mu\pi\delta + \pi\sigma\mu)\lambda - \pi\mu\delta = 0 \tag{7}$$

where λ represents the characteristic roots. Since the constant term is negative, the product of the roots is positive ; and since the coefficient of λ^2 is positive, the sum of the roots is negative. Therefore, one root must be positive, and two roots are negative ; and the system is unstable because of the positive root. However, as is well known, the latter occurs here because of the rational expectations assumption in equation (1). Since the other two roots are negative, there exists a stable saddle−point path for the system.

To obtain the stable saddle−point path, one must next find the exchange rate expectations scheme that is satisfied along the path. As shown in the Appendix, the following scheme turns out to be the compatible one under rational expectations :

$$\dot{e} = \theta_1(\bar{e} - e) + \theta_2\dot{p} \tag{8}$$

where $\bar{e}$ denotes the long−run equilibrium exchange rate, and the signs of θ_1 and θ_2 are yet to be determined. If (8) is combined with (1), one obtains

$$r = r^* + \theta_1(\bar{e} - e) + \theta_2\dot{p} \tag{1'}$$

This equation, along with (2) and (3′), comprise the saddle−point path in r, e, and p, with the following characteristic eqation :

$$\beta\theta_1\lambda^2 + (\beta\mu\theta_1 + \pi\sigma\theta_1 - \beta\mu\pi\delta\theta_2)\lambda + \mu\pi(\beta\delta\theta_1 + \sigma\theta_1 + \delta) = 0 \tag{9}$$

If the two negative roots of (7) were set equal, one at a time, to the two characteristic roots of (9), one would obtain from (9) two linear equations in θ_1 and θ_2, which could be solved for the unique rational expectations set, θ_1^* and θ_2^*. As shown in the Appendix, this set is based on the following relations that will be used below :

$$\theta_1 = \frac{-\pi\mu\theta_2[\theta_1(\delta\beta + \sigma) + \delta]}{\theta_1} \tag{10}$$

and

$$\theta_2 = \frac{-\pi\sigma\theta_2 - 1 - \beta\mu\theta_2 - \beta\pi\mu\theta_2^2(\delta\beta + \sigma)}{\beta\theta_1} \tag{11}$$

To see if monetary expansion causes exchange rate overshooting, as in the original Dornbusch model, consider a long-run equilibrium position of the system that is suddenly disturbed by monetary expansion. From (2) the immediate effect on the domestic interest rate is given by

$$\mathrm{dr} = (-\frac{1}{\beta})\mathrm{dm} \tag{12}$$

since prices adjust only gradually. However, the initial rate of inflation will not be zero but will be given from (3) by

$$\dot{\mathrm{p}} = \pi \mathrm{di} \tag{13}$$

That is, the fall in interest rates will stimulate investment spending, producing excess demand in the goods market and in turn inflation. Combining (12), (4), and (13), the initial rate of inflation can be written

$$\dot{\mathrm{p}} = \frac{\pi\sigma}{\beta}\mathrm{dm} \tag{13'}$$

Next, observe from (1) that the initial rate of change of the exchange rate, $\dot{e}$, coincides with dr. Hence,

$$\dot{e} = (-\frac{1}{\beta})dm \tag{14}$$

initially. Combining the expectations scheme (8) with (14) then produces the following relationship at the initial point of the system's path :

$$(-\frac{1}{\beta})dm = \theta_1(\bar{e} - e) + \theta_2\dot{p} \tag{15}$$

Finally, substituting p from (13′) into (15) yields

$$\bar{e} - e = \frac{-(\theta_2\pi\sigma + 1)}{\theta_1\beta}dm \tag{16}$$

at the initial point on the path. Consequently, if θ_1 is positive (as will be demonstrated below), overshooting occurs if and only if

$$\theta_2 > \frac{-1}{\pi\sigma} \tag{17}$$

To see if condition (17) is satisfied, first obtain the solution for θ_2 by eliminating θ_1 between (10) and (11). The result is

$$\theta_2^3[\beta\pi\mu(\delta\beta + \sigma)(\pi\sigma + \beta\mu) + \pi\mu\delta\beta^2] + \theta_2^2[(\pi\sigma + \beta\mu)^2 + \beta\pi\mu(\delta\beta + \sigma)] + 2\theta_2(\pi\sigma + \beta\mu) + 1 = 0 \tag{18}$$

