

## Open-Economy Response to a Terms of Trade Shock in a Growth Context

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The issue of how open economies do and should respond to adverse movements in their terms of trade is one that has received considerable attention in the past decade. Recently, however, a new dimension has been added to the discussion by authors such as Maurice Obstfeld (1980, 1982), Jeffrey Sachs (1983), and Lars E O Svensson and Assaf Razin (1983), who have argued that the issue should be studied in the framework of an intertemporal optimizing model. Using an Uzawa-type (1968) utility function, for example, Obstfeld (1982) obtains the unusual result that an economy that is specialized in the production of its export good, and that is initially in stationary equilibrium, will respond to a sudden, unanticipated, and permanent terms-of-trade deterioration by initially reducing its consumption level and generating a current-account surplus.

The logic behind this result is, as Obstfeld points out, 'easy to grasp'. The stationary equilibrium is characterized by the attainment of a utility level such that the associated rate of time preference equals the exogenous rate of interest on bonds. At this equilibrium, the economy's savings are zero. A terms-of-trade shock impairs the ability of the economy to maintain this utility level, and it must consequently accumulate additional assets through saving in order to raise its income to the level that permits the restoration of the original utility level.

While Obstfeld (and Sachs) have adopted an infinite-horizon framework, other authors have examined the issue in the context of two-period models (or, in the case of Marion and Svensson (1984a), a three-period model, one of which is a past period). They point out that other outcomes are possible. A current-account deficit could result initially if the intertemporal utility function were not additively separable (Marion-Svensson, Svensson-Razin),

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The author would, without implicating, like to acknowledge his deep indebtedness to Professor Ronald Findlay for his most helpful suggestions in the course of preparing this paper. Comments of an anonymous referee are also deeply appreciated.

if wages were sticky (Marion-Svensson), if a nontraded good were incorporated into the model (Marion(1984)), if the technology were of the putty-clay variety (van Wijnbergen (1984b)), or if the oil-importing nation or bloc were large in the world oil market (Marion-Svensson (1984b), as well as Sachs). Other, related works include those of Bruno and Sachs (1979, 1985), and, in a monetary context, Buitert (1978).

This paper seeks to examine another possible rationale for the frequently-observed phenomenon of countries initially running current-account deficits in response to unanticipated and permanent deteriorations in their terms of trade. Intuitively, one might hypothesize that an economy that is experiencing continued growth over time might wish to run a current-account deficit immediately subsequent to being subjected to a terms-of-trade shock, for the purpose of achieving a 'smoother' time-profile of consumption than would otherwise prevail – even if the utility function were additively separable.<sup>1</sup> This is particularly likely to be the case if the future growth prospects of the economy are at least partially 'dissociated' from its current savings performance – on account, for example, of autonomous growth in factor productivity.

Accordingly, we construct here a simple model that will enable us to investigate this possibility. Many ingredients of the basic model, as presented in Section I, are familiar from the optimal growth literature, and it becomes correspondingly easier to relate our analysis to that standard literature. In Section II, we depart somewhat from the basic model, for the purpose of obtaining a consumption profile that appears to be even more consistent with the observed behaviour of various real-world economies. Concluding observations are provided in Section III.

## I

The setting of our analysis is that of a small open economy that engages in both current and capital account transactions with the rest of the world. Let  $y$  represent the level of production of the single home good, and we shall assume that this requires inputs of

<sup>1</sup> Our use of the expression 'continued growth over time' indicates that we propose to study the issue within an infinite-horizon context, thereby obviating the doubts that inevitably arise in regards to the analytical 'robustness' of conclusions arrived at in finite-horizon models. Obstfeld's infinite-horizon model is characterized by the property that the initial and final equilibria of the economy are stationary ones. Both he and Svensson and Razin have noted, though, that under certain restrictions on the Uzawa time preference function, the economy would not converge to a stationary equilibrium. However, they do not characterize the nonstationary equilibrium, or analyse the effect of a terms-of-trade shock on it. On the other hand, Sachs does permit the economies in his model to grow continually: however, his model is soluble only by means of simulation techniques, and, moreover, as pointed out above, his results are contingent upon the assumption that the economies concerned are large in the world oil market.