The left-hand side of (18) is shown as a function of θ_2 in Figure 1 for the case of one real root.[3] Notice that this function is negative when θ_2 equals $\frac{-1}{\pi\sigma}$, and it is positive when θ_2 equals zero.[4] Consequently, the root must occur between zero and $-\frac{1}{\pi\sigma}$, and it follows that condition (17) is satisfied. Therefore, overshooting occurs, provided θ_1 is positive. This last condition presumably is satisfied since it means that speculators' exchange rate expectations are formed regressively in (8), and hence their actions would be stabilizing. If θ_1 were negative, we would expect the system to

[3]The possibility of one real root can be demonstrated numerically. For example, consider the parameter values $\beta = 1$; $\mu = 1$; $\pi = 1$; and $\sigma = 1$; . Then the solution of (18) yields one real root for θ_2, namely –.4250.

[4]When θ_2 equals $-\frac{1}{\pi\sigma}$, the left-hand side of (18) reduces to $-\frac{\beta^2\mu\delta(\beta\mu + 1)}{\pi^2\sigma^3} < 0$.

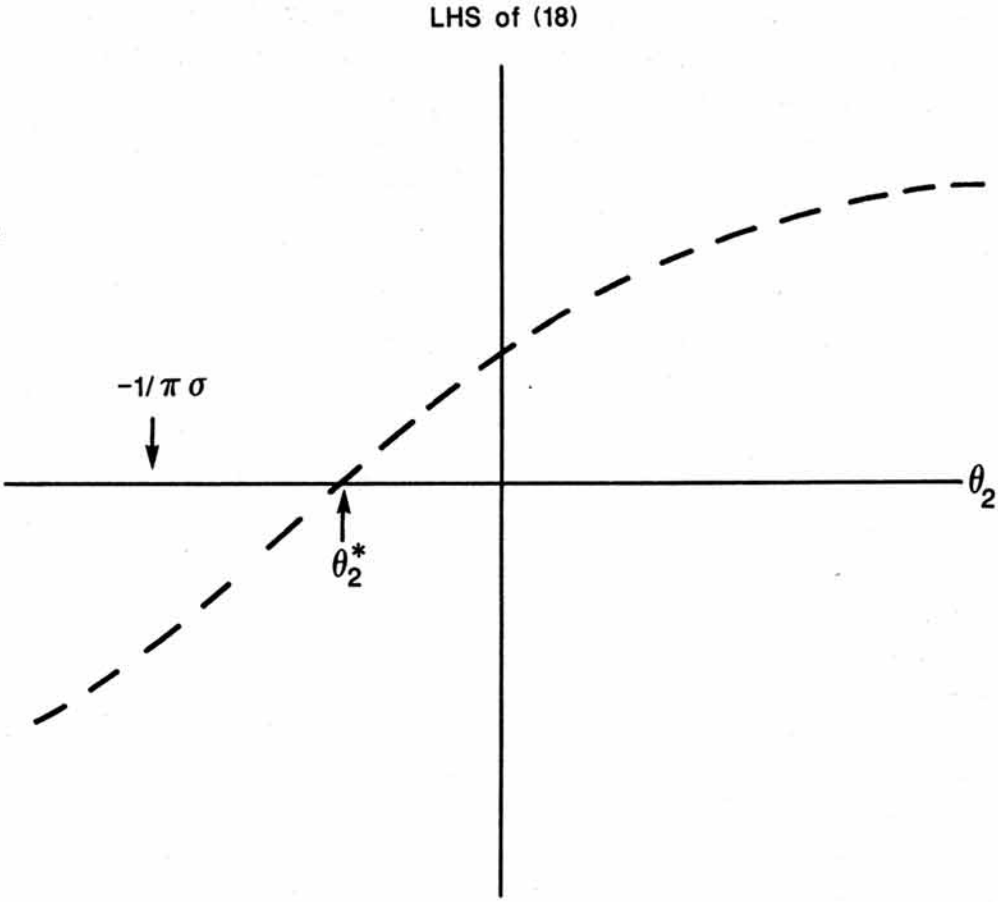

Figure 1 **Determination of θ_2^***

behave explosively since exchange rate expectations would be formed perversely. This conjecture, however, must now be examined more rigorously.

Consider the characteristic equation (9) for the saddle−point path. Since dynamic stability requires that the coefficients of λ^2 and λ and the constant term all have the same sign, one may first determine the sign of the constant term. The latter obviously is positive if θ_1 exceeds $\frac{-\delta}{\beta\delta + \sigma}$. At this point, it is necessary to turn to Figure 2, which graphs equations (10) and (11), the two relations involving θ_1 and θ_2. Equation (10) is shown by the two branches of the hyperbola labelled I, and similarly equation (11) is shown by the hyperbola labelld II. Notice in particular that the upper branches of the hyperbolas do not intersect because they have parallel asymptotes; but the lower branches do intersect. Furthermore, the lower branch of hyperbola I is asymptotic to the line $\theta_1 = \frac{-\delta}{\beta\delta + \sigma}$.

Hence, the intersection of the hyperbolas must occur at a value of θ_1 greater than $\frac{-\delta}{\beta\delta + \sigma}$. It follows that the constant term in characteristic equation (9) must be positive. Dynamic stability then requires the coefficient of λ^2 in equation (9) also to be positive, implying that θ_1 must be positive. It follows that exchange rate overshooting occurs in response to monetary expansion, as in the original Dornbusch model. Notice, however, that the inflation that initially occurs starts to reverse the direction of interest rates because the rising price level increases the demand for money. This leads to an expected appreciation of the home currency in response to incipient capital inflows. But this effect on exchange rate expectations is not large enough to prevent the overshooting of the exchange rate.

The case of three real roots for θ_2 in equation (18) can now be considered.[5] Since the left-hand side of (18) is positive for all positive θ_2, the roots must all be negative. Furthermore, in Figure 2 the case of three real roots would occur if the vertex of the lower branch of hyperbola II was moved sufficiently in a northwesterly direction. In that event two intersections between the hyperbolas would occur to the left of the θ_2 axis; and one intersection would remain to the right of the θ_2 axis.[6] However, the latter is the only stable point since it involves a positive θ_1. Furthermore, from the nature of hyperbola II, this intersection point involves the algebraically largest value for θ_2 among the three intersection points. Clearly, this value for θ_2 must lie between $-\frac{1}{\pi\sigma}$ and 0 as it has already been demonstrated in Figure 1 that at least one root for θ_2 will be in this upper range. Thus, condition (17) is once again satisfied, and overshooting again occurs.

Now consider fiscal expansion and its effect on the exchange rate. In the original Dornbusch model with no trade flow lags, fiscal expansion causes an immediate

[5]The possibility of three real roots can be demonstrated numerically. For example, consider the parameter values $\beta = 1$; $\mu = .00001$; $\pi = 1$; $\sigma = 1$; and $\delta = 1$. Then the solution of (18) yields the roots $-.9969$, -1.0032, and $-33,332.4443$.

[6]This follows from the fact that hyperbola II is asymptotic to the θ_1 axis and therefore would intersect hyperbola I in the southeast quadrant.