capital  $K$ , labour  $L$ , and an imported intermediate input  $M$ , as well as depending on a productivity growth factor. Unfortunately, econometric studies have yet to arrive at a consensus as regards the most appropriate form for the aggregate production function: if the imported input were energy, for example, then Vinals (1982) cites some studies finding a complementarity relationship between energy and capital, and others finding a substitutability relationship. Since our interest in this paper centers essentially on the qualitative characterization of the short-run adjustment process (and since we wish primarily to investigate the *possibility* that the current account might initially deteriorate after a terms-of-trade shock), it appears reasonable to postulate a fixed-coefficients production function:

$$y = \min. [aK, fM, cLe^{\theta t}], \quad (1)$$

where  $\theta$  is the rate of Harrod-neutral technical progress,  $t$  is an index of time, and  $a, f$ , and  $c$  are positive parameters. For simplicity, we assume that  $L$  is constant. Likewise the price of the home good and the price of the capital good (which is imported but which is assumed not to generate the terms-of-trade shock) are constant, and are normalised to unity: the only price that is permitted to change is the (relative) price of the intermediate input  $P_M$ .

The home good is assumed to be the only consumption good in the model. Next, we assume the existence of an international bond market on which a fixed rate of interest of  $r$  prevails. We further suppose that our economy enjoys perfect international mobility of bonds, and that bonds and physical capital (which is assumed to be non-depreciating for simplicity) are viewed as perfect substitutes. Assuming competitive behaviour on the part of producers, and bearing in mind our normalization conventions, it follows that the wage rate for 'effective' labour  $w$  must equal:

$$w = c \left[ 1 - \frac{r}{a} - \frac{P_M}{f} \right] \quad (2)$$

If effective labour is always fully employed, it then follows that  $y$ , and hence  $K$  and  $M$  as well, will always grow at the rate  $\theta$ . This rate is independent of domestic savings behaviour, since domestic entrepreneurs can, if necessary, finance capital accumulation by selling bonds internationally.

The wealth  $W$  of domestic residents is defined as:

$$W = K + b = \frac{y}{a} + b \quad (3)$$

where  $b$  comprises the net value of the stock of foreign bonds held by residents, and may be positive (if the country is a net international creditor) or negative (if it is a net international debtor). The rate of saving, or  $\dot{W}$  (with a dot denoting a time derivative), is given by:

$$\dot{W} = \left[ 1 - \frac{r}{a} - \frac{P_M}{f} \right] y + Wr - C. \quad (4)$$

where  $C$  is the level of consumption of the home good. The first term on the right-hand side constitutes income net of payments for capital services and for intermediate imports: earnings from capital and ownership of foreign bonds (which may be positive or negative) comprise the second term.

At the same time, however, equation (3) may be differentiated with respect to time to yield:

$$\dot{W} = \frac{\dot{y}}{a} + \dot{b} = \frac{\partial y}{a} + \dot{b} \quad (5)$$

Eliminating  $W$  and  $\dot{W}$  between equations (3), (4) and (5), and re-arranging terms, we obtain

$$\dot{b} = \left( 1 - \frac{P_M}{f} \right) y - \frac{\partial y}{a} + rb - C. \quad (6)$$

Let us consider next the specification of the objective function  $U$  of the economy. For the purpose of obtaining closed-form solutions, it is necessary to specify the function explicitly, and the most convenient form is the well-known iso-elastic one:

$$u = \int_0^\infty \frac{c^{1-\nu}}{1-\nu} dt, \quad (7)$$

where convergence of the integral requires that we stipulate that  $\nu > 1$ . Not only is this functional form algebraically tractable, it also has what is for our purposes the important property that, as Chakravarty (1969, p. 27) points out, ' $\nu$  can be regarded as a very convenient index of . . . . egalitarian bias . . . , since the higher  $\nu$  is the sharper is the relative rate of fall in the marginal utility of consumption when consumption is going to rise in the future.'