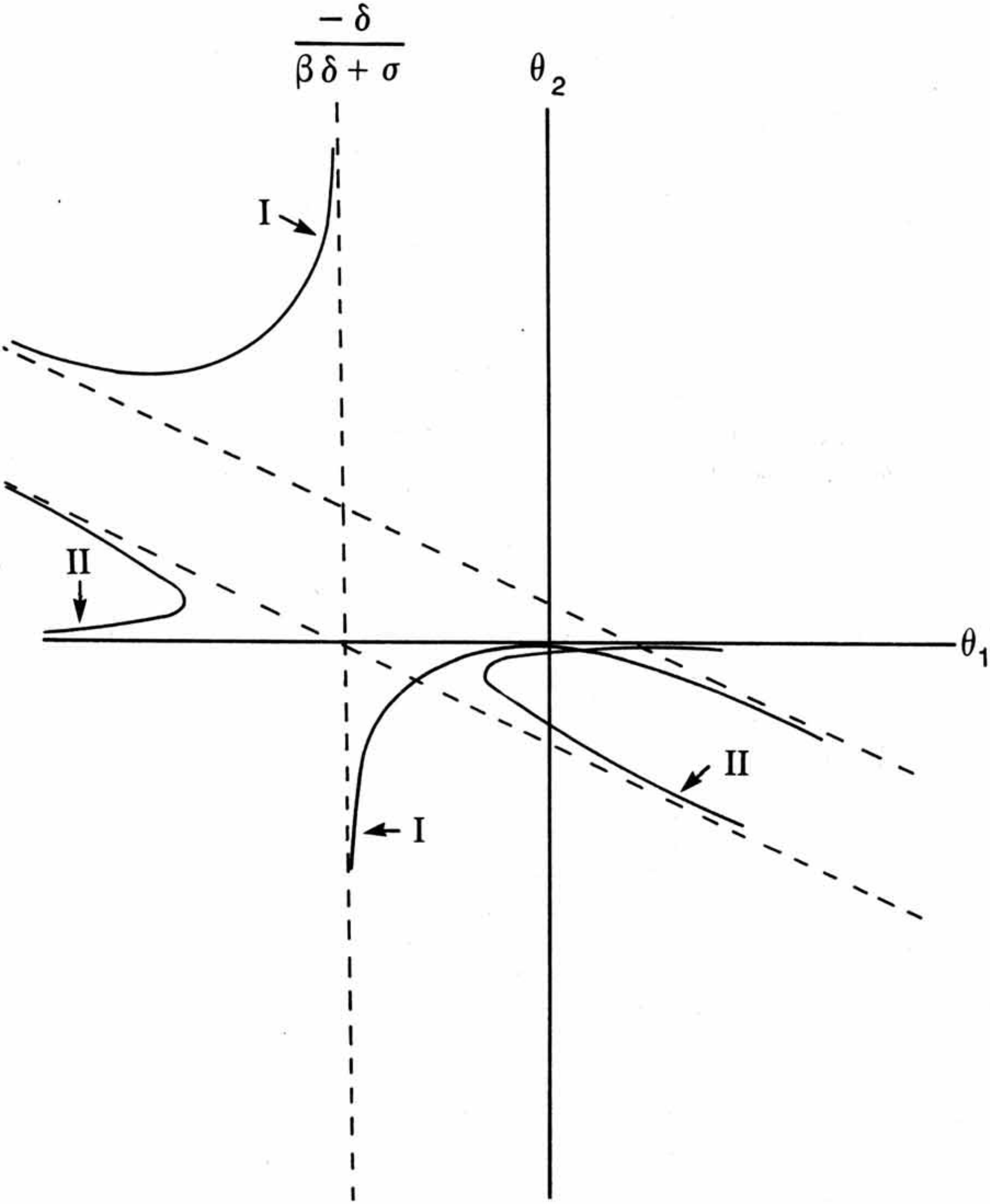

Figure 2 **Joint Determination of θ_1^* and θ^{**}**

depreciation of the foreign currency to its new long-run equilibrium level. Because of this exchange rate movement, aggregate demand remains unchanged despite the fiscal expansion, and the system immediately reaches full equilibrium.[7] However, in the case of trade flow lags, a movement of the exchange rate has no immediate effect on aggregate demand because of the spending lags described by (5) and (6). Therefore, fiscal expansion now produces excess demand in the goods market, and inflation initially occurs at the rate

$$p = \pi dg \tag{19}$$

The interest rate, however, initially equals its equilibrium level, r^*, because the fiscal expansion does not immdeiately alter the real money supply. Thus, from (1) the initial rate of change of the exchange rate is zero, so that from (19) and the expectations scheme (8) we obtain

$$\bar{e} - e = -\frac{\theta_2}{\theta_1}\pi dg \tag{20}$$

Since fiscal expansion causes in equilibrium exchange rate on the foreign currency to fall,[8] overshooting occurs if and only if

$$\frac{\theta_2}{\theta_1} < 0 \tag{21}$$

As it has already been demonstrated in Figure 2 that $\theta_1 > 0$ and $\theta_2 < 0$, this condition is satisfied, and fiscal expansion does produce exchange rate overshooting. In essence, the inflation that initially occurs because of the trade flow lags produces expectations that the foreign currency will depreciate.[9] This must be offset by an overshooting of the foreign currency, producing expectations that the latter will appreciate and exactly nullifying the expectations generated by the inflation.