Since  $y$  will always grow at rate  $\theta$ , we may re-write equation (6) as

$$\dot{b} = \left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0 e^{\theta t} + rb - C, \quad (6')$$

where  $y_0$  denotes the value of  $y$  at period 0. Our optimal control problem may therefore be stated as: maximize  $U$  as specified in equation (7) with respect to the instrument variable  $C(t)$ , and subject to the equation of motion (6') in the state variable  $b$ , and to its given initial value  $b_0$ . Since the solution procedure is fairly straightforward, we relegate the details to an Appendix: it is only noteworthy here that the transversality conditions require that  $\theta$

be less than  $r$ .<sup>2</sup>

Our solutions are:

$$C = r \left( 1 - \frac{1}{\nu} \right) \left[ b_0 + \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} \right] e^{\frac{r}{\nu} t} \quad (8)$$

$$b = \left[ b_0 + \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} \right] e^{\frac{r}{\nu} t} - \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} e^{\theta t} \quad (9)$$

with the existence of an optimal solution requiring that the squarebracketed expression in the above equations be positive.

It is readily shown that the economy's current account surplus is equal to the rate of accumulation of foreign bonds  $\dot{b}$ , which equals the difference between saving and domestic capital formation. Suppose now that at time 0 there occurs an unanticipated, discrete, and permanent increase in the price of the imported intermediate input  $P_M$ . From equation (8), this is seen to lead to a discrete fall in the level of consumption at time 0, followed by resumption of its growth at rate  $r/\nu$ . Differentiating equation (9) with respect to time, we have

$$\begin{aligned} \dot{b} &= \frac{r}{\nu} \left[ b_0 + \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} \right] e^{\frac{r}{\nu} t} - \theta \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} e^{\theta t} \\ &= \frac{r}{\nu} \left[ b_0 + \frac{\left( 1 - \frac{\theta}{a} \right) y_0}{r - \theta} \right] e^{\frac{r}{\nu} t} - \theta \frac{\left( 1 - \frac{\theta}{a} \right) y_0}{r - \theta} e^{\theta t} - \frac{P_M}{r - \theta} y_0 \left( \frac{r}{\nu} e^{\frac{r}{\nu} t} - \theta e^{\theta t} \right) \end{aligned} \quad (10)$$

Since by assumption  $r/\nu > \theta$ , an increase in  $P_M$  will have a negative effect on the value of  $\dot{b}$  at time 0. Indeed,  $\dot{b}$  at that time will become negative if (letting  $P_M$  now denote its post-shock value which, clearly, cannot be so large as to drive  $w$  to zero)

$$\frac{r}{\nu} \left[ b_0 + \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta} \right] < \theta \frac{\left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y_0}{r - \theta}$$

<sup>2</sup> In fact, we shall impose the stronger condition that  $\theta$  be less than  $r/\nu$ , which ensures that  $b$  will eventually become (increasingly) positive. If  $r/\nu < \theta < r$ , not only will the economy remain a net international debtor, but in the limit its indebtedness will exceed the value of its capital stock. To obviate this, one could impose the constraint that the economy's wealth be non-negative: when this constraint becomes binding (at a zero value of wealth), consumption would thenceforth equal  $(1 - r/a - p_M/f)y$ , and savings would be zero. Since the focus of our inquiry is on the possibility of a 'cyclical' adjustment of the current account to a terms-of-trade shock (which involves its initial deterioration and subsequent recovery), the case  $\theta > r/\nu$  appears to be of peripheral interest, and will not be discussed further.



A necessary condition for this is that  $b_0$  be negative, or that the country be a net international debtor at time 0. Moreover, differentiating equation (10) with respect to time

$$\dot{b} = \left(\frac{r}{\nu}\right)^2 \left[ b_0 + \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{r - \theta} \right] e^{\frac{r}{\nu}t} - \theta^2 \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{r - \theta} e^{\theta t} \quad (11)$$

This too could possibly be negative at time 0 (although a necessary but not sufficient condition for this is that  $\dot{b} < 0$  at that time).

Although fairly simple, therefore, the foregoing model is rich in economic implications. It is possible for a permanent, unanticipated terms-of-trade shock to generate an initial current account deficit: a necessary condition for this to occur is that  $\theta$ , the 'non-savings' component of growth, be strictly positive, even though asymptotically  $C$  and  $b$  will grow at rate  $r/\nu$ . Moreover, the higher the value of  $\nu$ , the greater the likelihood of the emergence of an initial current account deficit, which is again in accordance with one's intuition, since, as Chakravarty points out, a higher  $\nu$  implies a preference for a more gradual consumption time-profile (and a smaller initial drop in  $C$  for any given increase in  $P_M$ ).