[7]See Williamson [9, p.233] for an exposition of this case.

[8]Specifically, from (5) $dc = (\eta_M - 1)d\bar{e}$; and from (6) $dx = \eta_X d\bar{e}$. Thus, $dc + dx = \delta d\bar{e}$; and $d\bar{e} = (\frac{1}{\delta})(dc + dx) = (-\frac{1}{\delta})dg$.

[9]The connection in the model once again is that inflation eventually raises interest rates because of the effect of rising prices on the real money supply. Rising interest rates in turn require the foreign currency to depreciate to maintain asset market equilibrium.

Ⅱ. Conclusions

In Dornbusch's basic full-employment model, monetary expansion causes exchange rate overshooting, whereas fiscal expansion causes the exchange rate to move to its new long-run equilibrium level. When the model is modified to include trade flow lags in response to the eschange rate, it remains true that monetary expansion causes exchange rate overshooting. However, fiscal policy also causes the exchange rate to overshoot the new long-run equilibrium level. The reason is that with trade flow lags the fiscal expansion produces excess demand in the goods market, causing an immediate inflation of domestic prices. The latter generates expectations that the home currency will appreciate because of the effect of rising prices on interest rates. This is turn causes the exchange rate to appreciate beyond its new long-run equilibrium level, thereby neutralizing the expectations of future appreciation induces by the inflation and restoring asset market equilibrium. This result suggests that fiscal policy may be a more important source of exchange rate variability than is commonly believed.[10]

APPENDIX

To see that expectations scheme (8) is satisfied along the saddle point path, observe from differentiating (1′) on p.5 that

$$\acute{e} = -\frac{1}{\theta_1}\mathrm{r} + \frac{\theta_2}{\theta_1}\acute{p} \tag{A1}$$

Using (2) differentiated for r and (3′) in deviation form for $\acute{p}$ yields

$$\begin{aligned} e = &\left\{\frac{-\pi\sigma\theta_2 - 1 - \beta\mu\theta_2}{\beta\theta_1}\right\}p - \pi\mu\sigma\frac{\theta_2}{\theta_1}(r - r^*) \\ &+ \mu\pi\delta\frac{\theta_2}{\theta_1}(\bar{p} - p) - \mu\pi\delta\frac{\theta_2}{\theta_1}(\bar{e} - e) \end{aligned} \tag{A2}$$

where $\bar{p}$ denotes the long-run equilibrium value for p. Finally, using

[10]For a modification of the model in Dornbusch's appendix to include trade flow lags, see Levin [6]. In this model output is permitted to adjust instantaneously to aggregate demand, and the price of domestic goods moves according to a simple Phillips curve relation. it turns out that, because of the trade folw lags, monetary policy may or may not cause exchange rate overshooting ; whereas fiscal policy always does, as in this paper.

$$\bar{p} - p = -\beta(r - r^*) \tag{A3}$$

from (2) to eliminate $\bar{p} - p$ and (1′) to eliminate $r - r^*$, one obtains

$$e = \frac{-\pi\mu\theta_2}{\theta_1}[\theta_1(\delta\beta + \sigma) + \delta](\bar{e} - e)$$

$$+ \frac{-\pi\sigma\theta_2 - 1 - \beta\mu\theta_2 - \beta\pi\mu\theta_2^2(\delta\beta + \sigma)}{\beta\theta_1} p \tag{A4}$$

Comparing (A4) with (8) shows that θ_1 is given by the coefficient of $(\bar{e} - e)$ in (A4) and θ_2 by the coefficient of p in (A4).

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