It is also noteworthy that not only might the current account turn into deficit initially, the deficit might thereafter widen even further for some time (as evinced by a negative value of  $\dot{b}$  at time 0), before the current account finally recovers into a (growing) surplus (as it must if  $r/\nu > \theta$ ). This too is in accord with observed experience: according to data presented in various of the International Monetary Fund's *Annual Reports*, many countries have experienced substantial and frequently widening current account deficits in the first two years after the oil shocks of 1973-74 and 1979-80, and it is only in the third succeeding year that improvements have been effected. This suggests that intertemporal models of open-economy response to terms-of-trade shocks which allow for only two periods, the present and a single future period (as in some of the papers cited earlier), might by virtue of their very structure be incapable of reflecting the complete pattern of observed dynamic behaviour.

$$\frac{\left(\frac{\theta}{a} - \frac{r}{\nu} - 1\right)}{\theta - \frac{r}{\nu}} > 0 \quad \text{II} \quad \frac{\left(\frac{\theta}{a} - \frac{r}{\nu} - 1\right)}{\theta - \frac{r}{\nu}} + \theta^2 > 0$$

Further economic implications of the analysis are discussed subsequently. At this juncture, it may be instructive to consider one consequence of the above model which may not be entirely consistent with actual experience. This is that consumption at time zero registers a discrete fall in response to the terms-of-trade shock. This would appear to conflict with the following general observation of W.M. Corden (1977, p. 101): 'governments

in countries hard-hit by the oil-price rise ran budget deficits, and so dissaved. In general this reflected a deliberate attempt to sustain, or at least avoid drastic falls in, national consumption levels'.

Such behaviour may be rationalised on the grounds of 'ratchet', and 'habit persistence', effects, which figure prominently in analyses of consumer behaviour. Infinite-horizon models have almost universally ignored these phenomena, owing to the fact that the assumption of additive separability of the objective function precludes their incorporation: symptomatic of this is Obstfeld's (1982, p. 263) statement that 'when no (further) price changes are expected, the family smooths the path of its future consumption stream; previous expenditure levels are irrelevant'. The omission is not as severe as it might appear, since in our model, as pointed out earlier, the higher the value of  $\nu$  the lower the initial drop in consumption for a given increment in  $P_M$ . However, conceptually this too is based upon 'forward-looking', and not 'backward-looking', considerations: moreover, some countries might wish to avoid any fall in consumption altogether, and instead have future consumption grow at a slower rate, which is certainly not inconsistent with generally accepted notions of intertemporal equity.

Allowing for intertemporal interdependence of consumption in full generality is, however, a daunting analytical task, as the elegant paper of Ryder and Heal (1973) demonstrates. Since our purpose here is essentially to illustrate the differences that may arise once intertemporal interdependence of consumption preferences is allowed for, we shall adopt a modified version of the Chakravarty-Manne (1968) function, in which utility depends upon the change in consumption:

$$\bar{U} = \int_0^{\infty} \frac{(A + \dot{C})^{1-\nu}}{1-\nu} dt, \quad A > 0, \nu > 1 \quad (12)$$

our functional differs from Chakravarty and Manne's, however, in the presence of the parameter  $A$ , which they effectively set equal to zero: this implies that in their model the instantaneous marginal utility of  $\dot{C}$  approaches infinity as  $\dot{C}$  approaches zero from above. This appears unduly restrictive for our purposes, and by specifying  $A$  to be a rather large positive number, we permit  $\dot{C}$  to take on negative values up to the limiting value of  $-A$ .

The remainder of the model is unchanged from that presented in Section I. From the viewpoint of setting up the optimal control problem, however, since it is the derivative of consumption that appears in the objective function, it is helpful to follow Turnovsky (1973), and introduce an additional equation:

$$\dot{C} = g \quad (13)$$

The control problem now has two state variables,  $b$  and  $C$ , with the equations of motion

being (6) and (13), the 'control variable' being  $g$  and the initial values of  $b_0$  and  $C_0$  (the value of  $C$  at time 0) given.<sup>3</sup> Maximization of  $\hat{U}$  under these conditions proceeds entirely analogously to the optimization in respect of the model of Section I, and our solutions are:

$$C(t) = C_0 - At + r(V-1) \left\{ b_0 - \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} - \frac{C_0 - \frac{A}{r}}{r} \right\} (e^{\frac{r}{V}t} - 1) \quad (14)$$

$$b(t) = \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} e^{\theta t} - \frac{A}{r}t + \frac{C_0 - \frac{A}{r}}{r} + (V-1) \left\{ b_0 - \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} - \frac{C_0 - \frac{A}{r}}{r} \right\} \left[ \frac{e^{\frac{r}{V}t}}{1 - \frac{1}{V}} - 1 \right] \quad (15)$$

A discrete step increase in  $P_M$  at time 0 produces a downward jump in  $(1 - P_M/f - \theta/a)y_0$ , which (since  $\theta < r$ ) can be seen to cause the optimal values of  $\dot{C}$  and  $\dot{b}$  to decline at time 0.  $\dot{C}(0)$  will be negative if

$$\frac{r^2(V-1)}{V} \left\{ b_0 - \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} - \frac{C_0 - \frac{A}{r}}{r} \right\} < A,$$

and  $\dot{b}(0)$  will be negative if

$$r \left\{ b_0 - \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} - \frac{C_0 - \frac{A}{r}}{r} \right\} < \frac{A}{r} + \theta \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{r - \theta},$$

which is simply equivalent to

$$\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0 + rb_0 < C_0:$$

$\dot{b}(0)$  will be negative if

$$\frac{r^2}{V} \left\{ b_0 - \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{\theta - r} - \frac{C_0 - \frac{A}{r}}{r} \right\} < \theta^2 \frac{\left(1 - \frac{P_M}{f} - \frac{\theta}{a}\right)y_0}{r - \theta}$$

(the expression in braces in the above inequalities is required to be positive for the existence of an optimal solution).

It may be seen, therefore, that the behaviour of consumption is somewhat different from that obtaining in the preceding model. It does not jump downward at time 0: al-

<sup>3</sup> A jump in  $C$  at time 0 is ruled out, because it would amount to an infinite instantaneous rate of change of  $C$ , which can readily be shown to be non-optimal.



though it may with some justice be argued that this result follows from the inherent structure of the model, in which  $C_0$  is in effect required to be specified as an initial condition, the fact remains that the rate of change in consumption is an important determinant of many individuals' perceived sense of well-being, at least in short-run situations. The 'price' of not having to lower  $C_0$  discretely is that the rate of change of consumption immediately thereafter has to be reduced below its preexisting level, and may even have to be negative, whereas in the preceding model it remains at  $r/\nu$  after the initial discrete fall in  $C$ . Intuitively, one might suspect that if countries had to choose between a slower but positive rate of growth of  $C$  and a discrete reduction in  $C$  combined with a resumption of its original growth rate, most of them would choose the former: if, however, they were required to choose between an initially *negative* rate of growth of  $C$  and a discrete reduction in it, then the choice would have to be a highly subjective one. At any rate, it appears that cross-country differences in consumption response-patterns following upon terms-of-trade shocks is consistent with differences in their underlying objective functions.

On the other hand, the possible patterns of behaviour of the current account  $\dot{b}$  are very similar to those generated by the preceding model. The behaviour of  $\dot{b}(0)$  is in a sense 'parametrically given' since  $C_0$  does not jump<sup>4</sup>: however, it is interesting to note that even  $\dot{b}(0)$  can be negative, and that a necessary condition for this is, once again, that  $\theta$  be positive (and that  $\dot{b}(0)$  be negative).

### III

The results of Sections I and II lead to one positive and one prescriptive conclusion. At the positive level, they suggest that a country will borrow heavily abroad in response to a terms-of-trade shock only if it perceives its likely future growth potentiality, as measured by  $\theta$ , to be high. If it is correct in its perception, it will experience little difficulty in servicing and eventually discharging its debt. However, if its expectations in regards to  $\theta$  turn out to be over-optimistic, it will in due course run into severe debt-servicing difficulties, and will be constrained to seek debt-relief assistance, and institute adjustment measures domestically.

On the normative plane, our analysis leads to the conclusion that countries confronted by a sudden terms-of-trade shock can mitigate the adverse consequences that this would entail for consumption levels in the immediately succeeding periods by attempting to ac-

<sup>4</sup> Although it should be pointed out that, unlike the previous case,  $\dot{b}(0)$  here can be negative even if  $b_0 \geq 0$  — the country is not required to be a net international debtor before its current account turns into deficit.

celerate their rates of productivity growth. It may be argued that the latter should be aimed for in any event, independently of the incidence of terms-of-trade shocks. It appears to be true, however, that policy-making in many less developed countries is characterized by considerable 'X-inefficiency', and 'satisficing' behaviour. A shock to real income brought about by an adverse terms-of-trade movement may provide the impetus necessary to induce such countries to undertake reforms aimed at eliminating distortions in pricing policies, domestic interest rate structures, exchange rates, and in the fiscal and monetary fields generally, that inhibit attainment of their full productive potentialities.<sup>5</sup>

In summary, therefore, the analysis of this paper has hopefully illustrated the crucial importance of analysing economy-wide responses to terms-of-trade shocks within a continuing growth framework. Our model, abstract though it may be, generates insights that are clearly of relevance to real-world economies, especially those whose growth is partly attributable to improvements in factor productivity.<sup>6</sup> The concomitant identification in our model of alternative patterns of consumption response to terms-of-trade shocks may be of assistance in enabling one to ascertain the nature of the implicit underlying objective function of policy-makers in particular empirical contexts: we have shown that consumption initially drops sharply (but not so sharply as to inevitably generate an initial current-account surplus—indeed the 'consumption-smoothing' function of initial current-account deficits is the essential feature of our argument) and subsequently grows at a constant rate under an additively separable utility function, but not otherwise. Finally, we have demonstrated that it is not sufficient simply to allow for growth within a two-period model: the complete configuration of dynamic response only manifests itself within a multi-period framework. Indeed, one might even wish to argue that the intertemporal dimension that earlier authors have viewed as fundamental can only be conclusively incorporated into the analysis in an infinite-horizon setting: as is well-known, the results of any finite-horizon exercise must be viewed as tentative in view of the inevitable arbitrariness involved in the selection of horizon length.

## Appendix

The necessary conditions for a solution to the optimal control problem formulated in Section I above may be derived through an application of the Pontryagin maximum principle (Arrow and Kurz (1970), Chap-

<sup>5</sup> To the extent that such reforms increase the efficiency of use of capital and the intermediate input, instead only of labour, their effects could for analytical convenience be represented by increases in the coefficients  $a$  and  $f$ .

<sup>6</sup> This last-mentioned feature is absent from Sachs' simulation-based growth model, and he relies instead upon an endogenously-generated decline in the world real rate of interest to obtain an initial current account deficit.

## ter 2). Forming the Hamiltonian

$$H = \frac{C^{1-\nu}}{1-\nu} + \phi(t) \left[ \left( 1 - \frac{P_M}{f} - \frac{\theta}{a} \right) y + rb - C \right] \quad (A1)$$

where  $\phi(t)$  is the co-state variable, it is necessary that at the maximizing value of  $C(t)$  the following conditions hold:

$$\frac{\partial H}{\partial C} = 0 \quad (A2)$$

$$\dot{\phi}(t) = -\phi(t)r \quad (A3)$$

In addition, we retain equation (6') with  $C$  replaced by its optimal value, obtained by solving the first-order condition (A2):

$$C^{-\nu} = \phi \quad (A2a)$$

Finally, we have the transversality conditions:

$$\begin{aligned} \lim_{t \rightarrow \infty} \phi(t) &\geq 0 \\ \lim_{t \rightarrow \infty} \phi(t) b(t) &= 0 \end{aligned} \quad (A4)$$

It then becomes a straightforward matter to solve equation (A3) for  $\phi(t)$ , equation (A2a) for  $C$  and, using the transversality conditions and the initial condition on  $b$ , equation (6') for  $b$  (thereby also solving for the unknown constant in the solution for  $C$ ), to give us equation (8) and (9) in the text. Finally, it is not difficult to show, using the fact that the objective function is concave in  $C$ , that the foregoing conditions are also sufficient for a maximum.

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